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EVALUATION OF THE IMPULSIVE SOLUTION SPACE OF LINEAR MULTIVARIABLE HOMOGENEOUS IMPLICIT SYSTEMS

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A closed formula is given, which allows the determination of the impulsive solutions of Linear Homogeneous Matrix Differential Equations (L.H.M.D.E.) directly in terms of finite and infinite spectral data of the associated polynomial matrix. Specifically the notions of finite and infinite Jordan pairs for a general polynomial matrix are defined and it is pointed out the strong relationship among them and the impulsive solutions of L.H.M.D.E.

1. INTRODUCTION

In the recent years there has been a growing interest in the system-theoretic problems of generalized state space or singular systems due to the extensive applications of this kind of systems in large-scale, singular perturbation theory, circuits, robotics, economics, demography, control theory and other areas. Generalized state space systems are systems described by:

$$E\ \rho x(t) = Ax(t) + Bu(t), \tag{1}$$

where $\rho \doteq \frac{\mathrm{d}}{\mathrm{dt}}$ is the differential operator, $E \in \Re^{r\times r}$ $\operatorname{rank}_{\Re} E \leq r$, $A \in \Re^{r\times r}$, $B \in \Re^{r\times m}$, whereas (regular) state space systems are the systems described by:

$$\rho x(t) = Ax(t) + Bu(t) \tag{2}$$

which represent a particular case of (1) with $E = I_r$, the identity matrix. Regular state space systems were studied explicitly in the past decades (see e. g. [10]). On the other hand many researchers in the recent years explored special properties of generalized state space systems and found many connections between them and regular ones. Generalized state space systems were studied in the frequency domain (see [11, 14–16]) as well as in time-domain (see [2–6, 8–9]). Generalized state space systems as (1) represent a particular case of Polynomial Matrix Descriptions (PMDs) i. e. physical systems whose dynamics can be described by a linear matrix differential equation having the form:

$$A(\rho)\beta(t) = B(\rho)\, u(t), \tag{3}$$

where $A(\rho) = \sum_{i=0}^{q_1} A_i \rho^i \in \Re^{r\times r}[\rho]$, $A_i \epsilon\, \Re^{r\times r}$, $i = 0, 1, 2, \ldots, q_1 \geq 1$ with $\mathrm{rank}_{\Re} A_{q_1} < r$, $B(\rho) = \sum_{i=0}^{\sigma} B_i \rho^i \in \Re^{r\times m}[\rho]$, $B_j \in \Re^{r\times m}$, $j = 0, 1, 2, \ldots, \sigma \geq 0$, $\beta(t)$ the *pseudo-state* of the system and $u(t)$ the control input to the system. In a recent paper [13] various known results regarding the smooth and impulsive solutions of homogeneous generalized state space systems have been translated to the more general case of PMDs. A new treatment of PMDs using Algebraic methods is given in [12]. Among other topics the author gives a formula for the determination of the impulsive solutions of L.H.M.D.E. in terms of the notions of fast and slow states at $t = 0^-$ which are introduced. Campell in [2–3] has found the solution of systems of the form (1) using the notion of Drazin inverse. Also in [4–6] Cobb used a different approach which utilizes the Weierstrass canonical form. Comparing those methods to ours we remark the following: Each method has its advantages and disadvantages for different problems. The main advantage of our method is that it treats systems of the form (3) which seem to be generalizations of systems of the form (1). The possible disadvantage is that the proposed solution is associated with the evaluation of several matrices which are derived by means of some previously established algorithms. On the other hand the approach based on the canonical decomposition which separates the system behavior in the finite and infinite frequencies provides deep insight into the structure of singular systems in systems analysis. However, it does not always provide a useful framework for actual computation. The canonical form also uses a change of the internal variable which is unconvenient in practical situations since the original variables are chosen to have their own significance. The main drawback of the method proposed by Cambell is the computational complexity of the proposed algorithm. Our approach differs from that given in [12] because we give a formula such that the set of the Dirac impulses and its derivatives which are used in the determination of the impulsive solutions of L.H.M.D.E., as well as, the set of finite and infinite spectral data of the associated polynomial matrix are presented in a – easy to use – closed form.

