ON ALMOST QUASICONTINUOUS FUNCTIONS

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Summary. A function $f\colon X \to Y$ is said to be almost quasicontinuous at $x \in X$ if $x \in \operatorname{Cl}\operatorname{Int}\operatorname{Cl} f^{-1}(V)$ for each neighbourhood V of $f(x)$. Some properties of these functions are investigated.

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Let X and Y be topological spaces. For a subset A of a topological space denote $\operatorname{Cl} A$ and $\operatorname{Int} A$ the closure and the interior of A, respectively. The letters $\mathbb{N}$, $\mathbb{Q}$ and $\mathbb{R}$ stand for the set of natural, rational and real numbers, respectively.

A set A is called ***semi-open*** [8] (quasi-open [11]), if $A \subset \operatorname{Cl}\operatorname{Int} A$, ***pre-open*** [10] (nearly open [18]), if $A \subset \operatorname{Int}\operatorname{Cl} A$, ***$\beta$-open*** [1] (semi-preopen [2]), if $A \subset \operatorname{Cl}\operatorname{Int}\operatorname{Cl} A$, ***somewhat nearly open*** [18], if $\operatorname{Int}\operatorname{Cl} A \neq \emptyset$.

Let $f\colon X \to Y$ be a function and $x \in X$. A function f is called ***quasicontinuous at x*** [9], if $x \in \operatorname{Cl}\operatorname{Int} f^{-1}(V)$, ***almost continuous at x*** [5] (nearly continuous at x [18]), if $x \in \operatorname{Int}\operatorname{Cl} f^{-1}(V)$, ***almost quasicontinuous at x*** [3], [15], if $x \in \operatorname{Cl}\operatorname{Int}\operatorname{Cl} f^{-1}(V)$, for each neighbourhood V of $f(x)$.

A function $f\colon X \to Y$ is quasicontinuous (almost continuous, almost quasicontinuous), if it is such at every point. A function f is called semi-continuous [8] (pre-continuous [10], β-continuous [1]), if $f^{-1}(V)$ is semi-open (pre-open, β-open) for each open set V in Y. A function f is somewhat continuous [6] (somewhat nearly continuous [18]), if $\operatorname{Int} f^{-1}(V) \neq \emptyset$ ($f^{-1}(V)$ is somewhat nearly open) for each open V in Y such that $f^{-1}(V) \neq \emptyset$. Evidently, f is pre-continuous iff f is almost continuous and f is semi-continuous iff f is quasicontinuous [14].

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The notion of almost quasicontinuity is a simultaneous generalization of almost continuity and of quasicontinuity. Properties of almost quasicontinuous functions are studied in [1], [3], [15], [16]. In this paper we shall show further properties of these functions. We also give answers to three Piotrowski's questions.

Immediately we see that f is almost quasicontinuous if and only if it is β-continuous. This is also true "pointwise".

Theorem 1. *Let $f\colon X \to Y$ and $x \in X$. Then the following conditions are equivalent:*

(1) *f is almost quasicontinuous at x,*
(2) *for each neighbourhood V of $f(x)$ and each neighbourhood U of x, $f^{-1}(V) \cap U$ is not a nowhere dense set,*
(3) *for each neighbourhood V of $f(x)$ there is a β-open set U such that $x \in U$ and $f(U) \subset V$.*

Proof. We shall prove (2) $\Rightarrow$ (3). Other implications are obvious.

Let V be a neighbourhood of $f(x)$. Then for each neighbourhood U of x there is a nonempty open set $G_U \subset U$ such that $G_U \subset \operatorname{Cl} f^{-1}(V)$. Denote $H_U = G_U \cap f^{-1}(V) \neq \emptyset$. Let $H = \bigcup\{H_U\colon U \text{ is a neighbourhood of } x\}$. Then $x \in H$ and $f(H) \subset V$. Let $z \in \operatorname{Cl} G_U$ and let T be an open neighbourhood of z. Then $T \cap G_U$ is a nonempty open set. Let $u \in T \cap G_U$. Then $u \in \operatorname{Cl} f^{-1}(V)$ and hence $\emptyset \neq (T \cap G_U) \cap f^{-1}(V) = H_U \cap T$. This yields $z \in \operatorname{Cl} H_U$ and $\operatorname{Cl} G_U \subset \operatorname{Cl} H_U$. Since evidently $\operatorname{Cl} H_U \subset \operatorname{Cl} G_U$, we have $\operatorname{Cl} G_U = \operatorname{Cl} H_U$. Hence for each neighbourhood U of x we have $H_U \subset G_U \subset \operatorname{Int}\operatorname{Cl} G_U = \operatorname{Int}\operatorname{Cl} H_U \subset \operatorname{Int}\operatorname{Cl} H$.

