

Martin Gavalec

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THE DIMENSION OF THE SET OF ALL FINITE SUBSETS OF THE CONTINUUM

MARTIN GAVALEC, Košice

H. Komm [1] proved that the dimension of the set $P(M)$ of all subsets of a set M is $\text{card } M$. From the proof it can be easily seen that the set $P^n(M)$ of all finite subsets of M and their complements has the same dimension. A question arises whether the dimension of the set $P'(M)$ of all finite subsets of M is also equal to $\text{card } M$. This question is answered negatively in this note. Let us denote by $P^n(M)$ the set of all the subsets of M which have the cardinality less than n .

Theorem 1. *Let m, n be infinite cardinals, $m \geq n$. If $\text{card } M \leq 2^m$, then $\dim P^n(M) \leq \text{card } P^n(m)^{(1)}$.*

Proof. On the set ${}^m 2$ of all functions from m to 2 we define a base b for a topology t as follows: if $m \in P^n(m)$ and if φ is a function from m to 2, then $u_\varphi = \{f \in {}^m 2; \varphi \subseteq f\}$ belongs to b . If $x \in P^n({}^m 2)$, then the complement $-x$ of x in ${}^m 2$ is an open set in the topology t and denoting $\bar{x} = \{u_\varphi \in b; u_\varphi \subseteq -x\}$ we have $-x = \cup \bar{x}$. For $u_\varphi \in b$ we define $f_\varphi(x) = 0$ if $u_\varphi \in \bar{x}$ and $f_\varphi(x) = 1$ if $u_\varphi \notin \bar{x}$. Evidently f_φ is a homomorphism of $P^n({}^m 2)$ into 2. If $x' \not\subseteq x$, then $-x' \not\supseteq -x$, so there is $u_\varphi \in b$ such that $u_\varphi \subseteq -x$, $u_\varphi \not\subseteq -x'$, i. e. $u_\varphi \in \bar{x}$, $u_\varphi \notin \bar{x}'$. Therefore $f_\varphi(x') = 1 \not\leq f_\varphi(x) = 0$ and the system $\{f_\varphi; u_\varphi \in b\}$ realises the order in $P^n({}^m 2)$ in the sense of [2]. Evidently the cardinality of the realising system is the same as the cardinality of $P^n(m)$, which completes the proof.

¹ Cardinal number m is supposed to be a determined set of power m .

Theorem 2. *If $\aleph_0 \leq \text{card } M \leq 2^{\aleph_0}$, then $\dim P'(M) = \aleph_0$.*

Proof. It follows from the mentioned Komm's theorem and from Theorem 1 if we put $n = m = \aleph_0$.

REFERENCES

- [1] Komm H., *On the Dimension of Partially Ordered Sets*, Amer. J. Math. 70 (1948), 507–520.
- [2] Novák V., *On the Pseudodimension of Ordered Sets*, Czechoslovak Math. Journ. 13 (88) (1963), 587–593.

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*Katedra matematiky
Přirodovedecké fakulty UPJŠ
Košice*