Book Reviews

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BOOK REVIEWS

F. Schipp, W. R. Wade, P. Simon, with assistance from J. Pál: WALSH SERIES. AN INTRODUCTION TO DYADIC HARMONIC ANALYSIS. Akadémiai Kiadó, Budapest 1990, x + 560 pp.

Let us denote by r_n, $n \in \mathbb{N}$, the Rademacher functions, that is,

$$r_n(x) = \operatorname{sign} \sin 2^n \pi x, \qquad x \in [0,1),$$

and define the Walsh (Walsh-Paley) functions in the following way: if $n \in \mathbb{N}$ has a binary expansion $n = \sum_{k=0}^{\infty} n_k 2^k$, $n_k \in \{0,1\}$, then we set

$$w_n = \prod_{k=0}^{\infty} r_k^{n_k}.$$

The orthogonal system $\{w_n,\ n \in \mathbb{N}\}$ obtained is of considerable interest from both the applied (e.g. data transmission, pattern recognition) and theoretical (e.g. the construction of Schauder bases in function spaces) points of view. The treatise under review aims at providing a thorough and detailed information about the Walsh-Fourier analysis, concentrating upon the theoretical aspects. Let us describe the contents of the book briefly. In the first two chapters some fundamental facts about the Walsh system, its identification with the characters of the dyadic group $\{0,1\}^{\mathbb{N}}$, and about the basic properties of Walsh-Fourier coefficients are presented. In the third chapter, the probabilistic techniques (dyadic martingales and martingale transforms, dyadic Hardy spaces) essential for authors' approach to the topic are developed; the chapter culminates with the proof of almost everywhere convergence of the Walsh-Fourier series of an L^p-function, $p > 1$. The convergence problem is investigated further in Chapters 4 and 6, both the convergence in norm and the almost everywhere convergence and summability being addressed. In the fifth chapter, two main topics are dealt with: the approximation by Walsh polynomials and the construction and equivalence of bases in various Banach spaces (L^p-spaces, Hardy spaces, VMO etc.) by means of Walsh and closely related systems (Haar, Schauder, Franklin functions). Moreover, the results obtained are used for a construction of a separable Banach space without the compact approximation property. In Chapters 7 and 8 some problems related to general Walsh series are considered (the uniqueness problem, representation of an arbitrary measurable function by a Walsh series). The final chapter is about the Walsh-Fourier transform. In seven appendices, some results on interpolation of operators, locally compact abelian groups etc., which are frequently used in the text, are recalled.

The monograph is written in a careful manner and is amended by detailed historical notes, a rich bibliography, and by author, subject and notation indices, which facilitates its use as a reference book.

Jan Seidler, Praha

H. M. Edwards: LINEAR ALGEBRA. Birkhäuser, Basel, 1995, xiii + 184 pages, price sFr 58,–.

This undergraduate textbook is based on a purely algorithmic approach to linear algebra, the theory of matrices.

The book consists of the following chapters: Matrix multiplicaton, Equivalence of matrices, Matrix division, Determinants, Testing of equivalence, Matrices with rational number entries, The method of least squares, Matrices with polynomial entries, Similarity of matrices, The spectral theorem, and an appendix on linear programming.

The philosophy of the author is algebraic, based mainly on avoiding the concept of real numbers; the first five chapters deal purely with matrices with integer entries, rational entries are allowed in the sixth chapter and reals are unavoidable in the chapter on the spectral theorem. The statements given in the book are mainly algorithms which can be easily translated into computer language.

The book is full of instructive examples and exercises. Some general algebraic concepts are given in supplementary units to the chapters on determinants, tests of equivalence and matrices with rational entries.

This textbook represents in some sense an experiment showing that the basic concepts of linear algebra can be presented without set theory and other things disturbing the pure theory. It is a challenge to teachers to try this approach.

Štefan Schwabik, Praha

Tamer Basar, Pierre Bernhard: H^∞-OPTIMAL CONTROL AND RELATED MINIMAX DESIGN PROBLEMS. (A Dynamic Game Approach), 224 pages, Birkhäuser, Boston, Basel, Berlin, 1991.

