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ON THE PROJECTIVE TENSOR PRODUCT OF VECTOR-VALUED MEASURES

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1. The aim of this article is to consider the following problem. Let measurable spaces $(S, \mathscr{S})$ and $(T, \mathscr{T})$, locally convex topological vector spaces X and Y, and (countably additive) vector-valued measures $\mu : \mathscr{S} \to X$ and $\nu : \mathscr{T} \to Y$ be given. Let us denote by symbols $\mathscr{S} \otimes \mathscr{T}$, $\mathscr{S} \otimes_\sigma \mathscr{T}$ ($\mathscr{S} \otimes_\delta \mathscr{T}$) the ring, the σ-ring (the δ-ring), respectively, generated by the sets of the form $E \times F$, $E \in \mathscr{S}$, $F \in \mathscr{T}$. Let $X \hat{\otimes} Y$ denote the projective tensor product of the spaces X and Y. We ask under which conditions imposed on the space X there exists a vector-valued measure $\lambda : \mathscr{S} \otimes_\sigma \mathscr{T} \to X \hat{\otimes} Y$ such that the relation

$$(1) \qquad \lambda(E \times F) = \mu(E) \otimes \nu(F),\ E \in \mathscr{S},\ F \in \mathscr{T},$$

holds.

We give the following definition.

A locally convex topological vector space X is called an admissible factor if, for any locally convex topological vector space Y and for every vector-valued measure $\mu : \mathscr{S} \to X$ and every vector-valued measure $\nu : \mathscr{T} \to Y$, there exists a vector-valued measure $\lambda : \mathscr{S} \otimes_\sigma \mathscr{T} \to X \hat{\otimes} Y$ such that the relation (1) *holds.* (We suppose that $\mathscr{S}$ and $\mathscr{T}$ are σ-algebras.)

Thus if a vector-valued measure takes its values in an admissible factor, we can construct from it and from any other the projective tensor product.

It is true that every nuclear locally convex topological vector space is an admissible factor. This proposition is proved in [4] in this form:

Let $\mathscr{S}$ and $\mathscr{T}$ be σ-rings (δ-rings). Let X and Y be locally convex topological vector spaces and let X be nuclear. Then there exists a unique vector-valued measure $\lambda : \mathscr{S} \otimes_\sigma \mathscr{T}$ ($\mathscr{S} \otimes_\delta \mathscr{T}$) $\to X \hat{\otimes} Y$ such that (1) *holds.*

An adaptation [10] of the example given in [13] provides us with the normed spaces X and Y, the bounded bilinear operation $z = x \circ y$, $x, y \in Y$, $z \in X$, such that the vector-valued measure $\mu : \mathscr{S} \to Y$ ($\mathscr{S}$ is the σ-algebra of all subsets of the set of nonnegative integers) can be defined, for which the function λ,

$\lambda(E \times F) = \mu(E) \circ \mu(F) \in X, E, F \in \mathcal{S}$, extended on the algebra $\mathcal{S} \otimes \mathcal{S}$ by an additivity, is not bounded on $\mathcal{S} \otimes \mathcal{S}$.

Since every vector-valued measure defined on a σ-algebra is bounded (see [5, IV. 10. 2] or [6, Theorem 2.6]), we can see from the mentioned example that the question if a locally convex topological vector space is an admissible factor or not, is reasonable.

It is known that it is possible to divide all in functional analysis occurring concrete locally convex topological vector spaces (with exception of a few cases) into two classes. We have on the one hand the normed spaces that belong to the classical part of functional analysis and on the other we have the nuclear locally convex topological vector spaces (see e. g. [14]). Both classes have the trivial intersection because only the finite-dimensional locally convex spaces are both normable and nuclear. It follows that we must search for admissible factors in the class of the non-nuclear locally convex topological vector spaces.

In this paper we give some admissible factors. All given admissible factors have „sequence" character.

2. Let X and Y be locally convex topological vector (abbreviated locally convex) spaces. Let the topology of the space X be determined by the system of the seminorms $\{|\ |_\alpha\}_{\alpha \in A}$ and the topology of the space Y by the system of the seminorms $\{|\ |_\beta\}_{\beta \in B}$. X' and Y' are the dual spaces for X and Y, respectively. For $x' \in X'$ we denote $\|x'\|_\alpha = \sup\{|\langle x, x'\rangle| : |x|_\alpha \leqq 1\}$ for every $\alpha \in A$. Similarly for the space Y.

