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Discourse on One Way in which a Quantum-Mechanics Language on the Classical Logical Base Can Be Built up

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In this paper one of possible methods how to build up a quantum-mechanics language on the classical logical base is presented. If a time variable is introduced in the formulae of the language, then some statements on complementary magnitudes of quantum mechanics can be formulated without any occurence of well known paradoxes.

Quantum mechanics with its set of physical mathematical and semantic problems has been a source of basic discoussions and large generalizations. In this paper I will be dealing with a few questions concerned with the necessity of a special, non-classical "quantum logic" suggested by Hans Reichenbach. The question will be considered with from the semantic point of view.

This special logic, according to Reichenbach is to be the base of quantum-mechanical language that enables to describe also the results of empirical operations with complementary magnitudes in a sphere of microworld. Its suggestion originated from some well known empirical initial points that used to be frequently published in special as well as popular literature. When striving for explanation of these empirically found facts the physicists faced difficulties of basic importance. In order to solve all these problems a number of drastic changes in physical, mathematical, logical and philosophical aspects of modern physics were suggested.

Let us start our discourse with a short treatment of an empirical fact, which is — in view of many authors — of basic importance when discussing quantum mechanics. In the next part of this discourse I will try to show that — under certain assumptions — logical difficulties originating from this fact can be solved on the base of classical logic, too.

First I briefly explain the well known "two-slits experiment" (of. (3)).

Let us consider the function of the device consisting of a particle source Z, a diaphragm with the slit A and of a rear shield on which particules drop after passing through the slit A (Fig. 1).

Traces of particles that fall in the point X after passing through A may be broken in the region of A. This effect can be explained by interference of the particle and that

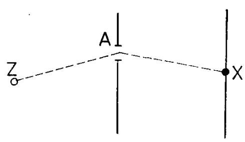

Fig. 1.

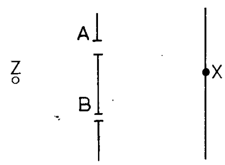

Fig. 2.

of the diaphragm in a neighborhood of A. The fall of a particle into a given region of the rear shield may be influenced by various factors varying in time. A particle can be but need not fall in the given point X. Probability that the particle from the source Z will fall in the point X after passing through the slit A, is by Fine $P(X) = P(A \wedge X)$, where $(A \wedge X)$ denots the conjunction of statements: "The particle from the source Z passed through the slit A" and "The particle fell into the point X".

In case of soft radiation from Z individual falls of particles can be observed as gleams on the rear shield if it is properly adjusted. If a photografic plate is placed on the rear shield and if the experiment is repeated with a great number of particles, the well known interference scheme is produced by the impact points of the particles.

Let us consider a similar experiment with a diaphragm with two slits A, B (Fig. 2).

Obviously any particle that fell on the rear shield passed through one and just one of the slits A, B. The probability that a particle from the source Z will fall into the point X of the rear shield after passing through one of the slits A, B is

$$P(X) = P((A \vee B) \wedge X) .$$

Using the distributive law of classical logic we can modify the term in brackets as follows

$$P(X) = P((A \wedge X) \vee (B \wedge X)) .$$

Using the probability theory rule on addition of probabilities of disjoint arguments we get:

$$P(X) = P(A \wedge X) + P(B \wedge X) - P(A \wedge X \wedge B \wedge X) .$$

The last term in the equation is obviously zero (it is impossible for a particle to pass through both of the slits A, B). Hence we get

$$P(X) = P(A \wedge X) + P(B \wedge X).$$

The term $P(A \wedge X)$ corresponds with the one-slit experiment (in case of two-slits diaphragm B can be sheltered). Similarly the term $P(B \wedge X)$ corresponds with the one-slit experiment with the slit B (A can be sheltered now). It follows from the last equation that the probability of the phenomenon that a particle will fall into the point X after passing through a two-slit diaphragm with the slits A, B is equal to the sum of probabilities of two distinct phenomena that a particle will fall into X after passing through just one of the slits A, B when the other one is sheltered.

This effect should be observable also in the case of experiment with many particles. The interference scheme that appears on the rear shield after passing through a two-slit diaphragm with both slits open should be identical with the superposition of two partial inteference schemes obtained by experiments with one of the slits sheltered.

The results of experiments performed did not comply with this conclusion at all. The superposition of schemes obtained from partial experiments does differ from the scheme obtained by experiments with both slits open.