Let $y \in H$. If $y \neq x$, then there is a neighbourhood U of x such that $y \in H_U$. Then $y \in \operatorname{Cl}\operatorname{Int}\operatorname{Cl} H$. If $y = x$ and U is a neighbourhood of x, then $\emptyset \neq H_U \subset U \cap \operatorname{Int}\operatorname{Cl} H$ and hence $x \in \operatorname{Cl}\operatorname{Int}\operatorname{Cl} H$. Therefore H is a β-open set. □

Evidently, every almost quasicontinuous function is somewhat nearly continuous.The converse is not true; however, we have

Proposition 1. *A function $f\colon X \to Y$ is almost quasicontinuous if and only if there is a base $\mathscr{B}$ of the space X such that $f|_B$ is somewhat nearly continuous for each $B \in \mathscr{B}$.*

Proof. Necessity follows from the obvious fact that the restriction of an almost quasicontinuous function to an open subspace is almost quasicontinuous.

Sufficiency. Let $x \in X$, let U be an open neighbourhood of $f(x)$ and let V be an open neighbourhood of x. Let $B \in \mathscr{B}$ be such that $x \in B \subset U$. Then $(f|_B)^{-1}(V) \neq \emptyset$ and hence $\emptyset \neq \operatorname{Int}\operatorname{Cl}(f|_B)^{-1}(V) \subset \operatorname{Int}\operatorname{Cl} f^{-1}(V) \cap \operatorname{Int}\operatorname{Cl} B$. From this we get $\operatorname{Int}\operatorname{Cl} f^{-1}(V) \cap B \neq \emptyset$ and hence $x \in \operatorname{Cl}\operatorname{Int}\operatorname{Cl} f^{-1}(V)$. □

Proposition 1 shows that a relation between almost quasicontinuity and somewhat nearly continuity is similar to that between quasicontinuity and somewhat continuity (see [12]). Next proposition shows a similar relation between almost quasicontinuity and almost continuity and between quasicontinuity and continuity (see [11]).

Proposition 2. *Let X be a first countable Hausdorff space and let Y be a first countable space. Let $x \in X$. Then $f: X \to Y$ is almost quasicontinuous at x if and only if there is a semi-open set A containing x such that $f|_A$ is almost continuous at x.*

Proof. Necessity. If $\{x\}$ is an open set, then we choose $A = \{x\}$. Let $\{x\}$ be not open, let (V_n) be a nonincreasing base of neighbourhoods of $f(x)$ and (U_n) a nonincreasing base of neighbourhoods of x. Then there is a nonempty open set $G_1 \subset U_1$ such that $G_1 \subset \mathrm{Cl}\, f^{-1}(V_1)$. Evidently $G_1 \neq \{x\}$. Since X is Hausdorff, there is $n_2 > 1$ such that $G_1 - \mathrm{Cl}\, U_{n_2} \neq \emptyset$. Further there is an open nonempty set $G_2 \subset U_{n_2}$ such that $G_2 \subset \mathrm{Cl}\, f^{-1}(V_2)$. In this way, we construct an increasing sequence (n_k) of natural numbers (where $n_1 = 1$) and a sequence (G_k) of nonempty open sets such that $G_k \subset U_{n_k}$, $G_k \subset \mathrm{Cl}\, f^{-1}(V_k)$ and $G_k - \mathrm{Cl}\, U_{n_{k+1}} \neq \emptyset$. Denote $A = \bigcup_{k=1}^{\infty} (G_k - \mathrm{Cl}\, U_{n_{k+1}}) \cup \{x\}$. Then A is a semi-open set containing x. Since for each $i \in \mathbf{N}$ we have $A \cap U_{n_i} \subset \mathrm{Cl}\, f^{-1}(V_i)$, $f|_A$ is almost continuous at x.

