Mathematica Bohemica

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Mathematica Bohemica, Vol. 117 (1992), No. 3, 283–292

Persistent URL: http://dml.cz/dmlcz/126278

BINARY AND TERNARY RELATIONS

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(Received November 5, 1990)

Summary. Two operators are constructed which make it possible to transform ternary relations into binary relations defined on binary relations and vice versa. A possible graphical representation of ternary relations is described.

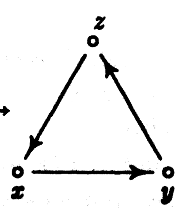
Keywords: ternary structure, double binary structure, cyclically ordered set

AMS classification: 04A05, 06A99

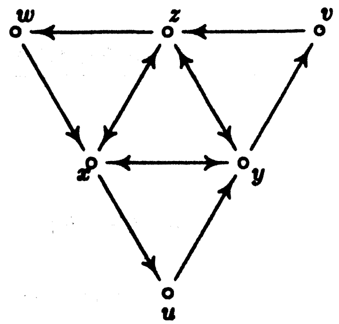
1. Introduction

In recent years ternary relations have often been studied in literature—cyclically ordered sets and groups ([1], [5], [6], [3], [8], [9]), but also the betweenness relation on lattices, partially ordered sets or graphs ([4], [2], [7]). When constructing examples or counterexamples of such relations we meet with the problem of graphical representation of ternary relations (formal description by ordered triplets is not suitable). If t is a ternary relation on a set G which is cyclic (the definition see below), then we can use oriented triangles:

$(x, y, z) \in t,\ (y, z, x) \in t,\ (z, x, y) \in t \longrightarrow$



Nonetheless, even in this case we can get into troubles: if $G = \{x, y, z, u, v, w\}$, $t = \{(x, y, z), (x, u, y), (y, v, z), (z, w, x)\}$ and t^c is a cyclic hull of t, then the graph of t^c is as follows:



We have obtained an oriented triangle corresponding to triplets (x,z,y), (z,y,x), (y,x,z) which are not in t^c. With general ternary relations the difficulties increase. If t is a ternary relation which is not cyclic, then $(x,y,z) \in t$ cannot be represented by a triangle, but by an oriented "broken line" composed of two vectors: the first of them having the initial point x and the end point y, the second having the initial point y and the end point z (the cases when either $x = y$ or $y = z$ are not excluded). Two different oriented broken lines may include the same vector, so that different broken lines must be distinguished, e.g. by different colours. However, this way of representation is not practically applicable. These considerations lead us to replacing a ternary relation by a binary relation whose carrier is a binary relation.

2. Definitions and examples

Let G be a set and t a ternary relation on G (i.e. $t \subseteq G^3$). The pair $\mathbf{G} = (G,t)$ will be called a *ternary structure*. Let $\mathbf{G} = (G,t)$ be a ternary structure. The relation t (and the structure $\mathbf{G}$) is said to be

symmetric, iff $(x,y,z) \in t \Rightarrow (z,y,x) \in t$;

asymmetric, iff $(x,y,z) \in t \Rightarrow (z,y,x) \notin t$;

cyclic, iff $(x,y,z) \in t \Rightarrow (y,z,x) \in t$;

transitive, iff $(x,y,z) \in t$, $(z,y,u) \in t \Rightarrow (x,y,u) \in t$.

If the last condition is satisfied only for $y = z$, i.e., if

$$(x,y,y) \in t,\ (y,y,z) \in t \Rightarrow (x,y,z) \in t,$$

then t (and $\mathbf{G}$) is called *weakly transitive*.

A ternary structure $\mathbf{G} = (G,t)$ is called a *cyclically ordered set*, iff it is asymmetric, cyclic and transitive.

If G is a set and $\alpha = (x,y) \in G^2$, then we put $\alpha^{-1} = (y,x)$.