From the facts obtained in an empirical way several alternative conclusions can be drawn:

The first conclusion possible is to resign on the identification of propabilities $P(A \wedge X)$, resp. $P(B \wedge X)$ with partial schemes obtained from one-slit experiments (with slits A and B respectively). That means that we admit in this case that a particle considered in the two-slits experiment is "different" from a particle considered in both partial experiments. In other words the fact that the slit B is open provides influence on the passage of a particle through the slit A and vice versa. Thus the passage of a particle through one slit X is effected by the other distant slit. This conclusion is in contradiction with the physical principle on the contact energy transmission.

Let us point out here that the anomaly is not encountered if particles are treated as prolifering waves and not "flying bodies". On the other hand this interpretation is in contradiction with those individual particles that create observable individual gleams on the rear shield. When adopting the wave interpretation the last effect can be only explained by an infinitely fast contraction of the wave into a single point at the time when the wave reaches the shield and when the gleam appears. This explanation appears to be in contradiction with principles of physical again.

Another conclusion possible is a requirement to give up the rule on addition of disjoint arguments of the probability function P and to change the mathematics apparatus used in quantum physics profoundly.

The third conclusion possible is to neglect the distributive law. This law is a tautology if the classical sentential calculus that forms (together with the predicate-calculus logic) the logical base of the accurate description of reality. Hence the requirement to give up the distributive law in its classical form is linked with a demand to build up a new non-classical logical calculus as logical base of quantum mechanics language by many authors.

In the following I will be dealing with the last of the three alternatives mentioned. The requirement to give up the distributive law and to build up some special "Quantum logic" is motivated by other reasonings as well.

Let us present briefly the reasoning of E. Scheibe [7] reffering to the well known requirement of simultaneous measurements of locality and momentum of a particle. Let us consider a moving particle. We shall measure the value of its momentum at a given time. Making a use of that we shall form a sentence B on momentum of the particle. We want to identify a position of the particle through measurements at the same time: i.e. a point of its trajectory when the particle is located at the given time. Let us imagine now that we are to make the selection between two possible points, in one of them the particle must occur. Based on this we form two statements A, A' on the location in one or in the other point of the particle.

We can now formulate composed sentence:

$$[(B \wedge (A \vee A')) \leftrightarrow B] .$$

This sentence is true in view of the facts mentioned. B obviously holds as well as the alternative $(A \vee A')$, hence the whole equivalence holds true. On the other hand from the quantum-mechanics theory it follows that the sentence

$$(B \wedge A) \vee (B \wedge A')$$

obtained from the antecedente of the given equivalence through the distributive law does not hold. The non-validity of this disjuction is given empirically.

Due to the well known Heisenberg principle it is namely impossible to determine exactly the position and momentum of a particle at the same time. The greater accuracy of momentum measurements, the less exactness of position measurements and vice versa. Hence neither proposition $(B \wedge A)$ nor proposition $(B \wedge A')$ are true since both of them are describing simultaneous exact measurements of momentum and position in one or in the other alternative point. It follows that the disjunction is not true as well.

The theorem on impossibility of simultaneous exact measurements of position and momentum of a particle is one of the most important principles of quantum mechanics. (A similar theorem on simultaneous exact measurements of time point and energy of a particle holds.) These theorems are closely linked with the character of mathematical formalism used for the description of quantum-mechanical phenomena. Pairs of quantities the simultaneous exact value of which it is impossible to determine by taking measurements are frequently called "complementary quantities" (due to Born). Statements on simultaneous exact values of complementary quantities are frequently called "incompatible statements".

While an empirical statement on precise value of one of both complementary quantities of a given microobject is verifiable and can be incorporated into the language system of quantum mechanics built up on the logical base of the classical double valued logic (on a Boolean base), conjunction of two statements on exact values of complementary quantities is not verifiable and it cannot be incorporated into the mentioned language system in view of many authors.

The limitation mentioned concerning the logical base of the quantum mechanics language is considered as definite and insurmountable by good many physicists. By them this fact is a consequence of the character of the mathematical formalism used in quantum mechanics (cf. arguments of Kochen and Specker in [4]).

On the other hand some other theoreticians insist that from the immeasureability of some magnitudes it does not follow that physical objects really do not have exact values of such magnitudes – independently of the measurement itself.

A lot of physicists put stress – in accordance with this sort of reasonings – on that it is impossible to exclude the existence of "hidden parameters" ("hidden variables") that once – after they will be found – will make it possible to evaluate simultaneous exact values of mutually complementary magnitudes (5).

Hans Reichenbach suggests in his basic work "Philosophic foundations of quantum mechanics" a solution of the difficulties treated above by a transformation to the three valued nonclassical logical base of the language of quantum mechanics. This language was used when results of measurements of quantum mechanical magnitudes were described.