Sufficiency. Let U and V be open neighbourhoods of x and $f(x)$, respectively. Then there is an open neighbourhood H of x such that $A \cap H \subset \mathrm{Cl}(f|_A)^{-1}(V) \subset \mathrm{Cl}\, f^{-1}(V)$. Since $x \in \mathrm{Cl}\,\mathrm{Int}\, A$, $G = \mathrm{Int}\, A \cap H \cap U$ is a nonempty open set and $G \subset U \cap \mathrm{Cl}\, f^{-1}(V)$. □

Remark 1. It is shown in [15] that almost quasicontinuous functions are closed with respect to uniform convergence. This is not true for pointwise convergence. In fact, every function $f: \mathbf{R} \to \mathbf{R}$ is a sum of two almost quasicontinuous functions and a limit of a sequence of almost quasicontinuous functions. By [4; p. 5] we can write $f = g + h$, where g and h are Darboux functions such that $g^{-1}(c)$ and $h^{-1}(c)$ are dense sets for each $c \in \mathbf{R}$. Similarly, we can write $f = \lim_{n\to\infty} f_n$, where f_n are Darboux functions such that $f_n^{-1}(c)$ are dense sets for each $c \in \mathbf{R}$. Evidently, g, h, f_n are almost quasicontinuous functions.

Remark 2. There is a Darboux function, which is not almost quasicontinuous. By [4; p. 13] there is a Darboux function f which is zero on the complement of the Cantor set, but not identically zero. This function is not almost quasicontinuous.

A subset A of X is called β-closed [1] (semi-preclosed [2]), if $X - A$ is β-open, i.e. if $\mathrm{Int}\,\mathrm{Cl}\,\mathrm{Int}\, A \subset A$. We say that a function $f: X \to Y$ has a β-closed graph if the graph of f, i.e. the set $G(f) = \{(x,y) \in X \times Y : y = f(x)\}$ is a β-closed subset of the product $X \times Y$.

Proposition 3. *Let Y be a Hausdorff space and let $f: X \to Y$ be an almost quasicontinuous function. Then f has a β-closed graph.*

Proof. Let $(x,y) \in X \times Y - G(f)$. Then there are disjoint open sets A_{xy} and B_{xy} in Y such that $f(x) \in A_{xy}$ and $y \in B_{xy}$. The almost quasicontinuity of f gives that $f^{-1}(A_{xy})$ is a β-open set in X. It is easy to see that $f^{-1}(A_{xy}) \times B_{xy}$ is a β-open set in $X \times Y$ and by [2] the set $T = \cup\{f^{-1}(A_{xy}) \times B_{xy} : (x,y) \in X \times Y - G(f)\}$ is β-open in $X \times Y$. We see that $X \times Y - G(f) = T$ and hence $G(f)$ is β-closed. □

Obviously, the converse assertion is not true. Denote by B_f the set of all almost quasicontinuity points of f. We characterize this set.

Lemma 1. *(See also [15].) Let Y be a second countable space. Let $f: X \to Y$. Then $X - B_f$ is a set of the first category.*

Lemma 2. *Let Y be a first countable Hausdorff space which has at least one accumulation point. Let $A \subset X$ be a set such that $X - A$ is a set of the first category. Then there is a function $f: X \to Y$ such that $B_f = A$.*

Proof. We can write $X - A = \bigcup_{n=1}^{\infty} A_n$, where A_n are nowhere dense pairwise disjoint sets. Let y_0 be an accumulation point of Y and let $\{y_n : n \in \mathbb{N}\}$ be a one-to-one sequence converging to y_0 such that $y_n \neq y_0$ for each $n \in \mathbb{N}$. Define a function $f: X \to Y$ as

$$f(x) = \begin{cases} y_n, & \text{for } x \in A_n, \\ y_0, & \text{for } x \in A. \end{cases}$$

We shall show that $B_f = A$. Let $x \in A$ and let V be a neighbourhood of $f(x) = y_0$. Then there is a finite set $K \subset \mathbb{N}$ such that $f^{-1}(V) = X - \bigcup_{i \in K} A_i$. Therefore $f^{-1}(V)$ is dense in X and $x \in B_f$.

Let $x \in A_n$ for some $n \in \mathbb{N}$. Let S and T be disjoint neighbourhoods of y_0 and y_n, respectively. Then there is a finite set $K \subset \mathbb{N}$ such that $y_i \in S$ for each $i \in \mathbb{N} - K$. Therefore $T \cap f(X) \subset \bigcup_{i \in K} \{y_i\}$ and $f^{-1}(T) \subset \bigcup_{i \in K} A_i$. This yields $x \notin B_f$. □

The condition Y is Hausdorff cannot be replaced by Y is T_1 as the following example shows.