Let G be a set, ϱ a binary relation on G. Further, let r be a binary relation on the set ϱ with the property

$$\alpha = (x,y) \in \varrho,\ \beta = (z,u) \in \varrho,\ (\alpha,\beta) \in r \Rightarrow y = z.$$

Then r is called a *binding relation* on ϱ.

Let G be a set, ϱ a binary relation on G and r a binding relation on ϱ. Then $\mathbf{G} = (G, \varrho, r)$ is called a *double binary structure*. An edge $\alpha \in \varrho$ in $\mathbf{G}$ is said to be *isolated* iff $(\alpha, \beta) \notin r$, $(\beta, \alpha) \notin r$ for any $\beta \in \varrho$.

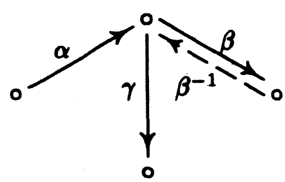
Let $\mathbf{G} = (G, \varrho, r)$ be a double binary structure. The relation r (and the structure $\mathbf{G}$) is said to be

inversely symmetric, iff $(\alpha, \beta) \in r \Rightarrow (\beta^{-1}, \alpha^{-1}) \in r$;
inversely asymmetric, iff $(\alpha, \beta) \in r \Rightarrow (\beta^{-1}, \alpha^{-1}) \notin r$;
reversely transitive, iff $(\alpha, \beta) \in r, (\beta^{-1}, \gamma) \in r \Rightarrow (\alpha, \gamma) \in r$;
transferable, iff $(\alpha, \beta) \in r \Rightarrow$ there exists $\gamma \in \varrho$ with $(\beta, \gamma) \in r$, $(\gamma, \alpha) \in r$.

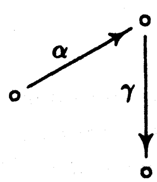
We now give two ways of representing finite double binary structures.

Representation 1. Let $\mathbf{G} = (G, \varrho, r)$ be a finite double binary structure (i.e., the set G is finite). Let i be an injection of G into a plane. We can identify x and $i(x)$, i.e., suppose that G is a finite subset of a plane. Any $\alpha = (x, y) \in \varrho$ will be represented by the vector with the initial point x and the end point y. The relation r is a subset of ϱ^2 such that $(\alpha, \beta) \in r$ implies that the end point of α coincides with the initial point of β. Thus, any element $(\alpha, \beta) \in r$ may be represented by an oriented broken line obtained as the union of the vectors α, β.

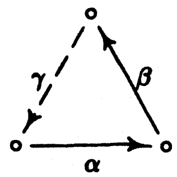
This representation is appropriate for interpreting various conditions for $\mathbf{G}$. The inverse symmetry of $\mathbf{G}$ means that for any oriented broken line (α, β) in r the inversely oriented broken line $(\beta^{-1}, \alpha^{-1})$ is in r, too. The inverse asymmetry means that for any oriented broken line (α, β) in r the inversely oriented broken line $(\beta^{-1}, \alpha^{-1})$ never is in r. The reverse transitivity means: If oriented broken lines (α, β), (β^{-1}, γ) are in r, then (α, γ) also is in r, i.e., if the configuration



is in r, then the configuration

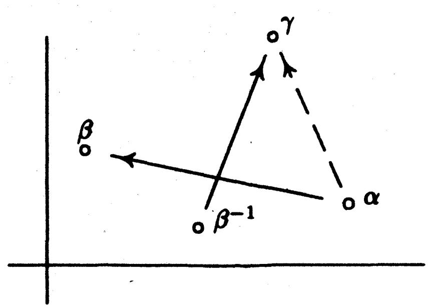


obtained by cancelling β and β^{-1} is in r as well. The transferability means the following. If $\alpha = (x, y) \in \varrho, \beta = (y, z) \in \varrho$ and the oriented broken line (α, β) is in r, then the oriented broken lines $(\beta, \gamma), (\gamma, \alpha)$ are in r as well, where clearly $\gamma = (z, x)$. Thus, $\gamma = (\alpha + \beta)^{-1}$ where $+$ means the usual addition of vectors.