The logical base of the language suggested contains propositions that can attain three values: "true" (T), "false" (F), "indeterminated" (N). Such a logical nonclassical calculus is built up by means of logical connectives introduced in tables. In the work by Reichenbach mentioned only a few connectives are introduced in tables and they are used further for constructions of formulae. By means of these formulae he shows that the discussed paradoxes can no more come in the language suggested by him.

First of all three connective of negation are introduced

$\sim$ is the connective of "cyclic negation"
$-$ is the connective of "diameter negation"
$\overline{}$ is the connective of "complete negation"

The table of these connectives:

A	$\sim A$	$-A$	$\bar{A}$
T	N	F	N
N	F	N	T
F	T	T	T

 Three-valued disjunction and conjunction, three kinds of implication and two kinds of equivalence are introduced further:

$\vee$ is the connective of three-valued disjunction
$.$ is the connective of three-valued conjunction
$\supset$ is the connective of standard implication
$\rightarrow$ is the connective of alternative implication
$\leftrightarrow$ is the connective of quasiimplication
$\equiv$ is the connective of standard equivalence
$\stackrel{*}{=}$ is the connective of alternative equivalence

The tables of these connectives:

A	B	$A \vee B$	$A . B$	$A \supset B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \equiv B$	$A \stackrel{*}{=} B$
T	T	T	T	T	T	T	T	T
T	N	T	N	N	F	N	N	F
T	F	T	F	F	F	F	F	F
N	T	T	N	T	T	N	N	F
N	N	N	N	T	T	N	T	T
N	F	N	F	N	T	N	N	F
F	T	T	F	T	T	N	F	F
F	N	N	F	T	T	N	N	F
F	F	F	F	T	T	N	T	T

In this way Reichenbach introduces only a less part of all possible connectives of the three-valued logic, the number of which is 3^9, i.e. 19 683.

These connectives make it possible to form several tautologies of the three-valued logic so that paradoxes in the quantum mechanics are avoided.

For example Reichenbach presents the following tautologies:

$A \equiv A$ ("the identity rule")
$A \equiv - - A$ ("the double diameter negation rule")
$A \equiv \sim \sim \sim A$ ("the triple cyclic negation rule")
$\bar{A} \equiv \bar{\bar{A}}$ ("the double complete negation rule")
$\bar{A} \equiv (\sim A \vee \sim \sim A)$ ("the relation between cyclic and complete negation)")
$(A \vee \sim A \vee \sim \sim A)$ ("the rule quartum non datur")
$(A \vee \bar{A})$ ("the rule pseudo-tertium non datur")
$-(A . B) \equiv (-A \vee -B)$ ("de Morgan rules for the diameter negation")
$-(A \vee B) \equiv (-A \vee -B)$
$(A . (B \vee C)) \equiv ((A . B) \vee (A . C))$
$(A \vee (B . C)) \equiv ((A \vee B) . (A \vee C))$ ("the distributive rules")

The concept of "complementary sentences A, B" (i.e. sentences giving statements on exact values of complementary magnitudes at a given moment of time) can be then introduced by means of the formula

$$(A \vee \sim A) \rightarrow \sim \sim B\,.$$

The value "true" is attained by simple sentences that are verifiable by an empirical (measuring) operation; the value "false" is attained by simple sentences that can be found as false by an empirical (measuring) operation; the value "indeterminated" is attained by simple sentences for which neither a verifying nor a falsifying empirical or measuring operation can be given.

Hence if we can realize a verifying or a falsifying operation for the sentence A speaking about one of the complementary magnitudes then such an operation cannot be realized for the sentence B describing the other complementary magnitude. The sentence B attains therefore the value "indeterminated".

In the work mentioned above Reichenbach shows on the example of the "two-slits experiment" that in the language constructed in this way the paradoxes mentioned above cannot be derived.

II.

In the second part of the treatment I will show that – under certain assumptions – the mentioned paradoxes can be solved without resorting to the nonclassical "quantum logic" as the base of the quantum mechanics language.

The basic starting point of the following reasonings is an assumption that physical objects posses objectively their properties at a given degree no mather whether these degrees of properties were measured yet or not. An obvious well known fact should be stressed now: measuring operations are not the only ways how to get information on real objective values of magnitudes of a given physical objects. Information on values of magnitudes of physical objects can also be obtained by analytic processes performed within the range of an exact language built on a given logical base and containing also terms and sentences of mathematics (containing mathematical formalism). Evaluations of values of magnitudes that were not measured directly by means of functions where in place of arguments other value of basic magnitudes that were actually measured is nothing now of course. These functions and their evaluation operations belong to the logic-mathematical component of the physical theory.