Example 1. Let $X = \mathbb{Q}$ with the usual topology. Let $Y = \mathbb{N}$ and let a set $S \subset Y$ be closed if S is a finite set or $S = \mathbb{N}$. Then Y is a first countable T_1-space without isolated points and $X - \emptyset$ is a set of the first category. Let $f\colon X \to Y$ be an arbitrary function. We shall show that $B_f \neq \emptyset$. We have two possibilities.

a) There is $y \in Y$ such that $f^{-1}(y)$ is not nowhere dense. Then there is a nonempty open set G such that $G \subset \mathrm{Cl}\, f^{-1}(y)$. Let $x \in G \cap f^{-1}(y)$, let V be a neighbourhood of $f(x)$ and let U be a neighbourhood of x. Then $f^{-1}(V) \cap U$ is dense in $G \cap U$ and hence $x \in B_f$.

b) For each $y \in Y$ the set $f^{-1}(y)$ is nowhere dense. Then for each nonempty open set V in Y the set $f^{-1}(Y - V)$ is nowhere dense and hence $G \cap f^{-1}(V)$ is nowhere dense for no nonempty open set G in X. Therefore $B_f = X$.

Theorem 2. *Let X be a topological space and let Y be a second countable Hausdorff space which has at least one accumulation point. Let $A \subset X$ be a set. Then $X - A$ is of the first category if and only if there is a function $f\colon X \to Y$ such that $A = B_f$.*

Similarly as almost quasicontinuity we may define "almost cliquishness".

Definition 1. Let (Y, d) be a metric space. We say that a function $f\colon X \to Y$ is almost cliquish at $x \in X$, if for each $\varepsilon > 0$ and for each neighbourhood U of x there is a nonempty open set $G \subset U$ and a set H such that H is dense in G and $d(f(y), f(z)) < \varepsilon$ for each $y, z \in H$. Denote by Z_f the set of all almost cliquishness points of f. If $Z_f = X$, we say that f is almost cliquish.

Easy we see that Z_f is a closed set and $B_f \subset Z_f$. Hence by Lemma 1 we have

Proposition 3. *Let X be a Baire space and let (Y, d) be a separable metric space. Then every function $f\colon X \to Y$ is almost cliquish.*

We recall that a family $\mathscr{A}$ of nonempty open sets in X is a pseudo-base [17] if every nonempty open subset of X contains some member of A. (The space $\beta\mathbb{N}$ has a countable pseudo-base, but it is not second countable [17]). For a function $f\colon X \times Y \to Z$ the symbols f_x, f^y denote its x-section or y-section, respectively, i.e. f_x is the function defined on Y such that $f_x(y) = f(x, y)$ for each $x \in X$ and analogically f^y.

We shall show that there is a function $f\colon \mathbb{R}^2 \to \mathbb{R}$, which is separately almost quasicontinuous but not almost quasicontinuous. However, the following statement is true

Theorem 3. *Let X be a Baire space, let Y possess locally a countable pseudo-base and let Z be an arbitrary topological space. Let $f\colon X \times Y \to Z$ be such that f^y is quasicontinuous for each $y \in Y$ and f_x is almost quasicontinuous with the exception of a set of the first category. Then f is almost quasicontinuous.*

Proof. Suppose that f is not almost quasicontinuous. Then there is a point $(a,b) \in X \times Y$ and open neighbourhoods G, U and V of $f(a,b)$, a and b, respectively, such that

$$(\star) \qquad \operatorname{Int} \operatorname{Cl} f^{-1}(G) \cap (U \times V) = \emptyset.$$

Without loss of generality we may assume that $\{V_n : n \in \mathbb{N}\}$ is a countable pseudobase in V. The quasicontinuity of f^b at a gives

$$A = \operatorname{Int}(f^b)^{-1}(G) \cap U \neq \emptyset.$$

Let $T = \{x \in A : f_x \text{ is almost quasicontinuous}\}$ and
$T_n = \{x \in T : V_n \subset \operatorname{Int} \operatorname{Cl}(f_x)^{-1}(G)\}$.

We shall show that $T = \bigcup_{n=1}^{\infty} T_n$. If $x \in T$, then $x \in A$ and hence $f^b(x) \in G$. Therefore $b \in (f_x)^{-1}(G) \cap V$ and the almost quasicontinuity of f_x at b gives $b \in \operatorname{Cl} \operatorname{Int} \operatorname{Cl}(f_x)^{-1}(G)$ and this yields $\operatorname{Int} \operatorname{Cl}(f_x)^{-1}(G) \cap V \neq \emptyset$. Hence there is $n \in \mathbb{N}$ such that $V_n \subset \operatorname{Int} \operatorname{Cl}(f_x)^{-1}(G)$ and $x \in T_n$.

