Mathematica Bohemica

Book Reviews

Mathematica Bohemica, Vol. 123 (1998), No. 3, 332–336

Persistent URL: http://dml.cz/dmlcz/126072

123 (1998) MATHEMATICA BOHEMICA No. 3, 332–336

BOOK REVIEWS

P. K. Kythe: FUNDAMENTAL SOLUTIONS FOR DIFFERENTIAL OPERATORS AND APPLICATIONS. Birkhäuser, Basel 1996, 424 pages, DM 118,–.

Many problems in mathematical physics and applied mathematics can be reduced to boundary value problems for differential and in some cases, integrodifferential equations. These equations are solved by using methods from the theory of ordinary and partial differential equations, variational calculus, functional theory etc.

The main purpose of this book is to provide a self-contained and systematic introduction to just one aspect of analysis which deals with the theory of fundamental solutions for differential operators and their applications to boundary value problems of mathematical physics, applied mathematics and engineering, with the related computational aspects.

The application of fundamental solutions to a recently developed area of boundary element methods has provided a distinct advantage in that an integral equation representation of a boundary value problem is often more easily solved by numerical methods than a differential equation with specified boundary and initial conditions. Recent advances in the area of boundary element methods where the theory of fundamental solutions plays a pivotal role, have gained this subject a prominent place in research in partial differential equations and related boundary value problems. Computational components receive special attention throughout the book (Appendix B).

There are 14 chapters in the book. Chapter 1 and Appendix A are devoted to the theory of distributions, Dirac delta function and Fourier and Laplace transformations. In Chapters 2, 3, 4 the linear elliptic, parabolic and hyperbolic operators are described. Chapter 5 deals with nonlinear operators (Einsten-Kolmogorov, Fokker-Plank, Klein-Gordon, biharmonic, quasihyperbolic etc.)

Elastostatics operators are studied in Chapter 6. In Chapter 7 the elastodynamics operators are investigated (wave structure, problems of elastoplasticity, anisotropic media, etc.). The Navier-Stokes equations of fluid dynamics with emphasis on aerodynamic and non-Newtonian fluid flows, especially those dealing with the viscoelastic and power-law fluids are investigated in Chapter 8. Chapter 9 deals with statistics and dynamic piezoelectric operators and presents the piezocrystal wave theory. Chapters 10 and 11 cover the applications to the boundary element methods and domain integrals. The von Karman operator for finite deflection of plates is considered in Chapter 12; boundary integrals equations are derived and the problem of large deflection is investigated. Chapter 13 deals with miscellaneous topics, which include operators in poroelasticity, heat conduction, thermoelasticity, etc. Quasilinear elliptic operators and especially the p-Laplacian are studied in Chapter 14. The book offers an accesible and up-to date survey for advanced students, researchers and scientists in applied mathematics, engineering and physical sciences. This is the first book of its kind devoted exclusively to this subject. There are 70 different differential operators and derivation of the fundamental solutions.

Šárka Matušů, Praha

H. Holden, B. Øksendal, J. Ubøe, T. Zhang: STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS. A Modeling, White Noise Functional Approach. Probability and Its Applications, Birkhäuser, Boston 1996, x+230 pages, ISBN 0-8176-3928-4, price DM 118,–.

Stochastic partial differential equations (SPDEs) appear as models of distributed parameter systems subjected to a noise e.g. in physics, biology or finance. Their theory has been developing rapidly in the last two decades, the fundamental results being summarized in the recent monographs by G. Da Prato and J. Zabczyk (*Stochastic equations in infinite dimensions*, Cambridge Univ. Press, 1992) and B. L. Rozovskii (*Stochastic evolution systems*, Kluwer, 1990), devoted to the semigroup approach to SPDEs and the variational one, respectively. Nevertheless, the applicability of the standard theory to particular models is still limited. Restrictions on the differential operators and/or the noise are inevitable if solutions are required to be function-valued stochastic processes. Distribution-valued processes are available for linear equations, but in the nonlinear case difficulties with the interpretation of nonlinear terms arise. In the book under review, a different point of view is adopted, solutions to SPDEs being constructed in spaces of white noise functionals (stochastic distributions). Therefore, new classes of equations are covered by this theory and a new interpretation is provided for equations that have been already dealt with.