The topology defined on the algebraic tensor product $X \otimes Y$ of the spaces X and Y by the system of the seminorms

$$(2) \qquad |\sum_{i=1}^{k} x_i \otimes y_i|^{\wedge}_{(\alpha,\beta) \in A \times B} = \inf \sum_{i=1}^{n} |u_i|_\alpha |v_i|_\beta, \quad (\alpha, \beta) \in A \times B,$$

where the infimum is taken over all expressions $\sum_{i=1}^{n} u_i \otimes v_i$, which belong to the same class as $\sum_{i=1}^{k} x_i \otimes y_i$, is called the projective tensor topology (denoted by $\hat{\otimes}$). The completion of the locally convex space $X \otimes Y$ under this topology is the projective tensor product $X \hat{\otimes} Y$ of the spaces X and Y. (These notions are introduced in [7], [12], [2], [14]).

3. To start with some propositions.

Let us remark that there exists only one vector-valued measure $\lambda : \mathcal{S} \otimes_\sigma \mathcal{T} \to X \hat{\otimes} Y$ such that the relation (1) holds (if it exists). Thus the problem is only as regards its existence.

Proposition 1. *Let X be an admissible factor and Y any locally convex space. Let $\mathcal{S}$ and $\mathcal{T}$ be σ-rings (δ-rings) and $\mu : \mathcal{S} \to X$ and $\nu : \mathcal{T} \to Y$ be vector-*

valued measures. Then there exists a unique vector-valued measure $\lambda : \mathscr{S} \otimes_\sigma \mathscr{T}(\mathscr{S} \otimes_\delta \mathscr{T}) \to X \hat{\otimes} Y$ *such that the relation* (1) *holds.*

Proof. Let $\mathscr{S}$ and $\mathscr{T}$ be σ-rings. For every $\alpha \in A$ and $\beta \in B$ there exist the sets $S_\alpha \in \mathscr{S}$ and $T_\beta \in \mathscr{T}$ such that $|\mu(E - S_\alpha)|_\alpha = 0$ for all $E \in \mathscr{S}$ and $|\nu(F - T_\beta)|_\beta = 0$ for all $F \in \mathscr{T}$ ([9], Theorem 3.1). Evidently, we can now proceed as in the case of σ-algebras.

If $\mathscr{S}$ and $\mathscr{T}$ are δ-rings, then to every set $G \in \mathscr{S} \otimes_\delta \mathscr{T}$ there exist the sets $E \in \mathscr{S}$ and $F \in \mathscr{T}$ such that $G \subset E \times F$. Further, the system of those sets $G \in \mathscr{S} \otimes_\delta \mathscr{T}$, for which $G \subset E \times F$, is the σ-algebra of the subsets in $E \times F$. Again we can proceed as in the case of σ-algebras. The proof is terminated.

It is known that for any topology $\tilde{\otimes}$ on the algebraic tensor product $X \otimes Y$ we have for $z \in X \otimes Y$ ($X \tilde{\otimes} Y$ is the completion of $X \otimes Y$ under $\tilde{\otimes}$)

$$|z|^{\sim}_{(\alpha,\beta)} \leqq |z|^{\wedge}_{(\alpha,\beta)}, \quad (\alpha, \beta) \in A \times B$$

(see [2], IV. § 2 (2)). Hence we have immediately

Proposition 2. *Let X be an admissible factor and Y any locally convex space. Let* $\mathscr{S}$ *and* $\mathscr{T}$ *be* σ*-algebras* (σ*-rings,* δ*-rings*) *and* $\mu : \mathscr{S} \to X$, $\nu : \mathscr{T} \to Y$ *vector-valued measures. Then on* $\mathscr{S} \otimes_\sigma \mathscr{T}(\mathscr{S} \otimes_\delta \mathscr{T})$ *there exists a unique vector-valued measure* λ *with values in* $X \tilde{\otimes} Y$ *such that the relation* (1) *holds.*

Proposition 3. *A subspace* X_1 *of an admissible factor is an admissible factor.*

The proof is evident.

Proposition 4. *Let locally convex spaces* X *and* X_1 *be topologically isomorphic. If* X *is an admissible factor, so is* X_1.