We shall prove that T_n is nowhere dense in A for each $n \in \mathbb{N}$. Let $n \in \mathbb{N}$ and let $S \subset A$ be an open set. Then, in regard of $(\star)$, there is a nonempty open set $K \subset S \times V_n$ such that $K \cap f^{-1}(G) = \emptyset$. We may assume that $K = K_1 \times K_2$, where $K_1 \subset S$ and $K_2 \subset V_n$ are nonempty open sets.

Let $x \in K_1$ and $y \in K_2$. Then $f(x,y) \notin G$ and thus $y \notin (f_x)^{-1}(G)$. This is true for each $y \in K_2$ and therefore $K_2 \cap (f_x)^{-1}(G) = \emptyset$. This yields $K_2 \cap \operatorname{Cl}(f_x)^{-1}(G) = \emptyset$ and therefore V_n is not a subset of $\operatorname{Int} \operatorname{Cl}(f_x)^{-1}(G)$. This is true for each $x \in K_1$ and therefore $K_1 \cap T_n = \emptyset$, i.e. T_n is nowhere dense in A. Then T is of the first category, a contradiction. □

Example 2. There is a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

(i) functions f_x, f^y are continuous with the exception of a set of the first category,
(ii) functions f_x, f^y are almost continuous,
(iii) the function f is not somewhat nearly continuous.

Let $\{t_n : n \in \mathbb{N}\}$ be a dense set in $\mathbb{R}^2$ such that $t_n = (p_n, q_n)$, where p_n and q_n are irrational numbers for each $n \in \mathbb{N}$. Let $\{u_n : n \in \mathbb{N}\}$, $\{v_n : n \in \mathbb{N}\}$ be one-to-one sequences of all rational numbers. Denote
$P_n = \{(x,y) \in \mathbb{R}^2 : y = v_n\}$ and
$Q_n = \{(x,y) \in \mathbb{R}^2 : x = u_n\}$.

Since $t_1 \notin P_1 \cup Q_1$, there is an open set $V_1 = (a_1, b_1) \times (c_1, d_1)$ such that a_1, b_1, c_1, d_1 are irrational numbers, $t_1 \in V_1$ and $V_1 \cap (P_1 \cup Q_1) = \emptyset$. Suppose that we have open sets $V_1, \ldots, V_k$ such that $V_j = (a_j, b_j) \times (c_j, d_j)$, where a_j, b_j, c_j, d_j are irrational numbers, $t_j \in V_j$ and $V_j \cap \left(\bigcup_{i=1}^{j} P_i \cup \bigcup_{i=1}^{j} Q_i \right) = \emptyset$ for each $j \in \{1, 2, \ldots, k\}$.

Since $t_{k+1} \notin \bigcup_{i=1}^{k+1} P_i \cup \bigcup_{i=1}^{k+1} Q_i$, there is an open set $V_{k+1} = (a_{k+1}, b_{k+1}) \times (c_{k+1}, d_{k+1})$, where $a_{k+1}, b_{k+1}, c_{k+1}, d_{k+1}$ are irrational numbers, such that $t_{k+1} \in V_{k+1}$ and $V_{k+1} \cap \left(\bigcup_{i=1}^{k+1} P_i \cup \bigcup_{i=1}^{k+1} Q_i \right) = \emptyset$.

Denote $T = \bigcup_{n=1}^{\infty} V_n$. Then T is an open dense set and hence $\mathbf{R}^2 - T$ is a nonempty nowhere dense set. Define a function $f\colon \mathbf{R}^2 \to \mathbf{R}$ as

$$f(x, y) = \begin{cases} 1, & \text{for } (x, y) \in \mathbf{Q} \times \mathbf{Q} - T, \\ 0, & \text{otherwise.} \end{cases}$$

The function f satisfies (i), (ii) and (iii).

In [18] there are three following questions:

Let X be a Baire space, let Y be a second countable space and let Z be a metric space. Let $f\colon X \times Y \to Z$ be a function such that

(α) f is separately somewhat continuous or

(β) f is separately almost continuous or

(γ) f is separately somewhat nearly continuous.

Must f be jointly somewhat nearly continuous?

The example 2 shows that the answer is negative in the cases (β) and (γ). Now we shall show that the answer is positive in the case (α).