Representation 2. Let $G = (G, \varrho, r)$ be a finite double binary structure. We identify the elements of G with points of the coordinate axis. It follows that any $\alpha = (x, y) \in G^2$ is a point in a plane; $\alpha^{-1} = (y, x)$ is the point symmetric to α with respect to the line $y = x$. Thus, ϱ is a finite subset of a plane. If $\alpha \in \varrho$, $\beta \in \varrho$ and $(\alpha, \beta) \in r$, then we represent the pair (α, β) by the vector with the initial point α and the end point β. Hence, r is the set of vectors whose initial points and end points are in ϱ. As $\alpha = (x, y) \in \varrho$, $\beta = (z, u) \in \varrho$, $(\alpha, \beta) \in r$ imply $y = z$, α may be connected with β by the vector $(\alpha, \beta) \in r$ only if the first coordinates of α^{-1} and β coincide.

Some properties of r may be also read in this representation. If r is inversely symmetric, then the set r is symmetric with respect to the line $y = x$. The reverse transitivity means the completion of the configuration formed by vectors (α, β), (β^{-1}, γ) by the vector (α, γ):



The transferability has the following meaning: If the vector $((x, y), (y, z))$ is in r, then the point (z, x) is in ϱ and the vectors $((y, z), (z, x))$, $((z, x), (x, y))$ are in r. This means the existence of the oriented triangle with vertices (x, y), (y, z), (z, x) in our representation of r.

Example 1. Let $G = \{x, y, z, u, v\}$, $\varrho_1 = \{(x, y), (y, z), (y, u), (y, v), (z, u), (u, v)\}$, $r_1 = \{((x, y), (y, z)), ((x, y), (y, u)), ((x, y), (y, v)), ((z, u), (u, v))\}$. Then the double binary structure $G_1 = (G, \varrho_1, r_1)$ has the following representation:

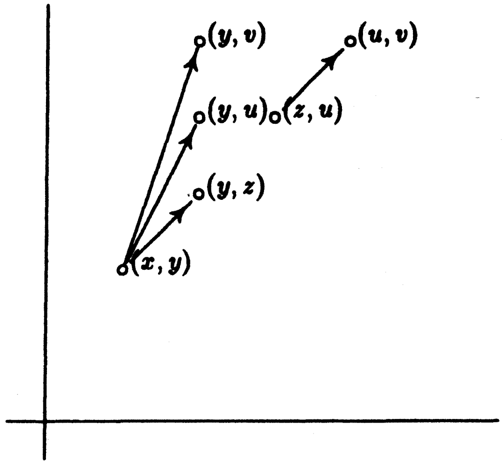


Fig. 1

Example 2. Let $G = \{x, y, z, u, v\}$, $\varrho_2 = \{(x, y), (y, z), (z, x), (y, u), (u, x), (y, v), (v, x), (z, u), (u, v), (v, z)\}$, $r_2 = \{((x, y), (y, z)), ((y, z), (z, x)), ((z, x), (x, y)), ((x, y), (y, u)), ((y, u), (u, x)), ((u, x), (x, y)), ((x, y), (y, v)), ((y, v), (v, x)), ((v, x), (x, y)), ((z, u), (u, v)), ((u, v), (v, z)), ((v, z), (z, u))\}$. Then $G_2 = (G, \varrho_2, r_2)$ has the following representation:

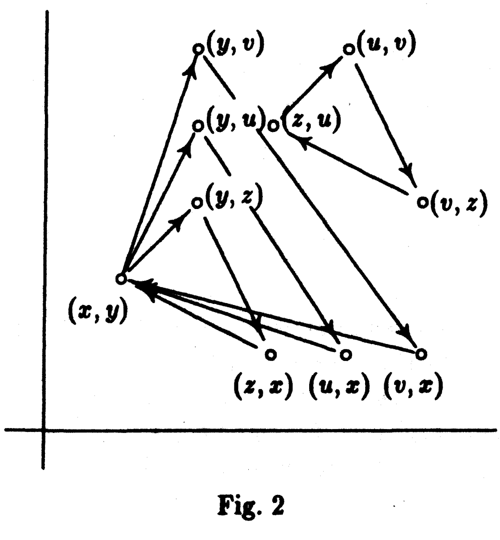


Fig. 2

3. Operators T and B

We denote by $\mathscr{B}$ the class of all double binary structures and by $\mathscr{T}$ the class of all ternary structures.

Let $\mathbf{G} = (G, \varrho, r)$ be a double binary structure. Let us define a ternary relation t on the set G by

$$(x, y, z) \in t \Leftrightarrow (x, y) = \alpha \in \varrho, \quad (y, z) = \beta \in \varrho \text{ and } (\alpha, \beta) \in r,$$

and denote the ternary structure (G, t) as $\mathbf{T}(\mathbf{G})$. Thus, $\mathbf{T}$ is an operator transforming a double binary structure into a ternary structure, i.e.

$$\mathbf{T}: \mathscr{B} \to \mathscr{T}.$$

Let $\mathbf{G} = (G, t)$ be a ternary structure. Let us define a binary relation ϱ on the set G by

$$(x, y) \in \varrho \Leftrightarrow \text{ there is } z \in G \text{ with either } (x, y, z) \in t \text{ or } (z, x, y) \in t;$$

let us define a binary relation r on the set ϱ by

$$(\alpha, \beta) \in r \Leftrightarrow \text{ there exist } x, y, z \in G \text{ such that } \alpha = (x, y),\ \beta = (y, z) \text{ and } (x, y, z) \in t.$$

Clearly, (G, ϱ, r) is a double binary structure; we denote it by $\mathbf{B}(\mathbf{G})$. Thus, $\mathbf{B}$ is an operator transforming a ternary structure into a double binary structure, i.e.

$$\mathbf{B}: \mathscr{T} \to \mathscr{B}.$$

Theorem 3.1. *Let $\mathbf{G}$ be a ternary structure. Then $(\mathbf{T} \circ \mathbf{B})(\mathbf{G}) = \mathbf{G}$, i.e., $\mathbf{T} \circ \mathbf{B} = \mathrm{id}_{\mathscr{T}}$.*

Proof. Let $\mathbf{G} = (G, t)$ and put $\mathbf{B}(\mathbf{G}) = (G, \varrho, r)$, $(\mathbf{T} \circ \mathbf{B})(\mathbf{G}) = \mathbf{T}(G, \varrho, r) = (G, t')$. If $(x, y, z) \in t$, then $\alpha = (x, y) \in \varrho$, $\beta = (y, z) \in \varrho$ and $(\alpha, \beta) \in r$ by the definition of $\mathbf{B}$. But then $(x, y, z) \in t'$ by the definition of $\mathbf{T}$; thus $t \subseteq t'$. Assume $(x, y, z) \in t'$. Then $\alpha = (x, y) \in \varrho$, $\beta = (y, z) \in \varrho$ and $(\alpha, \beta) \in r$ by the definition of $\mathbf{T}$ and, therefore, $(x, y, z) \in t$ by the definition of $\mathbf{B}$. Thus $t' \subseteq t$ and hence $t' = t$. □

Theorem 3.2. *Let $\mathbf{G} = (G, \varrho, r)$ be a double binary structure and let $(\mathbf{B} \circ \mathbf{T})(\mathbf{G}) = (G, \varrho', r')$. Then $\varrho' \subseteq \varrho$ and $r' = r$. If $\mathbf{G}$ contains no isolated edges, then $\varrho' = \varrho$. Thus, if $\mathscr{B}'$ is the class of all double binary structures without isolated edges, then $\mathbf{B} \circ \mathbf{T}_{\mathscr{B}'} = \mathrm{id}_{\mathscr{B}'}$.*