2. MAIN RESULTS

Consider the linear, homogeneous matrix differential equation:

$$A(\rho)\beta(t) = 0, \quad t \geq 0 \tag{4}$$

and let $S^{\infty}_{A(s)}(s)$ be the Smith–McMillan form at $s = \infty$ of $A(s) = A_0 + A_1 s + \cdots + A_{q_1} s^{q_1}$:

$$S^{\infty}_{A(s)}(s) = \mathrm{diag}\left[s^{q_1}, s^{q_2} \ldots, s^{q_k}, \frac{1}{s^{\hat{q}_{k+1}}}, \ldots \frac{1}{s^{\hat{q}_r}}\right]. \tag{5}$$

If $A(s)$ has at least one zero at $s = \infty$ then the Laurent expansion of $A^{-1}(s)$ can be written according to [12]:

$$A^{-1}(s) = H_{\hat{q}_r} s^{\hat{q}_2} + H_{\hat{q}_{r-1}} s^{\hat{q}_2 - 2} + \ldots + H_1 s + H_0 + H_{-1} s^{-1} + H_{-2} s^{-2} + \ldots = H_{pol}(s) + H_{spr}(s), \tag{6}$$

where $H_{pol}(s) \in \Re^{r\times r}[s]$ is the polynomial part of $A^{-1}(s)$ and $H_{spr}(s) \in \Re^{r\times r}(s)$ is the strictly proper part of $A^{-1}(s)$. Let (C, J) and (C_∞, J_∞) be a finite and an infinite Jordan pair respectively of the polynomial matrix $A(s)$. Then the matrices (C, J), (C_∞, J_∞) satisfy the conditions [17]:

$$A_{q_1} C J^{q_1} + A_{q_1 - 1} C J^{q_1 - 1} + \cdots + A_0 C = 0 \tag{7}$$

$$Q_n := \begin{bmatrix} C \\ CJ \\ \vdots \\ CJ^{n-1} \end{bmatrix} \in \Re^{rn\times n}, \ \operatorname{rank} Q_n = n \tag{8}$$

$$A_0 C_\infty J_\infty^{q_1} + A_1 C_\infty J_\infty^{q_1 - 1} + \cdots + A_{q_1} C_\infty = 0_{r,\mu} \tag{9}$$

$$Q_\mu^\infty := \begin{bmatrix} C_\infty \\ C_\infty J_\infty \\ \vdots \\ C_\infty J_\infty^{\mu-1} \end{bmatrix}, \ \operatorname{rank}_\Re Q_\mu^\infty = \mu, \tag{10}$$

where $n = \deg\det A(s)$, $\mu = \sum_{j=k+1}^{r} (\widehat{q}_j + 1) = \sum_{j=k+1}^{r} \widehat{q}_j + (r-k)$, where $\widehat{q}_j$, $j = k+1, \ldots, r$ are the orders of the zeros at $s = \infty$ of $A(s)$. Then we can write (see [7]) $A^{-1}(s) = C_\infty [sJ_\infty - I_\mu]^{-1} B_\infty + C[sI_n - J]^{-1} B$, where B, B_∞ are constant matrices defined by:

$$\begin{bmatrix} B \\ B_\infty \end{bmatrix} = [I_n, \ J_\infty^{\sigma-1}] \begin{bmatrix} S_{\sigma-2} \\ V \end{bmatrix}^{-1} [0, 0, \ldots, 0, I_r]^T, \tag{11}$$

where:

$$S_{\sigma-2} = \begin{bmatrix} C & C_\infty J_\infty^{\sigma-2} \\ CJ & C_\infty J_\infty^{\sigma-3} \\ \vdots & \vdots \\ CJ^{\sigma-2} & C_\infty \end{bmatrix} \tag{12}$$

$$V = \left[A_\sigma C J^{\sigma-1}, \ -\sum_{i=0}^{\sigma-1} A_i C_\infty J_\infty^{\sigma-1-i} \right]. \tag{13}$$