Theorem 4. *Let X be a Baire space, let Y possess a countable pseudo-base and let Z be arbitrary topological space. Let $f\colon X \times Y \to Z$ be such that f^y is somewhat continuous for each $y \in Y$ and f_x is somewhat continuous with the exception of a set of the first category. Then f is somewhat nearly continuous.*

Proof. Suppose that f is not somewhat nearly continuous. Then there is an open set G in Z such that $f^{-1}(G) \neq \emptyset$ and $\operatorname{Int} \operatorname{Cl} f^{-1}(G) = \emptyset$. Let $\{V_n : n \in \mathbf{N}\}$ be a countable pseudo-base in Y. Let $(a, b) \in f^{-1}(G)$. Since $(f^b)^{-1}(G) \neq \emptyset$, the somewhat continuity of f^b gives $A = \operatorname{Int}(f^b)^{-1}(G) \neq \emptyset$. Denote

$T = \{x \in A\colon f_x \text{ is somewhat continuous}\}$ and

$T_n = \{x \in T\colon V_n \subset \operatorname{Int}(f_x)^{-1}(G)\}$.

We shall show that $T = \bigcup_{n=1}^{\infty} T_n$. Let $x \in T$. Then $f^b(x) \in G$ and $b \in (f_x)^{-1}(G)$. This yields $\operatorname{Int}(f_x)^{-1}(G) \neq \emptyset$ and hence there is $n \in \mathbf{N}$ such that $V_n \subset \operatorname{Int}(f_x)^{-1}(G)$. Similarly as in Theorem 3 we can prove that T_n is nowhere dense in A. □

References

[1] *M. E. Abd El-Monsef, S. N. El-Deeb, R. A. Mahmoud*: β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ. *12* (1983), 77–90.

[2] *D. Andrijevic*: Semi-preopen sets, Mat. Vesnik *38* (1986), 24–32.

[3] *J. Borsík, J. Doboš*: On decomposition of quasicontinuity, Real Anal. Exchange *16* (1990–91), 292–305.

[4] *A. M. Bruckner*: Differentiation of real functions, Springer-Verlag, Berlin-Heidelberg-New York, 1978.

[5] *Z. Frolík*: Remarks concerning the invariance of Baire spaces under mappings, Czechoslovak Math. J. *11* (1961), 381–385.

[6] *K. R. Gentry, H. B. Hoyle*: Somewhat continuous functions, Czechoslovak Math. J. *21* (1971), 5–12.

[7] *T. Husain*: Almost continuous mappings, Comment. Math *10* (1966), 1–7.

[8] *N. Levine*: Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly *70* (1963), 36–41.

[9] *S. Marcus*: Sur les fonctions quasicontinues au sens de S. Kempisty, Colloq. Math. *8* (1961), 47–53.

[10] *A. S. Mashour, M. E. Abd El-Monsef, S. N. El-Deeb*: On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt *53* (1982), 47–53.

[11] *O. Náther, T. Neubrunn*: On characterization of quasicontinuous multifunctions, Časopis Pěst. mat. *107* (1982), 294–300.

[12] *T. Neubrunn*: A generalized continuity and product spaces, Math. Slovaca *26* (1976), 97–99.

[13] *T. Neubrunn*: Generalized continuity and separate continuity, Math. Slovaca *27* (1977), 307–314.

[14] *A. Neubrunnová*: On certain generalizations of the notion of continuity, Mat. časopis *23* (1973), 374–380.

[15] *A. Neubrunnová, T. Šalát*: On almost quasicontinuity, Math. Bohemica *117* (1992), 197–205.

[16] *T. Noiri, V. Popa*: Weak forms of faint continuity, Bull. Math. Soc. Sci. Math. Roumanie *34* (1990), 263–270.

[17] *J. C. Oxtoby*: Cartesian product of Baire spaces, Fund. Math. *49* (1961), 156–170.

[18] *Z. Piotrowski*: A survey of results concerning generalized continuity in topological spaces, Acta Math. Univ. Comenian. *52–53* (1987), 91–110.

Súhrn

O SKORO KVÁZISPOJITÝCH FUNKCIÁCH

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Funkcia $f\colon X \to Y$ je skoro kvázispojitá v $x \in X$, ak $x \in \operatorname{Cl}\operatorname{Int}\operatorname{Cl} f^{-1}(V)$ pre každé okolie V bodu $f(x)$. Vyšetrujú sa niektoré vlastnosti takýchto funkcií.

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