Proof. Let $T : X \to X_1$ be the topological isomorphism of the space X onto the space X_1. Then there exists the topological isomorphism U of the space $X \hat{\otimes} Y$ onto $X_1 \hat{\otimes} Y$ such that $U(z \otimes y) = (Tz) \otimes y$ for all $z \in X$, $y \in Y$.

$T^{-1} \circ \mu$ is the vector-valued measure [5, IV. 10.8] defined on $\mathscr{S}$ with values in X. ($T^{-1} \circ \mu(E) = T^{-1}(\mu(E))$, $E \in \mathscr{S}$). Form the vector-valued measure $\lambda_1 : \mathscr{S} \otimes_\sigma \mathscr{T} \to X \hat{\otimes} Y$ in order that

$$\lambda_1(E \times F) = T^{-1} \circ \mu(E) \otimes \nu(F).$$

Then we take $\lambda = U \circ \lambda_1$ (i. e. $\lambda(G) = U(\lambda_1(G))$, $G \in \mathscr{S} \otimes_\sigma \mathscr{T}$) and λ takes its values in $X \hat{\otimes} Y$ and $\lambda(E \times F) = \mu(E) \otimes \nu(F)$.

4.

Theorem 1. *A space* $l^1(I)$ *is an admissible factor.*

The space $l^1(I)$ is the Banach space of all unconditionally (in this case also absolutely) summable numerical functions $[\xi_i, I]$ defined on an index set I, where the norm is

$$\|[\xi_i, I]\| = \sum_{i \in I} |\xi_i| .$$

Proof. Since μ is the (countably additive) vector-valued measure defined on the σ-algebra $\mathscr{S}$ with values in $l^1(I)$, thus for every $E \in \mathscr{S}$ the element $\mu(E) \in l^1(I)$ has the form

$$\mu(E) = [\xi_i(E), I],$$

i. e. „components" of the function μ are the set scalar functions defined on the σ-algebra $\mathscr{S}$, additive, for $\mu(E_1 \cup E_2) = [\xi_i(E_1 \cup E_2), I] = \mu(E_1) + \mu(E_2) = [\xi_i(E_1), I] + [\xi_i(E_2), I] = [\xi_i(E_1) + \xi_i(E_2), I], E_1 \cap E_2 = \emptyset, E_1, E_2 \in \mathscr{S}$, continuous in an empty set (hence countably additive), for if we have $E_{n+1} \subset E_n$, $\bigcap_{n=1}^{\infty} E_n = \emptyset$, $E_n \in \mathscr{S}$, so $\|\mu(E_n)\| \to 0, n \to \infty$, means that

$$\sum_{i \in I} |\xi_i(E_n)| \to 0, n \to \infty,$$

i. e. all „components" $|\xi_i(E_n)| \to 0, n \to \infty$, uniformly in i.

For the sets of the form

$$G = \bigcup_{i=1}^{k} E_i \times F_i, \tag{3}$$

where the union in (3) is disjoint and $E_i \in \mathscr{S}$, $F_i \in \mathscr{T}$, with regard to the additivity and the condition (1) we put

$$\lambda(G) = \sum_{i=1}^{k} \mu(E_i) \otimes \nu(F_i). \tag{4}$$

It is easy to see that the function λ is in this way unambiguously defined on the algebra $\mathscr{S} \otimes \mathscr{T}$ of the sets of the form (3) and is additive (ef. [8] § 36 (8)).

We must prove that the function λ is countably additive and that it can be extended to a countably additive function defined on the σ-algebra $\mathscr{S} \otimes_\sigma \mathscr{T}$ with values in $l^1 \hat{\otimes} Y$.

We will show that for every $\beta \in B$ there exists the finite positive measure m^β defined on $\mathscr{S} \otimes_\sigma \mathscr{T}$ with the property that if $m^\beta(G) \to 0$, then $|\lambda(G)|_\beta^\wedge \to 0$, $G \in \mathscr{S} \otimes \mathscr{T}$.