Considering now the Laplace transformed equation (4) we obtain:

$$\hat{\beta}(s) = A^{-1}(s)\hat{a}(s) \in \Re^{r\times 1}, \tag{14}$$

where $\hat{a}(s) \in \Re^{r\times 1}[s]$ is the initial condition vector associated with the initial values of $\beta(t)$ and its $(q_1 - 1)$-derivatives at $t = 0^-$ i.e. $\beta(0^-)$, $\beta^{(1)}(0^-)$, ..., $\beta^{(q_1-1)}(0^-)$ given by [1]:

$$\hat{a}(s) = [s^{q_1-1} I_r, s^{q_1-2} I_r, \ldots, sI_r, I_r] \times F \tag{15}$$

$$F = \begin{bmatrix} A_{q_1} & 0 & \cdots & 0 \\ A_{q_1-1} & A_{q_1} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ A_1 & A_2 & \cdots & A_{q_1} \end{bmatrix} \begin{bmatrix} \beta(0^-) \\ \beta^{(1)}(0^-) \\ \vdots \\ \beta^{(q_1-1)}(0^-) \end{bmatrix}. \tag{16}$$

After some matrix manipulations between $A^{-1}(s)$ and $\hat{a}(s)$ we finally arrive at the following special form for $\hat{\beta}(s)$ which appeared originally in [12]:

$$\hat{\beta}(s) = [s^{\widehat{q_r}+q_1-1}I_r,\ s^{\widehat{q_r}+q_1-2}I_r, \ldots, sI_r, I_r \mid s^{-1}I_r, s^{-2}I_r, \ldots]$$
$$\times \begin{bmatrix} H_{\widehat{q_r}} & 0 & \cdots & 0 \\ H_{\widehat{q_r}-1} & H_{\widehat{q_r}} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ H_{\widehat{q_r}-(q_1-1)} & H_{\widehat{q_r}-(q_1-2)} & \cdots & H_{\widehat{q_r}} \\ \vdots & \vdots & & \vdots \\ H_{-(q_1-1)} & H_{-(q_1-2)} & \cdots & H_0 \\ \hline H_{-q_1} & H_{-(q_1-1)} & \cdots & H_{-1} \\ H_{-(q_1+1)} & H_{-q_1} & \cdots & H_{-2} \\ \vdots & \vdots & & \vdots \end{bmatrix} F \tag{17}$$

$$= [s^{\widehat{q_r}+q_1-1}I_r,\ s^{\widehat{q_r}+q_1-2}I_r, \ldots, sI_r, I_r] \begin{bmatrix} H_{\widehat{q_r}} & 0 & \cdots & 0 \\ H_{\widehat{q_r}-1} & H_{\widehat{q_r}} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ H_{\widehat{q_r}-(q_1-1)} & H_{\widehat{q_r}-(q_1-2)} & \cdots & H_{\widehat{q_r}} \\ \vdots & \vdots & & \vdots \\ H_{-(q_1-2)} & H_{-(q_1-3)} & \cdots & H_1 \\ H_{-(q_1-1)} & H_{-(q_1-2)} & \cdots & H_0 \end{bmatrix} F$$

$$+[s^{-1}I_r, s^{-2}I_r, \ldots] \begin{bmatrix} H_{-q_1} & H_{-(q_1-1)} & \cdots & H_{-1} \\ H_{-(q_1+1)} & H_{-q_1} & \cdots & H_{-2} \\ \vdots & \vdots & & \vdots \end{bmatrix} F = \hat{\beta}_{pol}(s) + \hat{\beta}_{spr}(s)$$

are respectively the polynomial and the strictly proper part of $\hat{\beta}(s)$. We examine the form of $\hat{\beta}_{pol}(s)$. Consider the identity: $A^{-1}(s)A(s) = I_r$, which can be written also:

$$\begin{gathered} A^{-1}(s)A(s) = \\ \left[H_{\widehat{q_r}}s^{\widehat{q_r}} + H_{\widehat{q_r}-1}s^{\widehat{q_r}-1} + \ldots + H_1 s + H_0 + H_{-1}s^{-1} + \ldots\right] \\ \left[A_{q_1}s^{q_1} + A_{q_1-1}s^{q_1-1} + \cdots + A_0\right] = I_r \end{gathered} \tag{18}$$

from which after multiplying the matrices involved, we obtain the relations:

$$
\begin{gathered}
H_{\widehat{q_r}} A_{q_1} = 0 \\
H_{\widehat{q_r}} A_{q_1-1} + H_{\widehat{q_r}-1} A_{q_1} = 0 \\
\vdots \\
H_{\widehat{q_r}} A_1 + H_{\widehat{q_r}-1} A_2 + H_{\widehat{q_r}} A_{q_1-1} + \ldots + H_{\widehat{q_r}-(q_1-1)} A_{q_1} = 0 \\
\vdots \\
H_1 A_0 + H_0 A_1 + \ldots + H_{-(q_1-1)} A_{q_1} = 0 \\
H_0 A_0 + H_{-1} A_1 + \cdots + H_{-q_1} A_{q_1} = I_r \\
H_{-1} A_0 + H_{-2} A_1 + \cdots + H_{-(q_1+1)} A_{q_1} = 0 \\
\vdots
\end{gathered}
\tag{19}
$$

The first q_1 equations of (19) can be written in matrix form as:

$$
\begin{bmatrix}
H_{\widehat{q_r}} & 0 & \cdots & 0 \\
H_{\widehat{q_r}-1} & H_{\widehat{q_r}} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
H_{\widehat{q_r}-(q_1-1)} & H_{\widehat{q_r}-(q_1-2)} & \cdots & H_{\widehat{q_r}}
\end{bmatrix}
\begin{bmatrix}
A_{q_1} & 0 & \cdots & 0 \\
A_{q_1-1} & A_{q_1} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
A_1 & A_2 & \cdots & A_{q_1}
\end{bmatrix}
= \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & 0
\end{bmatrix}
\tag{20}
$$

Now in light of (20),(16) we have:

$$
\hat{\beta}_{\text{pol}}(s) = [s^{\widehat{q_r}+q_1-1} I_r, \ldots, s^{\widehat{q_r}+1}, s^{\widehat{q_r}} I_r, \mid s^{\widehat{q_r}-1} I_r, \ldots, s I_r, I_r]
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & & 0 \\
---- & ---- & & ---- \\
H_{\widehat{q_r}-q_1} & H_{\widehat{q_r}-(q_1+1)} & \cdots & H_{\widehat{q_r}} \\
\vdots & \vdots & & \vdots \\
H_{-(q_1-2)} & H_{-(q_1-3)} & \cdots & H_1 \\
H_{-(q_1-1)} & H_{-(q_1-2)} & \cdots & H_0
\end{bmatrix} F
$$

$$
= \left[s^{\widehat{q_r}-1} I_r, \ldots, s I_r, I_r\right]
\begin{bmatrix}
H_{\widehat{q_r}-q_1} & H_{\widehat{q_r}-(q_1+1)} & \cdots & H_{\widehat{q_r}} \\
\vdots & \vdots & & \vdots \\
H_{-(q_1-2)} & H_{-(q_1-3)} & \cdots & H_1 \\
H_{-(q_1-1)} & H_{-(q_1-2)} & \cdots & H_0
\end{bmatrix} F
$$

$$
\begin{aligned}
&= \left[H_{\widehat{q_r}-q_1} s^{\widehat{q_r}-1} + \ldots + H_{-(q_1-2)} s + H_{-(q_1-1)} \mid \ldots\right] \\
&\qquad \left[\ldots \mid H_{\widehat{q_r}-2} s^{\widehat{q_r}-1} + \ldots + H_0 s + H_{-1} \mid H_{\widehat{q_r}-1} s^{\widehat{q_r}-1} + \ldots + H_1 s + H_0\right] F \\
&\qquad = [H_{\widehat{q_r}-1}, \ldots, H_1, H_0 \mid H_{-1}, H_{-2}, \ldots, H_{-(q_1-1)}] \\
&\qquad\qquad \begin{bmatrix} 0 & 0 & \cdots & 0 & s^{\widehat{q_r}-1} I_r \\ 0 & 0 & \cdots & s^{\widehat{q_r}-1} I_r & s^{\widehat{q_r}-2} I_r \\ \vdots & \vdots & & \vdots & \vdots \\ s^{q_1-1} I_r & s^{q_1-2} I_r & \cdots & s I_r & I_r \\ s^{q_1-2} I_r & s^{q_1-3} I_r & \cdots & I_r & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ s I_r & I_r & \cdots & 0 & 0 \\ I_r & 0 & \cdots & 0 & 0 \end{bmatrix} F
\end{aligned} \tag{21}
$$

