Bedřich Pondělíček

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$\mathcal{T}$-PRIME SUBSETS IN SEMIGROUPS

BEDŘICH PONDĚLÍČEK

Let S be a semigroup. Let $\mathcal{I}$ denote the set of all two-sided ideals, or simply ideals, of S. An ideal I of S is said to be *prime (completely prime)* if $A, B \in \mathcal{I}$ and $AB \subset I$ ($a, b \in S$ and $ab \in I$) imply $A \subset I$ or $B \subset I$ ($a \in I$ or $b \in I$).

In [1] G. Szász studies some properties of semigroups in which every ideal is prime (completely prime). In this paper we shall study analogous properties of semigroups in which every quasi-ideal is $\mathcal{Q}$-prime (completely $\mathcal{Q}$-prime). Further, we shall consider semigroups in which every right ideal is $\mathcal{R}$-prime (completely $\mathcal{R}$-prime).

Definition 1. *Let $\mathcal{T}$ be a non-empty class of non-empty subsets of S. A subset P of S is said to be $\mathcal{T}$-prime if $A, B \in \mathcal{T}$ and $AB \cap BA \subset P$ imply $A \subset P$ or $B \subset P$.*

Remark 1. In [2] the following is proved: A two-sided ideal I of S is a prime ideal of S if and only if $AB \cap BA \subset I$ implies that $A \subset I$ or $B \subset I$; A, B being two-sided ideals of S. This implies that a two-sided ideal I of S is prime if and only if it is $\mathcal{I}$-prime.

It is clear that there holds:

Lemma 1. *Let $\mathcal{T}_1$, $\mathcal{T}_2$ be two non-empty classes of non-empty subsets of S. If $\mathcal{T}_1 \subset \mathcal{T}_2$, then every $\mathcal{T}_2$-prime subset of S is $\mathcal{T}_1$-prime.*

Definition 2. *A non-empty subset $Q \subset S$ is called a quasi-ideal of S if $QS \cap SQ \subset Q$. Denote by $\mathcal{Q}$ the class of all quasi-ideals of S.*

Finally, let $\mathcal{R}(\mathcal{L})$ denote the class of all right (left) ideals of S.

Remark 2. Let Q be an arbitrary quasi-ideal of S. It is known (see Exercise 17b in § 2.7 of [3]) that there exist a right ideal R of S and a left ideal L of S such that $Q = R \cap L$.

Remark 3. The following example shows that a prime ideal of S need not necessarily be either $\mathcal{R}$-prime or $\mathcal{Q}$-prime. Let $S = \{0, a, b\}$ be a semigroup with the multiplication table

	0	a	b
0	0	0	0
a	0	a	a
b	0	b	b .

Here $\mathscr{Q} = \mathscr{R} = \{N, A, B, S\}$, where $N = \{0\}$, $A = \{0, a\}$ and $B = \{0, b\}$. We have $\mathscr{L} = \mathscr{I} = \{N, S\}$. Thus N is a prime ideal of S and it is also $\mathscr{L}$-prime. Since $AB = A$ and $BA = B$, we have $AB \cap BA = A \cap B = N$. But $A \not\subset N$ and $B \not\subset N$. Therefore N is neither $\mathscr{R}$-prime nor $\mathscr{Q}$-prime.

Definition 3. *A semigroup S is called right (left) uniform if any two right (left) ideals have a non-empty intersection.* (See [4].)

Definition 4. *A semigroup S is said to be $\mathscr{Q}$-uniform if any two quasi-ideals have a non-empty intersection.*

Definition 5. *Let $\mathscr{T}$ be a class of non-empty subsets of S with $S \in \mathscr{T}$ and let x be an element of S. By $\mathscr{T}(x)$ we denote the intersection of all sets of $\mathscr{T}$ containing x.*

Evidently, $\mathscr{I}(x)$ ($\mathscr{R}(x)$, $\mathscr{L}(x)$) is the principal (principal right, principal left) ideal generated by x. It is known that

$$\mathscr{I}(xy) = \mathscr{L}(x)\mathscr{R}(y) \tag{1}$$

for every pair of elements x, y of S.