Proof. Put $\mathbf{T}(G) = (G, t)$ and suppose $(x, y) \in \varrho'$. Then there exists $z \in G$ such that either $(x, y, z) \in t$ or $(z, x, y) \in t$. In either case we have $(x, y) \in \varrho$; thus $\varrho' \subseteq \varrho$. Let $(\alpha, \beta) \in r'$. Then there exist $x, y, z \in G$ such that $\alpha = (x, y), \beta = (y, z)$ and $(x, y, z) \in t$. By the definition of $\mathbf{T}$ we have $\alpha \in \varrho, \beta \in \varrho$ and $(\alpha, \beta) \in r$. Thus, $r' \subseteq r$. Conversely, let $(\alpha, \beta) \in r$. Then $\alpha = (x, y) \in \varrho, \beta = (y, z) \in \varrho$ and $(x, y, z) \in t$ by the definition of the relation t. It follows that $\alpha = (x, y) \in \varrho', \beta = (y, z) \in \varrho'$ and $(\alpha, \beta) \in r'$ by the definition of $\mathbf{B}$. Thus $r \subseteq r'$ and hence $r' = r$. If G contains no isolated edges and if $\alpha = (x, y) \in \varrho$, then there exists $\beta \in \varrho$ such that either $(\alpha, \beta) \in r$ or $(\beta, \alpha) \in r$. In the first case the edge β must have the form $\beta = (y, z)$ and $(x, y, z) \in t$ by the definition of t. Then $(x, y) = \alpha \in \varrho'$ by the definition of $\mathbf{B}$. In the second case we have $\beta = (z, x)$ and $(z, x, y) \in t$. It also follows that $\alpha = (x, y) \in \varrho'$. Thus $\varrho \subseteq \varrho'$ and hence $\varrho' = \varrho$. □

Theorem 3.3. *Let G be a double binary structure. Then the following assertions hold:*

(1) *G is inversely symmetric iff $\mathbf{T}(G)$ is symmetric;*

(2) *G is inversely asymmetric iff $\mathbf{T}(G)$ is asymmetric.*

Proof. Let $G = (G, \varrho, r)$ and $\mathbf{T}(G) = (G, t)$.

(1) Let r be inversely symmetric and let $(x, y, z) \in t$. Then $\alpha = (x, y) \in \varrho$, $\beta = (y, z) \in \varrho$ and $(\alpha, \beta) \in r$. Thus $(\beta^{-1}, \alpha^{-1}) \in r$, so that $\beta^{-1} = (z, y) \in \varrho, \alpha^{-1} = (y, x) \in \varrho$ and $(z, y, x) \in t$. Hence t is symmetric. Let t be symmetric and let $(\alpha, \beta) \in r$. Then there exist $x, y, z \in G$ such that $\alpha = (x, y)$, $\beta = (y, z)$ and $(x, y, z) \in t$. Consequently, $(z, y, x) \in t$ so that $(z, y) = \beta^{-1} \in \varrho$, $(y, x) = \alpha^{-1} \in \varrho$ and $(\beta^{-1}, \alpha^{-1}) \in r$. Hence r is inversely symmetric.

(2) Let r be inversely asymmetric and assume the existence of elements $x, y, z \in G$ with $(x, y, z) \in t$, $(z, y, x) \in t$. Then $\alpha = (x, y) \in \varrho$, $\beta = (y, z) \in \varrho$, $(\alpha, \beta) \in r$, and $\beta^{-1} = (z, y) \in \varrho, \alpha^{-1} = (y, x) \in \varrho$, $(\beta^{-1}, \alpha^{-1}) \in r$. This contradicts the inverse asymmetry of r and thus t is asymmetric. Let t be asymmetric and assume the existence of elements $\alpha, \beta \in \varrho$ such that $(\alpha, \beta) \in r$, $(\beta^{-1}, \alpha^{-1}) \in r$. Then there exist $x, y, z \in G$ with $\alpha = (x, y)$, $\beta = (y, z), (x, y, z) \in t$, and $\beta^{-1} = (z, y) \in \varrho, \alpha^{-1} = (y, x) \in \varrho$, $(z, y, x) \in t$. This contradicts the asymmetry of t and thus r is inversely asymmetric. □