Let me point out in this place that an argument of a kind "no basic magnitudes (hidden parameters) can exist, which would enable an exact evaluation of simultaneously existing values of complementary magnitudes since it is not possible due to the mathematical structure of the theory" leave aside the possibility of replacement of one mathematical "tool" by another more adequate, one. The history of science provides more ground for the pluralism of logic-mathematical language tools than for the statement of definiteness of the "tool" used in the empirical science today.

Let us consider some magnitudes C, D used in classical physics. Let these magnitudes reflect objectively (i.e. independently of our observing or measuring abilities) existing properties of physical objects.

Let v_C (v_D resp.) be the values of these magnitudes possesed by a physical object a at a time t. Statements on this fact are formulated in a symbolic way as:

$$v_C(a, t)\,, \quad v_D(a, t)\,.$$

For this formulation we used the language of sentences in which, apart from ordinary individual constants and variables, also time variables occur. In my other papers I called this language the "chronology language" or the "language constructed on a chronological base".

Admit further that at the time t a measurement was realized so that the values of magnitudes C (D resp.) were measured. These facts we describe by means of sentences

$$M_C(a, t)\,, \quad M_D(a, t)$$

i.e. by sentences: "Measurement designed for the statement of values C (D resp.) of the object a at the time t was performed".

The measurements mentioned certainly joint an evaluation of some function in place of which arguments is object a, time t (metric character of time structure is supposed) and whose value is a number giving the measured value of the magnitude C (D resp.):

$$f_C(a, t) = v_C\,, \quad f_D(a, t) = v_D\,.$$

In classical physics it was supposed that in a case of an "ideal sharp measurement" the value measured is equal to the value possessed by the object independently on the measuring operation.

For every magnitude C the validity of the sentence

(1) $$\forall a\, \forall t[(M_C(a, t) \wedge f_C(a, t) = v_C) \rightarrow v_C(a, t)]$$

was assumed.

In fact each measuring apparatus has its own (technically given) reliability interval I_C, so that we are forced to weaken the sentence (1) slightly:

(2) $$\forall a\, \forall t[(M_C(a, t) \wedge f_C(a, t) = v_C \wedge v'_C(a, t)) \rightarrow |v'_C - v_C| \leqq J_C]\,.$$

In classical physics it was generally assumed that in course of the historical development of measuring techniques the reliability interval will become narrower (i.e. that the reliability of apparatuses will improve). The problems concerned with this "optimistic supposition" are dealt within an other work of mine.

Classical physics supposed further that all empirical physical magnitudes of an object can always be measured and their exact values found ("they are compatible magnitudes"). So it was assumed that for any pair of magnitudes C, D the sentence

(3) $$\forall a\, \forall t\, \forall t' [(M_C(a,t) \wedge M_D(a,t') \wedge f_C(a,t) = v_C \wedge f_D(a,t') = v_D) \rightarrow \rightarrow (v_C(a,t) \wedge v_D(a,t'))]$$

holds.

By substitution t'/t we get the sentence:

(4) $$\forall a\, \forall t [(M_C(a,t) \wedge M_D(a,t) \wedge f_C(a,t) = v_C \wedge f_D(a,t) = v_D) \rightarrow \rightarrow (v_C(a,t) \wedge v_D(a,t))] .$$

This sentence states the realizability of simultaneous measurements of magnitudes C, D leading to findings of really existing values on these magnitudes in the object measured.

Both sentences can be further reshaped for a given reliability intervals I_C (I_D resp.) of measuring apparatuses and also an "optimistic" sentence on historically unlimited improvement of both simultaneously realizable operations (this sentence will not be precisely formulated here not to make the text extremely complicated).

Let us consider a pair K, L of quantum-mechanical magnitudes of a microobject e. In the sense of the assumption of simultaneous existence of values of both magnitudes of the same object made above, let us form the sentence

(5) $$[v_K(e,t) \wedge v_L(e,t)] :$$

"Object e has the values v_K (v_L resp.) of magnitude K (L resp.) at the same time t."

If measurements of both magnitudes of the object e were realized simultaneously, then the two following sentences would hold simultaneously:

$$M_K(e,t) , \quad M_L(e,t) .$$

Similarly as before the following sentences would speak on measurements realized:

$$f_K(e,t) = v_K , \quad f_L(e,t) = v_L .$$

In quantum mechanics on the other hand the empirical knowings lead to the conclusion that an analogy of sentences (1)–(4) does not hold for arbitrary pairs of magnitudes K, L.

If these magnitudes are complementary, then it is not possible to measure them simultaneously and to ascribe the results of measurements as atributes to the objects themselves.