or equivalently:

$$\hat{\beta}_{pol}(s) = L\,\Delta(s) F, \tag{22}$$

where:

$$L = [H_{\widehat{q_r}-1}, \ldots, H_1, H_0 \mid H_{-1}, H_{-2}, \ldots, H_{-(q_1-1)}] \in \Re^{r \times [r(\widehat{q_r}+q_1-1)]} \tag{23}$$

$$\Delta(s) = \begin{bmatrix} 0 & 0 & \cdots & 0 & s^{\widehat{q_r}-1} I_r \\ 0 & 0 & \cdots & s^{\widehat{q_r}-1} I_r & s^{\widehat{q_r}-2} I_r \\ \vdots & \vdots & & \vdots & \vdots \\ s^{q_1-1} I_r & s^{q_1-2} I_r & \cdots & s I_r & I_r \\ s^{q_1-2} I_r & s^{q_1-3} I_r & \cdots & I_r & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ s I_r & I_r & \cdots & 0 & 0 \\ I_r & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{[r(\widehat{q_r}+q_1-1)] \times r q_1}[s] \tag{24}$$

Remark 1. $\Delta(s)$ has the form (24) in the case $\hat{q}_r \geq q_1$. In the case $q_1 > \hat{q}_r$ $\Delta(s)$ has the following form:

$$\Delta(s) = \begin{bmatrix} 0 & 0 & \cdots & 0 & s^{\widehat{q_r}-1} I_r \\ 0 & 0 & \cdots & s^{\widehat{q_r}-1} I_r & s^{\widehat{q_r}-2} I_r \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & s^{\widehat{q_r}-1} I_r & \cdots & s I_r & I_r \\ s^{\widehat{q_r}-1} I_r & & \cdots & I_r & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ s I_r & I_r & \cdots & 0 & 0 \\ I_r & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{25}$$

The matrix L as defined in (23) can be written also:

$$\begin{aligned}
L &= [-C_\infty J_\infty^{\hat{q}_r-1}B_\infty,\dots,-C_\infty J_\infty B_\infty,\,-C_\infty B_\infty \mid CB, CJB,\dots,CJ^{q_1-2}B] \\
&= [-C_\infty J_\infty^{\hat{q}_r-1}B_\infty,\dots,-C_\infty J_\infty B_\infty,-C_\infty B_\infty \mid 0,0,\dots,0] \\
&\qquad +[0,\dots,0,0 \mid CB, CJB,\dots,CJ^{q_1-2}B] \\
&= -C_\infty[J_\infty^{\hat{q}_r-1}, J_\infty^{\hat{q}_r-2},\dots,J_\infty, I_\mu \mid 0,\dots,0]\,\mathrm{diag}\,[B_\infty, B_\infty,\dots,B_\infty] \\
&\qquad +C[0,\dots,0 \mid I_n, J, J^2,\dots,J^{q_1-2}]\mathrm{diag}[B,B,\dots,B] \\
&= [-C_\infty \mid C]\begin{bmatrix} J_\infty^{\hat{q}_r-1} & J_\infty^{\hat{q}_r-2} & \cdots & J_\infty & I_\mu & | & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & | & I_n & J & \cdots & J^{q_1-2} \end{bmatrix} \\
&\times \text{block diag}[B_\infty, B_\infty,\dots,B_\infty \mid B,B,\dots,B] = L.
\end{aligned} \tag{26}$$