Definition 6. *Let $x \in S$. Then $\mathscr{Q}(x)$ will be called the principal quasi-ideal generated by x.*

Remark 4. It follows from Remark 2 that

$$\mathscr{Q}(x) = \mathscr{R}(x) \cap \mathscr{L}(x) \tag{2}$$

for every element x of S. This implies

$$\mathscr{Q}(xy) \cap \mathscr{Q}(yx) \subset \mathscr{Q}(x) \cap \mathscr{Q}(y) \tag{3}$$

for every pair of elements x, y of S. Indeed, $\mathscr{Q}(xy) \cap \mathscr{Q}(yx) = \mathscr{R}(xy) \cap \mathscr{L}(xy) \cap \cap \mathscr{R}(yx) \cap \mathscr{L}(yx) \subset \mathscr{R}(x) \cap \mathscr{L}(y) \cap \mathscr{R}(y) \cap \mathscr{L}(x) \subset \mathscr{Q}(x) \cap \mathscr{Q}(y)$.

Lemma 2. *Let $\mathscr{T}$ be a class of non-empty subsets of S with $S \in \mathscr{T}$. If the class $\{\mathscr{T}(x);\ x \in S\}$ forms a chain under set inclusion, then $\mathscr{T}$ forms a chain under set inclusion, too.*

Proof. Suppose that $\{\mathscr{T}(x);\ x \in S\}$ forms a chain. Let $A, B \in \mathscr{T}$. If $A \not\subset B$ and $B \not\subset A$, then there exist $a \in A - B$ and $b \in B - A$. Then $\mathscr{T}(a) \subset A$ and $\mathscr{T}(b) \subset B$. Since all $\mathscr{T}(x)$ form a chain by hypothesis, we have $a \in \mathscr{T}(a) \subset \subset \mathscr{T}(b) \subset B$ or $b \in \mathscr{T}(b) \subset \mathscr{T}(a) \subset A$, a contradiction. Hence $A \subset B$ or $B \subset A$.

Theorem 1a. *The following conditions on a semigroup S are equivalent;*

1. *Every quasi-ideal of S is $\mathcal{Q}$-prime and S is $\mathcal{Q}$-uniform.*

2. *Every quasi-ideal of S is idempotent and the quasi-ideals of S form a chain under set inclusion.*

3. *Every principal quasi-ideal of S is idempotent and the principal quasi-ideals of S form a chain under set inclusion.*

Proof. $1 \Rightarrow 2$. Let S be $\mathcal{Q}$-uniform and let every quasi-ideal be $\mathcal{Q}$-prime. Let $A, B \in \mathcal{Q}$. Then by Definition 2 we have $AB \cap BA \subset AS \cap SA \subset A$ and $AB \cap BA \subset SB \cap BS \subset B$ and so $AB \cap BA \subset A \cap B$. Since the quasi-ideal $A \cap B$ is $\mathcal{Q}$-prime, we have $A \subset A \cap B \subset B$ or $B \subset A \cap B \subset A$. Hence the quasi-ideals of S form a chain under set inclusion.

Let A be an arbitrary quasi-ideal of S. It follows from Remark 2 that there exist $R \in \mathcal{R}$ and $L \in \mathcal{L}$ such that $A = R \cap L$. Since $R, L \in \mathcal{Q}$ and $\mathcal{Q}$ is a chain, we have $R \subset L$ or $L \subset R$ and so $A = R$ or $A = L$. This implies that A is a one-sided ideal of S and so A^2 is a quasi-ideal of S. Since A^2 is $\mathcal{Q}$-prime by hypothesis and $AA \cap AA \subset A^2$, we have $A \subset A^2$. It is clear that $A^2 \subset A$ and thus we have $A = A^2$.

$2 \Leftrightarrow 3$. This follows from Lemma 2 and Corollary 2 of Theorem 1 from [5].