The H^∞-optimization presents in the control theory a new very important approach to the problems of controller's design. Initiated in 1981 with the paper by George Zames this approach dominated the control theory research in the 1980's. It is based on the idea of worst-case design, originally in the frequency domain. The controller obtained minimizes the maximum H^∞ norm of the input-output operator, where the maximum is taken over the unknowns, such as disturbances. The symbol H^∞ stands for the Hardy space of all complex functions of a complex variable that are analytic and bounded in the open complex right-half plane. For a linear time invariant continuous time plant the H^∞ norm of the transfer matrix is the maximum of its largest singular values over all frequencies.

Great part of the book under review is dedicated to the relationship between the H^∞-optimal control theory and the differential game-theoretic approach. Beginning with the linear-quadratic zero sum dynamic game theory, the authors proceed by applying its results to different H^∞-optimal control problems, both for continuous and discrete time, for finite and infinite horizon.

The book consists of preface, seven chapters, two appendices, references and the list of corollaries, definitions, lemmas, propositions, remarks and theorems.

Chapter 1 shortly introduces the reader into the problem formulations and the notation. Chapter 2 presents some of the most important results from zero-sum static and dynamic game theory. Chapter 3 is dedicated to the discrete-time linear-quadratic dynamic games and to the use of the corresponding results to the discrete-time H^∞-optimal control problem with perfect or delayed state information, for both finite and infinite time horizon. Analogous problems for continuous-time systems are studied in Chapter 4. Systems with imperfect information about the state of the system are investigated in Chapter 5 (continuous-time systems) and in Chapter 6 (discrete-time systems). Chapter 7 discusses

some problems in filtering and smoothing. The authors study here the minimax estimators and their connection with Kalman filters.

Some of the mathematical results used are explained in two appendices, denoted as Chapter 8 "Conjugate Points" and Chapter 9 "Danskin's Theorem". The list of references contains 93 items.

The book, based mostly on recent work of the authors, is written on a good mathematical level. Many results in it are original, interesting and inspirative. The explanation is mostly self-contained, only some experience and knowledge of basic topics, such as LQ-theory, matrix Riccati equations, Wiener process, Kalman filter etc. is needed. The book can be recommended to speci. lists and graduate students working in the development of control theory or using modern methods for controller design.

Antonín Tuzar, Praha

Theodore W. Palmer: BANACH ALGEBRAS AND GENERAL THEORY OF $*$-ALGEBRAS. Volume I. Algebras and Banach algebras, Encyclopedia of Mathematics and its Applications, Vol. 49, Cambridge Univ. Press 1994.

The book is the first part of a two volume monograph devoted to Banach algebras and $*$-agebras.

This first volume gives a detailed (almost 800 pages) survey of the theory of Banach algebras. The topics are usually treated in maximal possible generality so that many results are stated and proved rather for some classes of algebras than for Banach algebras. The proofs are very detailed and no special knowledge is assumed.

The book contains also many results from the classical analysis that illustrate the theory, and an extensive bibliography. Especially interesting are numerous historical remarks and comments which appear in such an extend, as far as I know, for the first time.

On the other hand, the generality of approach does not always contribute to the clarity of the presentation. A little bit confusing is also the non-standard notation (for example the letter σ denotes a semi-norm and not the spectrum which is usual in the theory of Banach algebras). Also it is rather unusual for a textbook about Banach algebras to treat in the very first sections topics like the Arens multiplication or double centralizers.

Vladimír Müller, Praha

C. B. Huijsmans, M.A. Kaashoek, W.A.J. Luxemburg, B. de Pagter: OPERATOR THEORY IN FUNCTION SPACES AND BANACH LATTICES. Operator Theory: Advances and Applications, OT 75, Birkhäuser, Basel, 1994, 309 pages, DM 128,–.