If we take the inverse Laplace transform L_-^{-1} of $\Delta(s)$ as defined in (24) and taking in mind that: $L_-^{-1}[s^j] = \delta^{(j)}(t), j = 0,1,\dots,$ we obtain:

$$\Delta(t) = L_-^{-1}[\Delta(s)] = \begin{bmatrix} 0 & 0 & \cdots & 0 & \delta^{(\hat{q}_r-1)}(t)I_r \\ 0 & 0 & \cdots & \delta^{(\hat{q}_r-1)}(t)I_r & \delta^{(\hat{q}_r-2)}(t)I_r \\ \vdots & \vdots & & \vdots & \vdots \\ \delta^{(q_1-1)}(t)I_r & \delta^{(q_1-2)}(t)I_r & \cdots & \delta^{(1)}(t)I_r & \delta(t)I_r \\ \delta^{(q_1-2)}(t)I_r & \delta^{(q_1-3)}(t)I_r & \cdots & \delta(t)I_r & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ \delta^{(1)}(t)I_r & \delta(t)I_r & \cdots & 0 & 0 \\ \delta(t)I_r & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{27}$$

Thus we have found:

$$\beta_{pol}(t) = L_-^{-1}[\hat{\beta}_{pol}(s)] = L\,\Delta(t)F = \beta_0\delta(t) + \beta_1\delta^{(1)}(t) + \dots + \beta_{\hat{q}_r-1}\delta^{(\hat{q}_r-1)}(t), \tag{28}$$

where β_i $i = 0,1,\dots,\hat{q}_{r-1}$ are $r\times 1$ vectors obtained after some manipulations in the terms of (28). From equation (28) and the definition of F in (16) it follows that if the initial conditions $\beta(0^-)$ $,\dots,$ $\beta^{(\hat{q}_r-1)}(0^-)$ are appropriate then $\beta(t)$ has an "impulsive behavior" at $t = 0^-$ which consists of a Dirac impulse $\delta(t)$ and its $(\hat{q}_r - 1)$ distributional derivatives. In other words when the initial conditions are imposed on $\beta(t)$ and its $(\hat{q}_{r-1})$ derivatives at $t = 0^-$, $\beta(t)$ may exhibit an impulsive behavior at $t = 0^-$ which is a consequence of the fact that (4) forces $\beta(t)$ and $\beta^{(i)}(t)$, $i = 1,2,\dots,\hat{q}_{r-1}$ to satisfy certain constraints at $t = 0^-$. The exact derivation of these constraints and their relation to the structure at $s = \infty$ of $A(s)$ are examined in [13] explicitly. From equation (28) it is also clear that the impulsive solutions of L.H.M.D.E. are closely related — because of the form of L in (16)- to the finite and infinite Jordan pairs (C,J) and (C_∞, J_∞) respectively, of the polynomial matrix $A(s)$.

3. ILLUSTRATIVE EXAMPLE

Consider the following differential equation:

$$A(\rho)\beta(t)=0,\begin{bmatrix}\rho+1 & \rho^2\\ 0 & 1\end{bmatrix}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

and the corresponding polynomial matrix (in the s-domain): $A(s)=\begin{bmatrix}s+1 & s^2\\ 0 & 1\end{bmatrix}$ with $A^{-1}(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{-s^2}{s+1}\\ 0 & 1\end{bmatrix}$ and $S^{\infty}_{A(s)}(s)=\begin{bmatrix}s^2 & 0\\ 0 & \frac{1}{s}\end{bmatrix}$ i.e. $r=2$, $n=1$, $\mu=\widehat{q}_2+1=1+1=2$, $q_1=2$, $\widehat{q}_2=1$. We can find the finite and infinite Jordan pairs of $A^{-1}(s)$ (see [12,13]):

$$C=\begin{bmatrix}1\\ 0\end{bmatrix},\quad J=[-1],\quad C_\infty=\begin{bmatrix}-1 & 1\\ 0 & 1\end{bmatrix},\quad J_\infty=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}.$$

The forms of B, B_∞ are respectively: $B=[1,-1], B_\infty=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$. From (26) we have:

$$\begin{aligned}L&=[-C_\infty,C]\begin{bmatrix}I_2 & 0\\ 0 & I_1\end{bmatrix}\begin{bmatrix}B_\infty & 0\\ 0 & B\end{bmatrix}\\
&=\begin{bmatrix}1 & -1 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1\end{bmatrix}=\begin{bmatrix}0 & -1 & 1 & -1\\ 0 & -1 & 0 & 0\end{bmatrix}.\end{aligned}$$