$2 \Rightarrow 1$. Let every quasi-ideal of S be idempotent and let the quasi-ideals of S form a chain under set inclusion. Evidently, S is $\mathcal{Q}$-uniform. We shall prove that every quasi-ideal Q of S is $\mathcal{Q}$-prime. Let $A, B \in \mathcal{Q}$ such that $AB \cap \cap BA \subset Q$. Since $\mathcal{Q}$ is a chain under set inclusion, we have $A \subset B$ or $B \subset A$ which implies $A^2 \subset AB \cap BA \subset Q$ or $B^2 \subset AB \cap BA \subset Q$. Since every quasi-ideal of S is idempotent, we have $A \subset Q$ or $B \subset Q$ and so Q is $\mathcal{Q}$-prime.

Theorem 1b. *The following conditions on a semigroup S are equivalent;*

1. *Every right ideal of S is $\mathcal{R}$-prime and S is right uniform.*

2. *Every right ideal of S is idempotent and the right ideals of S form a chain under set inclusion.*

3. *Every principal right ideal of S is idempotent and the principal right ideals of S form a chain under set inclusion.*

The proof can be given by a simple adaptation of the proof of Theorem 1a.

Theorem 1c. *The following conditions on a semigroup S are equivalent;*

1. *Every ideal of S is prime.*

2. *Every ideal of S is idempotent and the ideals of S form a chain under set inclusion.*

3. *Every principal ideal of S is idempotent and the principal ideals of S form a chain under set inclusion.*

(See Theorem 2 in [1].)

Lemma 3. *If the right ideals of S form a chain under set inclusion and the lef ideals of S form a chain under set inclusion, then the idempotents in S form a chain.*

Proof. Let e, f be idempotents of S. Then $\mathscr{R}(e) \subset \mathscr{R}(f)$ or $\mathscr{R}(f) \subset \mathscr{R}(e)$. Suppose (without loss of generality) that $\mathscr{R}(e) \subset \mathscr{R}(f)$. This implies $e \in \mathscr{R}(f)$ and so $fe = f(fx) = fx = e$ (for some $x \in S$). Analogously we can prove that $\mathscr{L}(e) \subset \mathscr{L}(f)$ implies $ef = e$ and $\mathscr{L}(f) \subset \mathscr{L}(e)$ implies $fe = f$. Since the left ideals of S form a chain under set inclusion, we have $fe = e = ef$ or $e = fe = f$. This means that the idempotents in S form a chain.

Theorem 2. *Let a semigroup S be right and left uniform. If every right ideal of S is $\mathscr{R}$-prime and every left ideal of S is $\mathscr{L}$-prime, then the idempotents in S form a chain.*

The proof follows from Theorem 1b, its dual and Lemma 3.

Definition 7. *Let $\mathscr{T}$ be a class of non-empty subsets of S with $S \in \mathscr{T}$. A subset P of S is said to be completely $\mathscr{T}$-prime (or simply $\mathscr{C}\mathscr{T}$-prime) if $a, b \in S$ and $\mathscr{T}(ab) \cap \mathscr{T}(ba) \subset P$ imply $a \in P$ or $b \in P$.*

Remark 5. We show that every two-sided ideal I of S is completely prime if and only if it is completely $\mathscr{I}$-prime.

Proof. Let I be a completely prime ideal of S. Let $a, b \in S$ and let $\mathscr{I}(ab) \cap \cap \mathscr{I}(ba) \subset I$ hold. Then $abab \in \mathscr{I}(ab) \cap \mathscr{I}(ba) \subset I$ and so $(ab)^2 \in I$. From this it follows that $ab \in I$. Hence $a \in I$ or $b \in I$.

Let an ideal I be $\mathscr{C}\mathscr{I}$-prime. Let $a, b \in I$. If $ab \in I$, then $\mathscr{I}(ab) \cap \mathscr{I}(ba) \subset \subset \mathscr{I}(ab) \subset I$ and so $a \in I$ or $b \in I$. This means that I is a completely prime ideal of S.