The book is dedicated to A. C. Zaanen on the occasion of his 80th birthday.

Professor Zaanen is one of the most important experts in the theory of vector lattices and modern integration theory. He significantly influenced the development of functional analysis by his famous book Linear Analysis which was published in 1953.

Apart from the biographical notes the book contains a selection of original research papers. Special attention is paid to the spectral theory of operators in Banach lattices, but other papers cover a wide area of functional analysis and operator theory. Among the topics studied in the book there are the invariant subspace problem, system theory, optimization and best approximation problems, evolution equations, function spaces and integration theory.

Vladimír Müller, Praha

R.W.R. Darling: DIFFERENTIAL FORMS AND CONNECTIONS. Cambridge University Press 1994, ISBN 0-521-46259-2 (hardback), ISBN 0-521-46800-0 (paperback).

A concise and self-contained introduction to the theory of differential forms on smooth manifolds. The first two chapters contain basic definitions (exterior algebra and differential forms) which are then used to introduce rudiments of differential geometry (manifold, frame, riemannian metric, bundle). The next chapters are devoted to the integration of forms, including the proof of the Stokes theorem (but leaving aside the cohomological background). The last two chapters introduce connections on vector bundles and give some applications—gauge field theory, Yang-Mills theory, instantons.

The exposition is directed towards graduate students in mathematics, physics and engineering. Many examples are given to illustrate the text.

Martin Markl, Praha

Marc Aubry: HOMOTOPY THEORY AND MODELS. Based on lectures held at a DMV Seminar in Blaubeuren by H. J. Baues, S. Halperin and J.-M. Lemaire, Birkhäuser Verlag, 1995, 128 pages, ISBN 3-7643-5185-3 (Basel), ISBN 0-8176-5185-3 (Boston), 48,– DM.

The book provides a comprehensive introduction to rational homotopy theory. It begins with a brief recollection of fundamental notions of algebraic topology (homotopy and homology, (co)fibrations). Then the basic categorial framework – cofibration categories – is introduced and some relevant examples are discussed. The next chapters are devoted to rational homotopy theory of simply connected topological spaces. Two basic models (Sullivan and Quillen) are introduced and their Eckmann-Hilton-type relationship is discussed.

As an application, the celebrated dichotomy between elliptic and non-elliptic spaces is proved and some results about the algebraic Lusternik-Schnirelmann category are proven. The last chapter is devoted to integral computations aiming at an algebraic description of homotopy types of low dimensional CW complexes due to H. J. Baues.

A moderate size of the book allows to give only a few complete proofs, but the relevant bibliographic references are included.

Martin Markl, Praha

Julian L. Davis: WAVE PROPAGATION IN SOLIDS AND FLUIDS. Springer-Verlag, New York-Berlin-Heidelberg-London-Paris-Tokyo, 1988, X+386 stran, cena DM 148,–.

Kniha je věnována matematickému popisu šíření vln v pevných látkách, kapalinách a plynech. Jsou zde odvozeny matematické metody, které umožňují vyšetřování šíření vln, včetně nelineárních efektů a asymptotických vlastností, z různých fyzikálních hledisek: zvukové a rázové vlny v plynech, vlny na vodě i tlakové vlny v pevných látkách.

V různých částech knihy jsou popsány v odpovídajícím fyzikálním kontextu tři základní zákony zachování, ze kterých jsou odvozeny příslušné soustavy parciálních diferenciálních rovnic, v daném případě obecně kvazilineárních hyperbolických rovnic. Vyšetřování těchto rovnic je založeno na metodě chrakteristik. Knihu uzavírá kapitola týkající se variačních metod a aplikace Hamiltonovy-Jacobiho teorie na šíření vln. Je zde ukázáno, že tato teorie přirozeným způsobem překlenuje mezeru mezi klasickou mechanikou a geometricku optikou.

Kniha je svou povahou interdisciplinární, je určena matematikům, fyzikům a inženýrům zabývajícím se uvedenou tematikou.

Hana Petzeltová, Praha