Theorem 3.4. *Let G be a ternary structure. Then the following assertions hold:*

(1) *G is symmetric iff $\mathbf{B}(G)$ is inversely symmetric;*

(2) *G is asymmetric iff $\mathbf{B}(G)$ is inversely asymmetric.*

Proof. If $\mathbf{B}(G)$ is inversely symmetric, then $\mathbf{T}(\mathbf{B}(G))$ is symmetric by 3.3; but $\mathbf{T}(\mathbf{B}(G)) = G$ by 3.1. If $G = \mathbf{T}(\mathbf{B}(G))$ is symmetric, then $\mathbf{B}(G)$ is inversely symmetric by 3.3. This proves the assertion (1). Condition (2) follows from 3.3 and 3.1 analogously. □

Theorem 3.5. *Let G be a double binary structure. Then G is transferable iff $\mathbf{T}(G)$ is cyclic.*

Proof. Put $G = (G, \varrho, r)$ and $\mathbf{T}(G) = (G, t)$. Let r be transferable and let $(x, y, z) \in t$. Then $\alpha = (x, y) \in \varrho$, $\beta = (y, z) \in \varrho$ and $(\alpha, \beta) \in r$. Thus there exists $\gamma \in \varrho$ such that $(\beta, \gamma) \in r$, $(\gamma, \alpha) \in r$. If $\gamma = (u, v)$, then from $(\beta, \gamma) \in r$ we have $u = z$ and from $(\gamma, \alpha) \in r$ we have $v = x$. Thus, $\gamma = (z, x)$ and $(\beta, \gamma) \in r$ imply $(y, z, x) \in t$. Hence t is cyclic. Let t be cyclic and suppose $(\alpha, \beta) \in r$. Then $\alpha = (x, y)$, $\beta = (y, z)$ and $(x, y, z) \in t$. It follows that $(y, z, x) \in t$, $(z, x, y) \in t$ so that, if we put $(z, x) = \gamma$, we have $\gamma \in \varrho$ and $(\beta, \gamma) \in r$, $(\gamma, \alpha) \in r$. Thus r is transferable. □

Theorem 3.6. *Let G be a ternary structure. Then G is cyclic iff $\mathbf{B}(G)$ is transferable.*

Proof follows from 3.5 and from $G = \mathbf{T}(\mathbf{B}(G))$.

Theorem 3.7. *Let $G = (G, \varrho, r)$ be a double binary structure. If the binary relation r is transitive, then the ternary structure $\mathbf{T}(G)$ is weakly transitive.*

Proof. Put $\mathbf{T}(G) = (G, t)$ and let $x, y, z \in G$, $(x, y, y) \in t$, $(y, y, z) \in t$. Then $a = (x, y) \in \varrho$, $\beta = (y, y) \in \varrho$, $\gamma = (y, z) \in \varrho$ and $(\alpha, \beta) \in r$, $(\beta, \gamma) \in r$. Transitivity of r implies $(\alpha, \gamma) \in r$ and hence $(x, y, z) \in t$. Thus t is weakly transitive. □

Theorem 3.8. *Let G be a double binary structure. Then G is reversely transitive iff $\mathbf{T}(G)$ is transitive.*