In this place we reach an important phase of our reasonings. The fact that we distinguish between the results of measurements of the value of magnitude of an object and between the value of the magnitude referring to the object itself independently of the results of measurements, makes it possible to define the concept of "complementary magnitudes" not by means of the concept of "simultaneous immeasureability of magnitudes" but by means of the concept of "nonexistence of possibility of such measurements of magnitudes that give results exactly referring to reality in both cases".

So in the chronology language we can introduce the concept of "complementarity of magnitudes K, L" by means of a sentence of the form

$$(6)\quad \mathrm{Compl}\,(K, L) \leftrightarrow \forall e\, \forall t [\forall M_K\, \forall f_K\, \forall v_K (M_K(e, t) \wedge f_K(e, t) = v_K \wedge v_K(e, t)) \rightarrow \rightarrow \sim \exists M_L\, \exists f_L\, \exists v_L (M_L(e, t) \wedge f_L(e, t) = v_L \wedge v_L(e, t))] \,.$$

Reichenbach increases in his paper a demand not to consider necessarily as complementary such magnitudes that by chance were not simultaneously measured up to now. The sentence (6) suits demand very well; from the sentence

$$(7)\qquad \sim \exists t\, \exists e\, \exists M_K\, \exists M_L [M_K(e, t) \wedge M_L(e, t)]$$

we find easily that

$$\mathrm{Compl}\,(K, L)$$

cannot be derived as its consequence.

Reichenbach states an important theorem on impossibility of simultaneous measurements of complementary magnitudes.

Consistently with the assumptions adopted here I will modify the theorem so that it will express impossibility of simultaneous measurements and findings of objectively existing values of complementary magnitudes reffering to the same object.

$$(8)\quad \mathrm{Compl}\,(K, L) \leftrightarrow \sim \exists e\, \exists t\, \exists M_K\, \exists M_L\, \exists f_K\, \exists f_L\, \exists v_K\, \exists v_L [M_K(e, t) \wedge v_K(e, t) \wedge \wedge f_K(e, t) = v_K \wedge M_L(e, t) \wedge v_L(e, t) \wedge f_L(e, t) = v_L] \,.$$

It is easy to get the sentence (8) from the sentence (6). The sentences (6)–(8) speak on the values of a magnitude of a physical object at the same moment of time.

The last condition can be generalized: we will consider two distinct moments t, t' that differ no more than a given time interval Δt.

Let there be $\Delta t \geqq 0$ so that it holds:

$$(9)\quad \mathrm{Compl}\,(K, L) \leftrightarrow \forall e\, \forall t [\forall M_K\, \forall f_K\, \forall v_K (M_K(e, t) \wedge f_K(e, t) = v_K \wedge v_K(e, t)) \rightarrow \rightarrow \sim \exists M_L\, \exists f_L\, \exists v_L\, \underset{|t - t'| \leqq \Delta t}{\exists t'} (M_L(e, t') \wedge f_L(e, t') = v_L \wedge v_L(e, t'))] \,.$$

For $\Delta t = 0$ (i.e. for $t = t'$) the sentence obviously goes into the sentence (6).

For $\Delta t \geqq 0$ sentences analogues to the sentences (7), (8) can be easily formed now.

The sentence (9) in its interpretation is closely connected with the physical hypothesis of discrete quantified time. The complementarity of magnitudes of a physical object comes within a single time interval. Let us say now that it is impossible to measure and find objectively existing values of complementary magnitudes at moments with the period equal or less than a given time interval Δt. The length of this interval is given by principles of physics (through light propagation speed in vacuum and through the value of the Planck effective quantum of electromagnetic field particularly).

The language suggested allows the formulation of a sentence on "hidden parameters" i.e. the sentence on the existence of measurable magnitudes from whose values for a given object it is possible to reach – through an analytic process – conclusions on real values of complementary magnitudes of this object within a given time interval – as a possible assumption. I will formulate this theorem in the mixed language in the following way.

Let there exist mutually compatible magnitudes of a physical object $e : J_1, \ldots, J_m$.

Let the sentences $M_{J_1}(e, t_1), \ldots, M_{J_m}(e, t_m)$ speak on the realizations of measuring operations performed at moments $t_1, \ldots, t_m$, where the period of no pair of these moments is greater than the lasting of the given interval Δt.

Analogously let the sentences

$$f_{J_1}(e, t_1) = v_{J_1}, \ldots, f_{J_m}(e, t_m) = v_{J_m}$$

be the sentences speaking on the values of magnitudes $J_1, \ldots, J_m$ of an object e at time $t_{J_1}, \ldots, t_{J_m}$ measured in this way. Let the sentence: $v_K(e, t) \wedge v_L(e, t)$ speak on complementary magnitudes of the object e, that attain values v_K, v_L at moments t, t'. Let the moments t, t' be within the same time interval together with all moments $t_{J_1}, \ldots, t_{J_m}$.