In Chapter 1, the authors motivate the theory treated in the book by discussing a free boundary problem for a stochastic pressure equation describing a flow of fluid injected into a porous rock. In Chapter 2, the reader finds a self-contained introduction to (the basic parts of) the white noise calculus. The Kondratiev spaces $(S)_\varrho^{m;N}$, $-1 \leqslant \varrho \leqslant 1$, are introduced and the Wick product $\diamond$ in $(S)_{-1}$ is thoroughly investigated. The importance of these notions can be seen e.g. from the formula

$$\int_{\mathbb{R}} Y(t)\,\delta B(t) = \int_{\mathbb{R}} Y(t) \diamond W(t)\,\mathrm{d}t,$$

which plays an important role in the sequel. Here the integral on the left hand side is the usual Skorokhod integral of a stochastic process Y with respect to a Brownian motion B, while on the right hand side a Pettis integral in the Hida space $(S)^*$ is considered, W denoting the white noise. Further, the Hermite transform is treated in detail. In the next two chapters, the above developed tools are used to solve ordinary and partial stochastic differential equations, respectively. The employed methods are based on the Hermite transforms, so the nonlinear terms in the equations are to be interpreted by means of the Wick products; on the other hand, the methods work for anticipative equations as well and in many cases lead to explicit formulae for the solution. Finally, a close relation between SPDEs driven by a Gaussian white noise and those driven by a Poissonian noise is established. A few auxiliary results (the Bochner-Minlos theorem, the Itô formula and the Girsanov theorem, properties of Hermite polynomials) are deferred to Appendices; furthermore, a rather technical proof of independence of the Wick product of the choice of basis is sketched in Appendix D. The book is amended with a useful list of frequently used notation and symbols, and with an index.

Although the book is, in fact, a research monograph, it has also many features of a textbook: it is carefully written, the proofs are sufficiently detailed, and many examples and exercises (with hints) are included. No preliminary knowledge of either white noise analysis or SPDEs is assumed. The book is of a considerable interest for everybody who works or wants to work in the theory of stochastic partial differential equations.

Bohdan Maslowski, Praha

R. Jeltsch; M. Mansour (eds.): STABILITY THEORY. Hurwitz Centenary Conference. Birkhäuser, Basel 1996, 260 pages, DM 148,-.

The present book is an outgrowth of the international conference *Centennial Hurwitz on Stability Theory* held in honor of Adolf Hurwitz on May 21–26, 1995 in Monte Verità in Switzerland.

The basic background of the topic is the paper of A. Hurwitz: Über die Bedingungen unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt, published in the Mathematische Annalen in 1895. This paper contains the famous Hurwitz criterion for a polynomial having roots with negative real parts only. In the appendix to the volume this fundamental paper of A. Hurwitz is reprinted.

A brief historical review of the state of the problem in the 19th century (Hermite, Routh, Lyapunov and Hurwitz) is presented in a paper of E. I. Jury.

The research papers presented in this collection concern stability theory, robust stability, control theory and some computational (numeric) aspects of the problem. Some open problems are given at the end of the book.

This volume is of interest to mathematicians as well as to engineers working in control theory, theory of electrical systems and numerical mathematics. The contemporary state of art in this classical field of mathematics and applications is given there in an exhaustive and nicely comprehensible way.

Štefan Schwabik, Praha

A. Böttcher; I. Gohberg (eds.): SINGULAR INTEGRAL OPERATORS AND RELATED TOPICS. Joint German-Israeli Workshop, Tel Aviv, March 1–10, 1995. Birkhäuser Basel 1996, 324 pages, DM 148,-.

This volume contains a selection of papers presented at the workshop mentioned in the title of the book.

Symbol calculus, index formulas, projection and quadrature methods for Toeplitz and singular integral operators, various algebras are the main topic of the volume.

Leading and extremely productive personalities (A. Böttcher, I. Gohberg, B. Silbermann and others) in this interesting and flourishing field of operator theory and its applications are presenting their results in the volume.

The book is oriented to a wide audience of mathematicians working in operator theory and of applied scientists.

Štefan Schwabik, Praha

R. H. Cushman; L. M. Bates: GLOBAL ASPECTS OF CLASSICAL INTEGRABLE SYSTEMS. Birkhäuser, Basel 1997, 452 pages, DM 78,-.

A complete global geometric description of the two dimensional harmonic oscillator, the Kepler problem, the Euler top, the spherical pendulum and the Lagrange top is given in this book.

The usual model integrable systems are analyzed from the point of view of their global geometric features. Mainly symmetries of the models are used to reduce the corresponding system to a system with one degree of freedom. This makes it possible to gain a deep topological and geometrical insight into the problems and teaches the reader to use sophisticated techniques concerning the topology of a fiber corresponding to a singular value of the energy–momentum mapping.

Examples form the basic approach to a general theory. A uniform scheme for treating the examples is followed in the text.

Besides the main topic five appendices of theoretical nature are included (symplectic algebra and manifolds, Hamilton equations, systems with symmetry, Ehresmann connections, local angle coordinates and monodromy and the fundamentals of Morse theory).

The book can be extremely valuable for specialists in global analysis because of its presentation of physically relevant examples and also for theoretical physicists.

Štefan Schwabik, Praha

J. A. Adam; N. Bellomo (eds.): A SURVEY OF MODELS FOR TUMOR-IMMUNE SYSTEM DYNAMICS. Birkhäuser, Basel 1996, 360 pages, DM 138,–.

The book is an interdisciplinary survey on constructing models for tumor dynamics and the interaction between tumors and the immune system of an organism.

The contents of the book is in the best way described by its chapters.