Proof. Put $G = (G, \varrho, r)$, $\mathbf{T}(G) = (G, t)$. Let r be reversely transitive and let $(x, y, z) \in t$, $(z, y, u) \in t$. Then $\alpha = (x, y) \in \varrho$, $\beta = (y, z) \in \varrho$, $(\alpha, \beta) \in r$, and $\beta^{-1} = (z, y) \in \varrho$, $\gamma = (y, u) \in \varrho$, $(\beta^{-1}, \gamma) \in r$. The reverse transitivity of r implies $(\alpha, \gamma) \in r$ and, therefore, $(x, y, u) \in t$. Thus t is transitive. Let t be transitive and let $\alpha, \beta, \gamma \in \varrho$, $(\alpha, \beta) \in r$, $(\beta^{-1}, \gamma) \in r$. Then there exist $x, y, z, u \in G$ such that $\alpha = (x, y), \beta = (y, z)$, $(x, y, z) \in t$, $\beta^{-1} = (z, y)$, $\gamma = (y, u)$, $(z, y, u) \in t$. The transitivity of t yields $(x, y, u) \in t$. Thus $(\alpha, \gamma) \in r$ and r is revesely transitive. □

Theorem 3.9. *Let G be a ternary structure. Then G is transitive iff $\mathbf{B}(G)$ is reversely transitive.*

Proof follows from 3.8 and 3.1. □

Now, from 3.4, 3.6 and 3.9 we get

Theorem 3.10. *Let G be a ternary structure. Then G is a cyclically ordered set iff the double binary structure $\mathbf{B}(G)$ is inversely asymmetric, transferable and reversely transitive.*

Analogously, from 3.3, 3.5 and 3.8 we obtain

Theorem 3.11. *Let G be a double binary structure. Then G is inversely asymmetric, transferable and reversely transitive iff $\mathbf{T}(G)$ is a cyclically ordered set.*

As the operator $\mathbf{B}$ transforms a ternary structure into a double binary structure, it offers a possibility of graphical representation of a ternary relation.

Example 3. Let $G = \{x, y, z, u, v\}$, $t_1 = \{(x, y, z), (x, y, u), (x, y, v), (z, u, v)\}$, $\boldsymbol{H}_1 = (G, t_1)$. We construct $\mathbf{B}(\boldsymbol{H}_1) = (G, \varrho_1, r_1)$. By definition, we obtain

$$\varrho_1 = \{(x, y), (y, z), (y, u), (y, v), (z, u), (u, v)\},$$
$$r_1 = \{((x, y), (y, z)), ((x, y), (y, u)), ((x, y), (y, v)), ((z, u), (u, v))\}.$$

Thus $\mathbf{B}(\boldsymbol{H}_1)$ coincides with the double binary structure $\boldsymbol{G}_1$ from Example 1 where its representation can be found.

Example 4. Let $G = \{x, y, z, u, v\}$, $t_2 = \{(x, y, z), (y, z, x), (z, x, y), (x, y, u), (y, u, x), (u, x, y), (x, y, v), (y, v, x), (v, x, y), (z, u, v), (u, v, z), (v, z, u)\}$, $\boldsymbol{H}_2 = (G, t_2)$. Clearly, t_2 is the cyclic hull of t_1 from Example 3 and $\boldsymbol{H}_2$ is a cyclically ordered set. We construct $\mathbf{B}(\boldsymbol{H}_2) = (G, \varrho_2, r_2)$;

$$\varrho_2 = \{(x, y), (y, z), (z, x), (y, u), (u, x), (y, v), (v, x), (z, u), (u, v), (v, z)\},$$
$$r_2 = \{((x, y), (y, z)), ((y, z), (z, x)), ((z, x), (x, y)), ((x, y), (y, u)),$$
$$((y, u), (u, x)), ((u, x), (x, y)), ((x, y), (y, v)), ((y, v), (v, x)),$$
$$((v, x), (x, y)), ((z, u), (u, v)), ((u, v), (v, z)), (v, z), (z, u))\}.$$

Thus $\mathbf{B}(\boldsymbol{H}_2)$ coincides with $\boldsymbol{G}_2$ from Example 2.