Let the sentences: $V_1, \ldots, V_n$ be theorems of a given physical theory and speak on fundamental relations between magnitudes $J_1, \ldots, J_m$ and complementary magnitudes K, L.

Under these conditions the assumption regarding the existence of "hidden parameters" will be expressed in sentence (10).

From the assumption of the form

$$(10) \qquad \exists J_1, \ldots, \exists J_m(\prod_{i=1}^{m} M_{J_i}(e, t_{J_i}) \wedge f_{J_i}(e, t_{J_i}) = v_{J_i})] \wedge V_1 \wedge \ldots \wedge V_n)$$

it is possible to get the consequence of the form

$$(v_K(e, t) \wedge v_L(e, t')]$$

in an analytic way.

Let us remind on this occasion that the consequent of the sentence (10) is an analytic sentence of the language considered and that it does not say anything about any result of direct measuring activities of values of magnitudes K, L which remain to be complementary magnitudes due to the validity of sentence (9).

The sentence (10) is no proof of the existence of "hidden parameters" in itself by any means. It can either be incorporated into the language or it need not be incorporated.

The matter of the last reasoning was only to stress the fact that the language suggested allows its formulation.

The validity of the theorem regarding complementarity of magnitudes K, L docs not exclude the following statements of course.

For any pair t, t' of moments the distance between which is not greater than a given interval Δt at most one of the following theorems holds:

$$\forall e[(M_K(e, t) \wedge f_K(e, t) = v_K) \to v_K(e, t)] , \tag{11}$$
$$\forall e[(M_L(e, t) \wedge f_L(e, t) = v_L) \to v_L(e, t)] .$$

Each of these theorems belongs to a "partial language" dealing with results of measurements of values of just one of the two complementary magnitudes (many authors speak on "partial Boolean algebras" in this respect). Partial languages – in each of them just one such a theorem is formulated – can obviously be formed within the chronology language discussed.

I will show now that in a sphere of the chronology language suggested, which is constructed on the classical logical base and contains the assumptions mentioned, we can avoid the paradoxes discussed without being forced to change the classical character of the logical base.

Let us back to "two-slits experiment" first. We formulate the following sentences in the language considered:

$A(e, t)$ "The particle e passed through the slit A at the time t") (in place of this statement we shall write briefly A^t).

$B(e, t)$ "The particle e passed through the slit B at the time t" (in place of this briefly B^t).

Both last sentences are empirical statements and speak on the results of partial experiments where always one slit is open while the other one is closed.

$X(e, t)$ "The particle e fell into the point X at the time t" (briefly X^t).

$M_A(e, t)$ "The particle e was measured at the time t when passing through A" (briefly M_A^t).

$M_B(e, t)$ "The particle e was measured at the time t when passing through B" (briefly M_B^t).

$f_A(e, t)$ "Through measurements the particle e was located in the slit A at the time t" (briefly f_A^t).

$f_B(e, t)$ "Through measurements the particle e was located in the slit B at the time t" (briefly f_B^t).

$f_X(e, t)$ "The particle e was empirically found when it fall in the point X at the time t" (briefly f_X^t).

f_A, f_B, f_X are proposition functions in this case;

v_A, v_B, v_X are their values respectively.

For the passage of the particle through the slits the sentence holds:

(12) $\forall e \, \forall t[(M_A(e, t) \wedge f_A(e, t) \wedge A(e, t)) \rightarrow \sim(M_B(e, t) \wedge f_B(e, t) \wedge B(e, t))]$.

This sentence is obviously satisfied at any experiment; the particle cannot be in two slits simultaneously.

The sentence (12) can obviously be generalized for two moments t, t', and a given interval Δt:

(13) $\forall e \, \forall t \, \forall t'[(M_A(e, t) \wedge f_A(e, t) \wedge A(e, t)) \rightarrow \sim(M_B(e, t') \wedge f_B(e, t') \wedge B(e, t'))]$.

The particle cannot be in two slits within a given time interval.

It is obvious that both last sentences comply with the complementarity requirement.