1. A Brief History of Immunologic Thinking: Is it Time for Yin and Yang? (B. D. Curti and D. L. Longo). The viewpoint of immunologists.

2. General Aspects of Modelling Tumor Growth and Immune Response. (J. A. Adam). A link between the viewpoint of biologists and applied mathematicians.

3. Mathematical Modelling of Tumor Growth Kinetics. (Ž. Bajzer, S. Vuk-Pavlovič, M. Huzak).

4. Tumor Immune System Interactions: The Kinetic Cellular Theory. (N. Bellomo, L. Preziosi, G. Forni).

5. From Mutation to Metastasis: The Mathematical Modelling of the Stages of Tumour Development. (M. A. J. Chaplain).

6. Basic Models of Tumor-Immune System Interactions. Identification, Analysis and Predictions. (V. A. Kuznetsov).

7. Tumor Heterogeneity and Growth Control. (S. Michelson, J. T. Leith)

A Biological Glossary helping mathematically oriented, but biologically disoriented readers by F. Cavallo and K. Boggio is appended as a valuable help at the end of the book.

This book is a very helpful tool for mathematicians constructing models in biology and hopefully it can help also biologists to understand some philosophy of mathematical description of biological phenomena with all its pluses and minuses.

Štefan Schwabik, Praha

R. P. Kanwal: LINEAR INTEGRAL EQUATIONS. Birkhäuser, Basel 1996, 328 pages, DM 148,–.

The book represents a nice introductory text for linear integral equations at the level of a beginners' graduate course.

After an introductory chapter concerning classification and basic concepts integral equations with separable kernels, methods of solving by successive approximation and the classical Fredholm theory are presented.

This is followed by some classical applications to ordinary and partial differential equations (boundary value problems, Green's functions, representation of the solution of the Laplace and Poisson equations, Helmholtz equation, ...).

Equations with symmetric kernel are described, singular integral equations with various types of the kernel are presented.

The method of integral transform is described and some other interesting applications are presented at the end of the book.

This volume presents the basics of linear integral equations theory in a very comprehensive way. The most valuable part of the book is in its richness in examples and applications. This makes the book extremely useful for teachers and also researchers using techniques of integral equations.

Štefan Schwabik, Praha

A. Borodin; P. Salminen: HANDBOOK OF BROWNIAN MOTION—FACTS AND FORMULAE. Birkhäuser, Basel 1996, DM 168,–.

The book consists of two parts. In Part One the authors explain (without proofs) the fundaments of the theory of Markov processes, martingales, onedimensional diffusion processes (local time, additive functionals, excessive functions, ergodic properties, stochastic integration over the Wiener process). In the chapter on stochastic calculus they present the Ito formula, Cameron-Martin-Girsan transformation, fundaments of the theory of stochastic differential equations. Another chapter is devoted to the Wiener process (Brown motion)—local time, the process of excursions, length of excursion, Brown's bridge, etc. Attention is also paid to Bessel processes.

Part Two contains tables of distributions and mean values of various functionals of the Wiener process (Bessel and other processes). This part is divided into seven sections, each of which consists of four subsections. The individual sections can be characterized as follows: 1. Wiener process, 2. Wiener process with drift, 3. Wiener process with reflecting barrier, 4. Bessel process of zero order, 5. Bessel process of order 1/2, 6. Bessel process of order $\nu > 0$, and 7. Ornstein-Uhlenbeck process. The subsections are ordered according to the following topics: 1. exponential moment of the stopping time, 2. stopping at first hitting time, 3. stopping at the first exit time, 4. stopping at the inverse local time.

I consider the book very useful since it can save much time and effort when looking for the required notions and formulas.

Ivo Vrkoč, Praha

A. Bellaïche, J.-J. Risler (eds.): SUB-RIEMANNIAN GEOMETRY. Progress in Mathematics 144, Birkhäuser, Basel 1996, viii+393 pages, ISBN 3-7643-5476-3, DM 108,–.

This collection represents a kind of proceedings of a satellite meeting of the first European Congress of Mathematics with the title "Journées nonholonomes, Géométrie sous-riemannienne, théorie du contrôle, robotique", which was held at the Université Paris VI on June 30th and July 1st, 1992. It is devoted to the sub-riemannian geometry which is also called Carnot geometry in France and nonholonomic geometry in Russia. It contains the followimg 5 articles: *André Bellaïche*: The tangent space in sub-riemannian geometry (78 pp.), *Mikhael Gromov*: Carnot-Carathéodory spaces seen from within (245 pp.), *Richard Montgomery*: Survey of singular geodesics (15 pp.), *Héctor J. Sussmann*: A cornucopia of four-dimensional abnormal sub-riemannian minimizers (24 pp.) and *Jean-Michel Coron*: Stabilization of controllable systems (24 pp.). It is a compact collection (provided even with an index) of very clearly written articles some parts of which can serve as a good introduction to the subject while the main parts of them lead the reader to the contemporary level of the research and contain many new results. The collection seems to be quite indispensable for every specialist in the field.

Jiří Vanžura, Brno