4. Categories $\mathcal{T}$ and $\mathcal{B}$

The class $\mathcal{T}$ is a category if we define morphisms in $\mathcal{T}$ in the obvious way, i.e. if $\boldsymbol{G} = (G, t) \in \mathcal{T}$, $\boldsymbol{H} = (H, s) \in \mathcal{T}$ and $f \colon G \to H$, then f is a morphism of $\boldsymbol{G}$ into $\boldsymbol{H}$ iff

$$(x, y, z) \in t \Rightarrow (f(x), f(y), f(z)) \in s.$$

The class $\mathcal{B}$ is category if we define morphisms in $\mathcal{B}$ as mappings preserving both ϱ and r, i.e. if $\boldsymbol{G} = (G, \varrho, r) \in \mathcal{B}$, $\boldsymbol{H} = (H, \sigma, s) \in \mathcal{B}$ and $f \colon G \to H$, then f is a morphism of $\boldsymbol{G}$ into $\boldsymbol{H}$ iff

$$(x, y) \in \varrho \Rightarrow (f(x), f(y)) \in \sigma, \qquad ((x, y), (y, z)) \in r \Rightarrow ((f(x), f(y)), (f(y), f(z))) \in s.$$

Clearly, the class $\mathscr{B}'$ of all double binary structures without isolated edges is a full subcategory of $\mathscr{B}$.

The operator $\mathbf{T}: \mathscr{B} \to \mathscr{T}$ is a functor, if for a morphism $f: G \to H$ we define the morphism $\mathbf{T}(f): \mathbf{T}(G) \to \mathbf{T}(H)$ by $\mathbf{T}(f) = f$. We will show that if f is a morphism of G into H then f is a morphism of $\mathbf{T}(G)$ into $\mathbf{T}(H)$. Thus, let $G = (G, \varrho, r)$, $H = (H, \sigma, s)$, $\mathbf{T}(G) = (G, t)$, $\mathbf{T}(H) = (H, u)$ and let $(x, y, z) \in t$. Then $(x, y) \in \varrho$, $(y, z) \in \varrho$ and $((x, y), (y, z)) \in r$. This implies $(f(x), f(y)) \in \sigma, (f(y), f(z)) \in \sigma, ((f(x), f(y)), (f(y), f(z))) \in s$ and hence $(f(x), f(y), f(z)) \in u$. Thus, $f: \mathbf{T}(G) \to \mathbf{T}(H)$ is a morphism. Cleraly, $\mathbf{T}(1_G) = 1_{\mathbf{T}(G)}$ and $\mathbf{T}(g \circ f) = \mathbf{T}(g) \circ \mathbf{T}(f)$. Thus, $\mathbf{T}: \mathscr{B} \to \mathscr{T}$ is a functor.

Analogously, $\mathbf{B}: \mathscr{T} \to \mathscr{B}$ is a functor, if for a morphism $f: G \to H$ in $\mathscr{T}$ we define the morphism $\mathbf{B}(f): \mathbf{B}(G) \to \mathbf{B}(H)$ by $\mathbf{B}(f) = f$. Since for a ternary structure G the double binary structure $\mathbf{B}(G)$ contains no isolated edges, the functor $\mathbf{B}$ is in fact a functor from $\mathscr{T}$ into $\mathscr{B}'$. Theorems 3.1 and 3.2 show that the functors $\mathbf{T}: \mathscr{B}' \to \mathscr{T}$ and $\mathbf{B}: \mathscr{T} \to \mathscr{B}'$ are isofunctors. Thus, we have proved

Theorem 4.1. *The categories $\mathscr{T}$ and $\mathscr{B}'$ are isomorphic. The functors $\mathbf{B}: \mathscr{T} \to \mathscr{B}'$ and $\mathbf{T}: \mathscr{B}' \to \mathscr{T}$ are isofunctors.*

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