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H. Komm [1] proved that the dimension of the set $\mathrm{P}(M)$ of all subsets of a set M is card M. From the proof it can be easily seen that the set $\mathrm{P}''(M)$ of all finite subsets of M and their complements has the same dimension. A question arises whether the dimension of the set $\mathrm{P}'(M)$ of all finite subsets of M is also equal to card M. This question is answered negatively in this note. Let us denote by $\mathrm{P}^{\mathfrak{n}}(M)$ the set of all the subsets of M which have the cardinality less than $\mathfrak{n}$.

Theorem 1. *Let* $\mathfrak{m}$, $\mathfrak{n}$ *be infinite cardinals,* $\mathfrak{m} \geqslant \mathfrak{n}$. *If* card $M \leqslant 2^{\mathfrak{m}}$, *th*… dim $\mathrm{P}^{\mathfrak{n}}(M) \leqslant$ card $\mathrm{P}^{\mathfrak{n}}(\mathfrak{m})$(1).

Proof. On the set ${}^{\mathfrak{m}}2$ of all functions from $\mathfrak{m}$ to 2 we define a base b f… a topology t as follows: if $m \in \mathrm{P}^{\mathfrak{n}}(\mathfrak{m})$ and if φ is a function from m to 2, the… $u_\varphi = \{f \in {}^{\mathfrak{m}}2;\ \varphi \subseteq f\}$ belongs to b. If $x \in \mathrm{P}^{\mathfrak{n}}({}^{\mathfrak{m}}2)$, then the complement $-x$ of x in ${}^{\mathfrak{m}}2$ is an open set in the topology t and denoting $\bar{x} = \{u_\varphi \in b;\ u_\varphi \subseteq -x\}$ we have $-x = \cup\, \bar{x}$. For $u_\varphi \in b$ we define $f_\varphi(x) = 0$ if $u_\varphi \in \bar{x}$ and $f_\varphi(x) = 1$ if $u_\varphi \notin \bar{x}$. Evidently f_φ is a homomorphism of $\mathrm{P}^{\mathfrak{n}}({}^{\mathfrak{m}}2)$ into 2. If $x' \nsubseteq x$, then $-x' \nsupseteq -x$, so there is $u_\varphi \in b$ such that $u_\varphi \subseteq -x$, $u_\varphi \nsubseteq -x'$, i. e. $u_\varphi \in \bar{x}$, $u_\varphi \notin \bar{x}'$. Therefore $f_\varphi(x') = 1 \nleq f_\varphi(x) = 0$ and the system $\{f_\varphi;\ u_\varphi \in b\}$ realises the order in $\mathrm{P}^{\mathfrak{n}}({}^{\mathfrak{m}}2)$ in the sense of [2]. Evidently the cardinality of the realising system is the same as the cardinality of $\mathrm{P}^{\mathfrak{n}}(\mathfrak{m})$, which completes the proof.

1 Cardinal number $\mathfrak{m}$ is supposed to be a determined set of power $\mathfrak{m}$.

Theorem 2. *If* $\aleph_0 \leqslant$ card $M \leqslant 2^{\aleph_0}$, *then* dim $\mathrm{P}'(M) = \aleph_0$.

Proof. It follows from the mentioned Komm's theorem and from Theorem 1 if we put $\mathfrak{n} = \mathfrak{m} = \aleph_0$.

REFERENCES

[1] Komm H., *On the Dimension of Partially Ordered Sets*, Amer. J. Math. *70* (1948), 507–520.

[2] Novák V., *On the Pseudodimension of Ordered Sets*, Czechoslovak Math. Journ. *13 (88)* (1963), 587–593.

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