Remark 6. In the example of Remark 3 (above) the set N is a completely prime ideal but N is neither $\mathscr{C}\mathscr{R}$-prime nor $\mathscr{C}\mathscr{Q}$-prime. As a matter of fact, $\mathscr{Q}(ab) = \mathscr{R}(ab) = A$ and $\mathscr{Q}(ba) = \mathscr{R}(ba) = B$. Hence $\mathscr{Q}(ab) \cap \mathscr{Q}(ba) = \mathscr{R}(ab) \cap \cap \mathscr{R}(ba) = A \cap B = N$, but $a \notin N$ and $b \notin N$.

Theorem 3a. *Every completely $\mathscr{Q}$-prime quasi-ideal of a semigroup S is $\mathscr{Q}$-prime.*

Proof. Let Q be a $\mathscr{C}\mathscr{Q}$-prime quasi-ideal of S. Suppose that Q is not $\mathscr{Q}$-prime. Then there exist two quasi-ideals A, B of S such that $AB \cap BA \subset Q$ and $A \not\subset Q$, $B \not\subset Q$.

We first show that $A \cap B \subset Q$. If $A \cap B \not\subset Q$, then there exists an element c of S such that $c \in A \cap B$ and $c \notin Q$. Hence $c^2 \in AB \cap BA \subset Q$ and so $\mathscr{Q}(c^2) \subset Q$. Since Q is $\mathscr{C}\mathscr{Q}$-prime, we have $c \in Q$, which is a contradiction. Therefore, $A \cap B \subset Q$.

Since $A \not\subset Q$, $B \not\subset Q$, there exist elements a, b of S such that $a \in A - Q$, $b \in B - Q$. Then $\mathscr{Q}(a) \subset A$ and $\mathscr{Q}(b) \subset B$. It follows from (3) that $\mathscr{Q}(ab) \cap$

$\cap \mathscr{Q}(ba) \subset \mathscr{Q}(a) \cap \mathscr{Q}(b) \subset A \cap B \subset Q$. Since Q is $\mathscr{C}\mathscr{Q}$-prime, we have $a \in Q$ or $b \in Q$, which is a contradiction. Consequently Q is $\mathscr{Q}$-prime.

Using the same method of proof as in Theorem 3a, we obtain:

Theorem 3b. *Every completely $\mathscr{R}$-prime right ideal of a semigroup S is $\mathscr{R}$-prime.*

Theorem 3c. *Every completely prime ideal of a semigroup S is prime.*

(See Footnote 1 in [6].)

Theorem 4a. *Every $\mathscr{Q}$-prime two-sided ideal of a semigroup S is completely prime.*

Proof. Let I be a $\mathscr{Q}$-prime ideal of S. Let $a, b \in S$ and suppose $\mathscr{I}(ab) \cap \cap \mathscr{I}(ba) \subset I$. Then it follows from (2) and (1) that $\mathscr{Q}(a)\mathscr{Q}(b) \cap \mathscr{Q}(b)\mathscr{Q}(a) \subset \subset \mathscr{L}(a)\mathscr{R}(b) \cap \mathscr{L}(b)\mathscr{R}(a) = \mathscr{I}(ab) \cap \mathscr{I}(ba) \subset I$ therefore $\mathscr{Q}(a) \subset I$ or $\mathscr{Q}(b) \subset I$. Hence $a \in I$ or $b \in I$ and so I is a completely prime ideal of S.

Theorem 4b. *Let every left ideal of a semigroup S be a right ideal of S. Then every $\mathscr{R}$-prime two-sided ideal of S is completely prime.*

The proof follows by a simple adaptation of the proof of Theorem 4a.

Theorem 4c. *Let every one-sided ideal of a semigroup S be a two-sided ideal of S. Then every prime ideal of S is completely prime.*

(See Satz 4 in [1].)