In proving this Theorem we use the fact that ([2], IV. 3.5 or [14], 7.2.3) for every complete locally convex space Y the projective tensor product $l^1(I)^1 \hat{\otimes} Y$ can be identified with the complete locally convex space $l^1(I, Y)$ of all absolutely summable functions $[y_i, I]$ with values in Y, where the locally convex topology is given by the system of the seminorms

$$|[y_i, I]|_\beta = \sum_{i \in I} |y_i|_\beta, \ \beta \in B.$$

By the isomorphism $H\colon l^1(I) \hat{\otimes} Y \to l^1(I, Y)$ we have for $G \in \mathscr{S} \otimes \mathscr{T}$

$$H(\lambda(G)) = H(\sum_{r=1}^{n} \mu(E_r) \otimes \nu(F_r)) = H(\sum_{r=1}^{n} [\xi_i(E_r), I] \otimes \nu(F_r)) = [\sum_{r=1}^{n} \xi_i(E_r)\nu(F_r), I].$$

Since for every $i \in I$ the function ξ_i is the scalar measure defined on $\mathscr{S}$ and ν is the vector-valued measure, thus by [3] for every $i \in I$ there exists one and only one vector-valued measure $\xi_i \times \nu : \mathscr{S} \otimes_\sigma \mathscr{T} \to Y$ such that $\xi_i \times \nu(E \times F) = \xi_i(E)\nu(F)$, $E \in \mathscr{S}$, $F \in \mathscr{T}$. Whence it follows for $G \in \mathscr{S} \otimes \mathscr{T}$

$$H(\lambda(G)) = [\sum_{r=1}^{n} \xi_i(E_r)\nu(F_r), I] = [\xi_i \times \nu(G), I],$$

where (see e. g. [14], 7.2.3) $[\xi_i \times \nu(G), I]$ is the absolutely summable function with values in Y, defined on I, i. e. for every $\beta \in B$ we have for $G \in \mathscr{S} \otimes \mathscr{T}$

$$|[\xi_i \times \nu(G), I]|_\beta = \sum_I |\xi_i \times \nu(G)|_\beta < +\infty.$$

It is known that ([5], IV. 10.5, [9], 4.2, [6], 3.2) to every vector-valued measure defined on the σ-algebra, hence also for $\xi_i \times \nu$, $i \in I$, there exists the finite positive measure m_i^β, defined on $\mathscr{S} \otimes_\sigma T$, for every $\beta \in B$ such that $m_i^\beta(G) \leqq |\xi_i \times \nu(G)|_\beta$, $G \in \mathscr{S} \otimes_\sigma \mathscr{T}$, $i \in I$, and further, $|\xi_i \times \nu(G)|_\beta \to 0$, $i \in I$, for $m_i^\beta(G) \to 0$.

Let $\sigma \subset I$ be an arbitrary finite subset. Take for every $\beta \in B$ the finite sum for $G \in \mathscr{S} \otimes_\sigma \mathscr{T}$

$$\sum_{i \in \sigma} m_i^\beta(G) \leqq \sum_{i \in \sigma} m_i^\beta(S \times T) \leqq \sum_{i \in \sigma} |\xi_i \times \nu(S \times T)|_\beta =$$

$$= \sum_{i \in \sigma} |\xi_i(S)| |\nu(T)|_\beta = |\nu(T)|_\beta \sum_{i \in \sigma} |\xi_i(S)| \leqq |\nu(T)|_\beta \,.\, K < +\infty \ (K \text{ const.})$$

Define for every $\beta \in B$ the set function m^β on $\mathscr{S} \otimes_\sigma \mathscr{T}$ by the relation:

$$m^\beta(G) = \sum_{i \in I} m_i^\beta(G) = \sup \{\sum_{i \in \sigma} m_i^\beta(G) : \sigma \subset I\}.$$

The function m^β is the finite positive measure [1, I . 10] with this property:

If $m^\beta(G) \to 0$, then $\sup \{\sum_{i \in \sigma} m_i^\beta(G) : \sigma \subset I\} \to 0$, i. e. $m_i^\beta(G) \to 0$ uniformly with respect to i, i. e. $|\xi_i \times \nu(G)|_\beta \to 0$ also uniformly with respect to i, hence also $\sum_{i \in \sigma} |\xi_i \times \nu(G)|_\beta \to 0$ for an arbitrary $\sigma \subset I$, and thus also

$$\sum_{i \in I} |\xi_i \times \nu(G)|_\beta \to 0,$$

i. e. for every $\beta \in B$ there holds $|[\xi_i \times \nu(G), I]|_\beta \to 0$ for $m^\beta(G) \to 0$. Since, as we have remarked, we can identify $l^1(I) \hat{\otimes} Y$ with $l^1(I, Y)$, it follows that for every $\beta \in B$ there exists the finite positive measure m^{β} such that for $m^{\beta}(G) \to 0$, $G \in \mathscr{S} \otimes \mathscr{T}$, we have $|\lambda(G)|_{\beta}^{\wedge} \to 0$.