Probability that the particle falls in the point X after passing through either the first or the other slit will be now:

$$\begin{aligned} P(X^{t''}) &= P[((M_A^t \wedge f_A^t \wedge A^t) \vee (M_B^t \wedge f_B^t \wedge B^t)) \wedge X^{t''}] , \quad t < t'' \\ &= P[(M_A^t \wedge f_A^t \wedge X^{t''}) \vee (M_B^t \wedge f_B^t \wedge B^t \wedge X^{t''})] , \\ &= P(M_A^t \wedge f_A^t \wedge A^t \wedge X^{t''}) + P(M_B^t \wedge f_B^t \wedge B^t \wedge X^{t''}) - \\ &- P(M_A^t \wedge f_A^t \wedge A^t \wedge X^{t''} \wedge M_B^t \wedge f_B^t \wedge B^t \wedge X^{t''}) . \end{aligned}$$

It follows from (12) that the last (subtracted) term is equal to zero and so it holds

(14) $$P(X^{t''}) = P(M_A^t \wedge f_A^t \wedge A^t \wedge X^{t''}) + P(M_B^t \wedge f_B^t \wedge B^t \wedge X^{t''})$$

and hence the wanted probability will be equal to the sum of probability of the case when the particle fell into X after having passed through A and that when the particle fell into X after having gone through B. The sum given does not correspond with the sum of probabilities for partial experiments with repeatedly closed slits: these experiments are never performed simultaneously.

When dealing with partial experiment when one slit is open while the other one is closed we work with the probabilities

(15) $$P(M_A^t \wedge f_A^t \wedge A^t \wedge X^{t''})$$

(16) $$P(M_B^{t'} \wedge f_B^{t'} \wedge B^{t'} \wedge X^{t'''})$$

where $t \neq t'$, $t'' \neq t$, $t < t''$, $t' < t'''$.

On these grounds there is no base whatsoever to insist that in both cases the same particle is dealt with. Partial experiments are necessarily performed with different particles and under different conditions as well. Obviously nothing can ensure any identity of all conditions of two experiments at distinct times t, t' (t'' and t''' resp.) Hence there is no reason for assuming the equality of the probability from the sentence (15) and that in the left-hand term of the sentence (14) (probability from the sentence (16) and that in the right-hand term of sentence (14) resp.)

From this is follows that we are not forced to draw the controversial conclusion that a particle passing through one slit is influenced by an effect of the other slit just at the time of its passage. Thus also any reason for the rejection of the distributive law of classical logic is lost (that is actually what was the motivation of Reichenbach's construction of three valued "quantum logic").

In the following part of this discourse I will turn to the Scheibe discussion concerning simultaneous identification of the position and momentum of a particle. Let us consider two complementary magnitudes K, L – now position and momentum – again. Let v_K, v_L be objective values of these magnitudes possessed by the particle e at the moment t.

Next we will use abbreviation similar to those used earlier: Instead of writing

$M_K(e, t)$	we use abbreviation	M_K^t,
$M_L(e, t)$		M_L^t,
$f_K(e, t)$		f_K^t,
$f_L(e, t)$		f_L^t,
$f_L'(e, t)$		$f_L'^t$,
$v_K(e, t)$		v_K^t.

Let us suppose now that measurements of values of magnitudes K, L were performed and results v_K, v_L were gained. Let us suppose further that the real, objective value of the magnitude K corresponds exactly with the results of the measurement. In reference to the real value of the magnitude L we will consider three possibilities successively. Let v_L be the real value in the first case; let v_L^t (different from v_L) be the real value in the second case; let no of the values found through measurements be the real one. Then we will formulate exactly corresponding sentences – analogous to those from Scheibe discussion – accordingly: in place of writing:

$v_L(e, t)$	abbreviations	v_L^t,
$v_L'(e, t)$		$v_L'^t$.

Sentences corresponding with the first alternative mentioned are:

(17) $$[(M_K^t \wedge f_K^t \wedge v_K^t) \wedge ((M_L^t \wedge f_L^t \wedge v_L^t) \vee (M_L^t \wedge f_L'^t \wedge v_L^t))],$$

(18) $$[(M_K^t \wedge f_K^t \wedge v_K^t \wedge M_L^t \wedge f_L^t \wedge v_L^t) \vee (M_K^t \wedge f_K^t \wedge v_K^t \wedge M_L^t \wedge f_L'^t \wedge v_L^t)].$$

Both sentences are equivalent due to the distributive law. Due to the interpretation mentioned the first member of conjunction in the sentence (17) is true; the second part of conjunction in (17) is also true. The first member in the disjunction cannot be true due to the assumption about complementarity of magnitudes K, L. Therefore the first member of the disjunction of the sentence (18) appears to be false while the second member of this disjunction is true. All both sentences are true in the interpretation given.