Theorem 5a. *The following conditions on a semigroup S are equivalent;*

1. *Every quasi-ideal of S is completely $\mathscr{Q}$-prime and S is $\mathscr{Q}$-uniform.*
2. *The quasi-ideals of S form a chain under set inclusion and for every $a, b \in S$ we have $\mathscr{Q}(a) \cap \mathscr{Q}(b) = \mathscr{Q}(ab) \cap \mathscr{Q}(ba)$.*
3. *S is a chain of groups.*

Proof. $1 \Rightarrow 2$. Let every quasi-ideal of S be $\mathscr{C}\mathscr{Q}$-prime and let S be $\mathscr{Q}$-uniform. If $a, b \in S$, then $Q = \mathscr{Q}(ab) \cap \mathscr{Q}(ba)$ is a $\mathscr{C}\mathscr{Q}$-prime quasi-ideal of S and so $a \in Q$ or $b \in Q$. If $a \in Q$, then $\mathscr{Q}(a) \subset Q$. It follows from (3) that $Q \subset \mathscr{Q}(a) \cap \cap \mathscr{Q}(b) \subset \mathscr{Q}(b)$ and so $\mathscr{Q}(a) \subset \mathscr{Q}(b)$. If $b \in Q$, then we obtain analogously that $\mathscr{Q}(b) \subset \mathscr{Q}(a)$. This means that the principal quasi-ideals of S form a chain under set inclusion. It follows from Lemma 2 that the quasi-ideals of S form a chain.

Finally, it is clear that $\mathscr{Q}(a) \subset Q \subset \mathscr{Q}(b)$ or $\mathscr{Q}(b) \subset Q \subset \mathscr{Q}(a)$ and so $\mathscr{Q}(a) \cap \cap \mathscr{Q}(b) \subset Q = \mathscr{Q}(ab) \cap \mathscr{Q}(ba)$. It follows from (3) that $\mathscr{Q}(a) \cap \mathscr{Q}(b) = \mathscr{Q}(ab) \cap \cap \mathscr{Q}(ba)$.

$2 \Rightarrow 3$. Let the quasi-ideals of S form a chain under set inclusion and suppose that for every $a, b \in S$ we have $\mathscr{Q}(a) \cap \mathscr{Q}(b) = \mathscr{Q}(ab) \cap \mathscr{Q}(ba)$. Then, by (2), we have $a \in \mathscr{Q}(a) = \mathscr{Q}(a^2) = \mathscr{R}(a^2) \cap \mathscr{L}(a^2)$ and so S is right regular and left regular. Hence, by Theorem 4.3 of [3], S is a union of groups. Since every one-sided ideal of S is a quasi-ideal of S, the right (left) ideals of S form a chain under set inclusion. It follows from Lemma 3 that the idempotents in S form a chain and so the idempotents of S commute. According to Theorem 4.6 of [3], S is a semilattice of groups and so S is a chain of groups.

$3 \Rightarrow 1$. Let S be a chain of groups. Evidently, S is a semilattice of groups and so, by Remark to Theorem 2 [7], every quasi-ideal of S is a two-sided ideal of S. This implies that S is $\mathscr{Q}$-uniform and $\mathscr{R}(x) = \mathscr{Q}(x) = \mathscr{L}(x)$ for any $x \in S$. Now we shall prove that every quasi-ideal of S is $\mathscr{C}\mathscr{Q}$-prime. Let Q be a quasi-ideal of S and suppose that $\mathscr{Q}(ab) \cap \mathscr{Q}(ba) \subset Q$ holds for some $a, b \in S$. Let $e\,(f)$ be the identity of the maximal subgroup of S containing the element $a\,(b)$. Since the idempotents in S form a chain, we have either $e \leqq f$ or $f \leqq e$. If $e \leqq f$, then $ef = e = fe$ and so $a = ae = a(ef) = (ae)f = af = abb^{-1} \in \mathscr{R}(ab) = \mathscr{Q}(ab)$. Dually we obtain that $a \in \mathscr{L}(ba) = \mathscr{Q}(ba)$ and so $a \in \mathscr{Q}(ab) \cap \mathscr{Q}(ba) \subset Q$. If $f \leqq e$, then analogously we have $b \in Q$. Consequently, Q is a $\mathscr{C}\mathscr{Q}$-prime quasi-ideal.