We have proved that λ is the set function, continuous in an empty set, on $\mathscr{S} \otimes \mathscr{T}$, hence countably additive on $\mathscr{S} \otimes \mathscr{T}$, and further it can be extended [9](4.2) to the function $\bar{\lambda}$ on the σ-algebra $\mathscr{S} \otimes_{\sigma} \mathscr{T}$ such that the relation (1) holds, and $\bar{\lambda}$ takes its v values in $l^1(I) \hat{\otimes} Y$, $\bar{\lambda}(G) = \lambda(G)$, $G \in \mathscr{S} \otimes \mathscr{T}$. The proof is completed.

From Theorem 1 we obtain

Theorem 2. *A Banach space with an absolute basis is an admissible factor.*

A Banach space X has an absolute basis $[e_i, i \in I]$, where I is an index set if every element $x \in X$ can be written in the form

$$x = \sum_{i \in I} \xi_i e_i,$$

$\|e_i\| = 1$, and the scalar function $[\xi_i, I]$ is absolutely summable, i.e.

$$\sum_{i \in I} |\xi_i| < + \infty$$

in the sense as in Theorem 1. In other words, the function $[\xi_i e_i, I]$ with values in X is absolutely summable (cf. [2], IV. §1. 6 or [14], 1.4.1).

Proof. Since $[e_i, I]$ is an absolute basis of the space X with $\|e_i\| = 1$, then the mapping T: $l^1(I) \to X$, defined by the formula $T[x_i, I] = \sum_{i \in I} x_i e_i$ is the topological isomorphism of the space $l^1(I)$ with the space X (cf. [2], IV. §4 (1)). The result now follows from Proposition 4 and Theorem 1.

Theorem 3. *A perfect space of sequences Λ is an admissible factor.*

Λ denotes the space of sequences $\vec{\xi}_n = [\xi_n, n \in N]$ (N is the set of positive integers) of complex numbers, where the locally convex topology is defined by the system of seminorms $A = \{\alpha\}$

$$\alpha(\vec{\xi}_n) = \sum_{n=1}^{\infty} |\alpha_n \xi_n|, \quad \vec{\alpha}_n \in \Lambda^*$$

where Λ^* is the dual space of sequences which consists of all sequences $\vec{\alpha}_n$ for which $\sum_{n=1}^{\infty} |\alpha_n \xi_n| < + \infty$ holds for an arbitrary $\vec{\xi}_n \in \Lambda$. We suppose that Λ is perfect, i.e. $\Lambda = \Lambda^{**}$, where Λ^{**} is the dual space for Λ^* (see [11] and [15]). Let us remark that for example the space l_p, $p \geqq 1$, is the perfect space, other examples can be found in [11].

On the space Y we use the system of seminorms $B = \{\beta\}$ (see e.g. [12], II. 4. 13)

$$|y|_{\beta} = \sup \{|< y, y' >|, y' \in Y', \|y'\|_{\beta} \leqq 1\}$$

Proof. Similarly as in Theorem 1 we have for every $E \in \mathscr{S}$ $\mu(E) = \vec{\xi}_n(E) \in \Lambda$, ξ_n, $n \in N$ being the scalar measures defined on $\mathscr{S}$.

As in Theorem 1 we define the function λ on the algebra $\mathscr{S} \otimes \mathscr{T}$ with values in $\Lambda \hat{\otimes} Y$.

We must show that λ is countably additive on $\mathscr{S} \otimes \mathscr{T}$ and it can be extended to the function $\bar{\lambda}$ defined on $\mathscr{S} \otimes_\sigma \mathscr{T}$, $\bar{\lambda}(G) = \lambda(G)$, $G = \mathscr{S} \otimes \mathscr{T}$.