The second alternative mentioned corresponds with sentences:

$$(19) \qquad [(M_K^t \wedge f_K^t \wedge v_K^t) \wedge ((M_L^t \wedge f_L^t \wedge v_L'^t) \vee (M_L^t \wedge f_L'^t \wedge v_L'^t))]\,,$$

$$(20) \qquad [(M_K^t \wedge f_K^t \wedge v_K^t \wedge M_L^t \wedge f_L^t \wedge v_L'^t) \vee (M_K^t \wedge f_K^t \wedge v_K^t \wedge M_L^t \wedge f_L'^t \wedge v_L'^t)]\,.$$

When this interpretation is adopted, both sentences are true propositions again.

The third alternative corresponds with the sentences:

(21)

$$[(M_K^t \wedge f_K^t \wedge v_K^t) \wedge ((M_L^t \wedge f_L^t \wedge \sim v_L^t \wedge \sim v_L'^t) \vee (M_L^t \wedge f_L'^t \wedge \sim v_L^t \wedge \sim v_L'^t))]\,,$$

$$(22) \qquad \begin{aligned} &[(M_K^t \wedge f_K^t \wedge v_K^t \wedge M_L^t \wedge f_L^t \wedge \sim v_L^t \wedge \sim v_L'^t) \vee \\ &\vee (M_K^t \wedge f_K^t \wedge v_K^t \wedge M_L^t \wedge f_L'^t \wedge \sim v^t \wedge \sim v_L'^t)]\,. \end{aligned}$$

When this interpretation is adopted, both sentences are true propositions again.

Thus we have seen that the distributive law is not broken in the region of quantum-mechanical effects in our treatment of Scheibes "hypothetical experiment".

The apriori assumption of objective existence of given values of magnitudes of physical objects even when they were not being measured and found, appears to be a very important condition of our reasonings in the recent part of this paper. The capability to use time terms in exact formulations of our chronology language is another assumption of course. Let us note further that it is possible to generalize the sentences (15)–(22) for reasonings about measuring procedures and existence of values of measured magnitudes at distinct time t, t' the period of which is not greater than the length of a given time interval Δt.

In the current reasonings it has been shown that the chronology language makes it possible to "avoid" some typical difficulties involved in the interpretation of quantum-mechanical effects. In this respect I have mentioned the idea of three valued "quantum logic" by Reichenbach.

In the end of this discourse I want to emphasize that the reason for the suggestion of the chronological base of the quantum-mechanics is not a proof of its equivalence with three values logical base of the Reichenbach's language. It is of course possible to introduce through definitions the triple of Reichenbach proposition values:

$$T(\underline{v_K(e,t)}) \leftrightarrow \exists \underline{M}_K\, \exists \underline{f}_K\underline{(M_K(e,t) \wedge f_K(e,t) = v_K)}$$

$$F(\underline{v_K(e,t)}) \leftrightarrow \exists \underline{M}_K\, \exists \underline{f}_K\underline{(M_K(e,t) \wedge f_K(e,t) \neq v_K)}$$

$$N(\underline{v_K(e,t)}) \leftrightarrow \exists \underline{L} \exists \underline{M}_L\, \exists \underline{f}_L\underline{(\mathrm{Compl}\,(L,K) \wedge M_L(e,t) \wedge f_L(e,t) = v_L \wedge v_L(e,t))}\,.$$

into the metalanguage of the quantum mechanics language using the complementarity relation discussed above.

In this metalanguage we can introducte through terms T, F, N proposition forming functors corresponding with Reichenbach's connectives of nonclassical logical calculus and then describe the logical structure of Reichenbach's tautologies by means of them.

The proof of the equal degree of adequacy of the chronological base of Reichenbach's three values logical base for the construction of the quantum mechanics language has not been presented in this paper. I feel that Reichenbach's ideas from the work mentioned above do not provide the performance of such a proof for the time being: Reichenbach does not build his nonclassical logic as an axiomatic system, but he presents some of its tautologies as an example.

III.

Summary:

1. It was an aim of the discourse presented to show the possibility of construction of the quantum mechanics language on the classical logical base, so that it is possible to formulate some of the important information of quantum mechanics and not to come to controversial conclusions in a logical way.
2. The classical logical calculus creating the logical base of the language was suggested while syntaxis and semantics were modified slightly: time terms enter its formulae together with common predicate and individual terms. The language constructed in this way was called a "chronology language".
3. When building the suggested language an important assumption concerning the objective existence of values of physical magnitides of objects – no matter whether they were actually measured – was made (in the metalanguage). Using this assumption the concept of "complementarity of physical magnitudes" was made in the language constructed.
4. It was shown that within the suggested language it is possible to formulate some sentences on complementary magnitudes of quantum mechanics avoiding paradoxies. Reichenbach solved the paradoxies by resorting to a nonclassical quantum logical base of the language.

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