Theorem 5b. *The following conditions on a semigroup S are equivalent;*

1. *Every right ideal of S is completely $\mathscr{R}$-prime and S is right uniform.*
2. *The right ideals of S form a chain under set inclusion and for every $a, b \in S$ we have $\mathscr{R}(a) \cap \mathscr{R}(b) = \mathscr{R}(ab) \cap \mathscr{R}(ba)$.*
3. *The right ideals of S form a chain under set inclusion, S is right regular and for every $a, b \in S$ we have $ab \in \mathscr{R}(a^2b)$.*

Proof. $1 \Rightarrow 2$. The proof is analogous to the proof of Theorem 5a.

$2 \Rightarrow 3$. Let the right ideals of S form a chain under set inclusion and suppose that for every $a, b \in S$ we have $\mathscr{R}(a) \cap \mathscr{R}(b) = \mathscr{R}(ab) \cap \mathscr{R}(ba)$. This implies $a \in \mathscr{R}(a) = \mathscr{R}(a^2)$ for all $a \in S$ and so S is a right regular semigroup. Furthermore, we have $ab \in \mathscr{R}(a) \cap \mathscr{R}(ab) = \mathscr{R}(a^2b) \cap \mathscr{R}(aba) \subset \mathscr{R}(a^2b)$ for every $a, b \in S$.

$3 \Rightarrow 1$. Let the right ideals of S form a chain under set inclusion, let S be right regular and suppose that for every $a, b \in S$ we have $ab \in \mathscr{R}(a^2b)$. This implies that S is right uniform. We shall prove that every right ideal R of S is $\mathscr{C}\mathscr{R}$-prime. Let $\mathscr{R}(ab) \cap \mathscr{R}(ba) \subset R$ hold for some $a, b \in S$. Since the principal right ideals form a chain by hypothesis, we have $\mathscr{R}(a) \subset \mathscr{R}(b)$ or $\mathscr{R}(b) \subset \mathscr{R}(a)$. Assume first that $\mathscr{R}(a) \subset \mathscr{R}(b)$. Then $\mathscr{R}(a^2) \subset \mathscr{R}(ab)$. Since S is right regular, we have $a \in \mathscr{R}(a^2)$ and so $a \in \mathscr{R}(ab)$. It follows from $\mathscr{R}(a) \subset \mathscr{R}(b)$ that $a \in \mathscr{R}(b)$ and so $a = b$ or $a = bs$ for some $s \in S$. Let $a = b$. Since S is right regular, we have $a = b \in \mathscr{R}(b^2) = \mathscr{R}(ba)$. Let $a = bs$. Then, by hypothesis, $a = bs \in \mathscr{R}(b^2s) = \mathscr{R}(ba)$. This gives in both cases $a \in \mathscr{R}(ba)$ and so $a \in \mathscr{R}(ab) \cap \mathscr{R}(ba) \subset R$. In an analogous manner it can be proved that $\mathscr{R}(b) \subset \mathscr{R}(a)$ implies $b \in R$. Thus R is a $\mathscr{C}\mathscr{R}$-prime right ideal of S.

Corollary 1b. *A semigroup S is a chain of groups if and only if at least one of the conditions of Theorem 5b and at least one of the conditions of the dual of Theorem 5b are satisfied.*

Theorem 5c. *The following conditions on a semigroup S are equivalent;*

1. *Every ideal of S is completely prime.*
2. *The ideals of S form a chain under set inclusion and for every $a, b \in S$ we have $\mathscr{I}(a) \cap \mathscr{I}(b) = \mathscr{I}(ab)$.*
3. *The ideals of S form a chain under set inclusion and S is intraregular.*

Proof. $1 \Rightarrow 2$. This can be proved by an analogous argument as in the proof of Theorem 5a.

$2 \Rightarrow 3$. This can be proved analogously as in the proof of Theorem 5b.

$3 \Rightarrow 1$. See Satz 1 in [1].

Corollary 1c. *Let every one-sided ideal of a semigroup S be a two-sided ideal of S. Then S is a chain of groups if and only if at least one of the conditions of Theorem 5c or of Theorem 1c is satisfied.*

This follows from Theorem 4c.

See Theorem 2.2 in [8].

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Elektrotechnická fakulta ČVUT
290 35 Poděbrady