In proving the Theorem we use the fact that by [16] for every complete locally convex space Y the projective tensor product $\Lambda \hat{\otimes} Y$ can be identified with the complete locally convex space $\Lambda(Y)$ of all sequences $\vec{y}_n$, $y_n \in Y$, with the property that all series $\sum \alpha_n y_n$, $\vec{\alpha}_n \in \Lambda^*$ are absolutely convergent. The locally convex topology on $\Lambda(Y)$ is defined by the system of the seminorms

$$|\vec{y}_n|_{\vec{\alpha}_n,\beta} = \sum_{n=1}^{\infty} |\alpha_n|\, |y_n|_\beta, \quad \vec{\alpha}_n \in \Lambda^*, \ \beta \in B.$$

To the element $\lambda(G)$, $G \in \mathscr{S} \otimes \mathscr{T}$, $G = \bigcup_{i=1}^{\infty} E_i \times F_i$,

$$\lambda(G) = \sum_{i=1}^{k} \mu(E_i) \otimes \nu(F_i) = \sum_{i=1}^{k} \overrightarrow{\xi_n(E_i)} \otimes \nu(F_i) \in \Lambda \hat{\otimes} Y,$$

there corresponds the element $\overrightarrow{\sum_{i=1}^{k} \xi_n(E_i)\, \nu(F_i)} \in \Lambda(Y)$. By [3] we can write

$$\overrightarrow{\xi_n \times \nu(G)} = \overrightarrow{\sum_{i=1}^{k} \xi_n(E_i)\, \nu(F_i)}.$$

We have

$$|\overrightarrow{\xi_n \times \nu(G)}|_{\vec{\alpha}_n,\beta} = \sum_{n=1}^{\infty} |\alpha_n|\, |\xi_n \times \nu(G)|_\beta < +\infty, \quad \vec{\alpha}_n \in \Lambda^*, \beta \in B.$$

For every $n \in N$ and every $\beta \in B$ there exists by [9] the finite positive measure m_n^β such that

$$m_n^\beta(G) \leqq |\xi_n \times \nu(G)|_\beta, \ G \in \mathscr{S} \otimes \mathscr{T},$$

and

$$|\xi_n \times \nu(G)|_\beta \to 0, \quad \text{if } m_n^\beta(G) \to 0.$$

Consider for every $\beta \in B$ the sequence of measures $\overrightarrow{m_n^\beta}$. For an arbitrary $\overrightarrow{\alpha_n} \in \Lambda^*$ we have

$$\sum_{n=1}^{\infty} |\alpha_n|\, m_n^\beta(G) \leqq \sum_{n=1}^{\infty} |\alpha_n|\, m_n^\beta(S \times T) \leqq \sum_{n=1}^{\infty} |\alpha_n|\, |\xi_n \times \nu(S \times T)|_\beta =$$

$$= \sum_{n=1}^{\infty} |\alpha_n|\, |\xi_n(S)|\, |\nu(T)|_\beta = |\nu(T)|_\beta \sum_{n=1}^{\infty} |\alpha_n|\, |\xi_n(S)| \leqq |\nu(T)|_\beta K < +\infty.$$

Define for an arbitrary $\overrightarrow{\alpha_n} \in \Lambda^*$ the function m^β for every $\beta \in B$ on $\mathscr{S} \otimes_\sigma \mathscr{T}$ by the formula

$$m^\beta(G) = \sum_{n=1}^{\infty} |\alpha_n| m_n^\beta(G).$$

The function m^β is the finite positive measure defined on $\mathscr{S} \otimes_\sigma \mathscr{T}$ with the property: If $m^\beta(G) \to 0$, then

$$\sup |\alpha_n| m_n^\beta(G) \to 0, \quad \text{i.e.} \quad \sup_n |\alpha_n| \, |\xi_n \times \nu(G)|_\beta \to 0,$$

hence also

$$\sum_{n=1}^{\infty} |\alpha_n| \mid \xi_n \times \nu(G)|_\beta = |\xi_n \times \nu(G)|_{\overrightarrow{\alpha_n},\beta} \to 0 \text{ for } m^\beta(G) \to 0, \; G \in \mathscr{S} \otimes \mathscr{T}.$$

It follows that the function $\overrightarrow{\xi_n \times \nu}$ is countably additive, hence the function λ is also coutably additive on $\mathscr{S} \otimes \mathscr{T}$ and it can be by [9] extended to the function $\bar{\lambda}$ on $\mathscr{S} \otimes_\sigma \mathscr{T}$ with values in $\Lambda \hat{\otimes} Y$ such that the relation (1) holds. The proof is completed.

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