From (27) we have

$$\Delta(t)=\begin{bmatrix}0 & 0 & \delta(t) & 0\\ 0 & 0 & 0 & \delta(t)\\ \delta(t) & 0 & 0 & 0\\ 0 & \delta(t) & 0 & 0\end{bmatrix}.$$

From (16) we obtain:

$$\begin{aligned}F&=\begin{bmatrix}A_2 & 0\\ A_1 & A_2\end{bmatrix}\begin{bmatrix}\beta(0^-)\\ \beta^{(1)}(0^-)\end{bmatrix}\\
&=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\xi_1(0^-)\\ \xi_2(0^-)\\ \xi_1^{(1)}(0^-)\\ \xi_2^{(1)}(0^-)\end{bmatrix}=\begin{bmatrix}\xi_1(0^-)\\ 0\\ \xi_1(0^-)+\xi_2^{(1)}(0^-)\\ 0\end{bmatrix}.\end{aligned}$$

Finally from (28) we obtain that the so-called *impulsive solutions* of the differential equation are given by:

$$\beta_{\text{pol}}(t) = L\ \Delta(t)\, F$$

$$= \begin{bmatrix} 0 & -1 & 1 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \delta(t) & 0 \\ 0 & 0 & 0 & \delta(t) \\ \delta(t) & 0 & 0 & 0 \\ 0 & \delta(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1(0^-) \\ 0 \\ \xi_1(0^-) + \xi_2^{(1)}(0^-) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \delta(t)\xi_2(0^-) \\ 0 \end{bmatrix} = \beta_{\text{pol}}(t).$$

It is clear from the above that the system yields an impulsive solution in the case $\xi_2(0^-) \neq 0$.

4. CONCLUSIONS

In this paper the determination of the impulsive solutions of Linear Homogeneous Matrix Differential Equations has been investigated. By adopting the definitions of finite and infinite Jordan pairs of the associated polynomial matrix, a closed formula for the determination of the impulsive solutions has been presented.

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REFERENCES

[1] F. M. Callier and C. A. Desoer: Multivariable Feedback Systems. Springer–Verlag, New York 1982.
[2] S. L. Cambell: Singular Systems of Differential Equations. Pitman, London 1980.
[3] S. L. Cambell, C. D. Meyer and N. Rose: Applications of the Drazin inverse to linear systems of differential equations with singular constant coefficients. SIAM J. Appl. Math. *31* (1976), 3, 411–425.
[4] D. Cobb: Feedback and Pole placement in descriptor-variable systems. Internat. J. Control *33* (1981), 6, 1135–1146.
[5] D. Cobb: A further interpretation of inconsistent initial conditions in descriptor variable systems. IEEE Trans. Automat. Control *AC-28* (1983), 9.
[6] D. Cobb: On the solution of linear differential equations with singular coefficients. J. Differential Equations *46* (1982), 310–323.
[7] N. Cohen: Spectral analysis of regular matrix polynomials. Integral Equations Operator Theory *6* (1983), 161–183.
[8] F. Lewis: A survey of linear singular systems. Circuits Systems Signal Process. *5* (1986), 1.
[9] D. G. Luenberger: Dynamic systems in descriptor form. IEEE Trans. Automat. Control *AC-22* (1977), 312–321.

[10] H. H. Rosenbrock: State-Space and Multivariable Theory. Nelson, London 1970.

[11] H. H. Rosenbrock: Structural Properties of Linear dynamical systems. Internat. J. Control *20* (1974), 2, 191–202.

[12] A. I. G. Vardulakis: Linear Multivariable Control: Algebraic Analysis and Synthesis Methods. Wiley, New York 1991.

[13] A. I. G. Vardulakis and G. F. Fragulis: Infinite elementary divisors of polynomial matrices and impulsive solutions of linear homogeneous matrix differential equations. Circuits Systems Signal Process. *8* (1989), 3, 357–373.

[14] G. Verghese: Infinite-frequency Behaviour in Dynamical Systems. Ph.D. Dissertation, Dept. Electrical Engineering, Stanford Univ. 1978.

[15] G. Verghese, B. C. Levy and T. Kailath: A generalized state-space for singular systems. IEEE Trans. Automat. Control *AC-26* (1981), 4, 811–830.

[16] G. Verghese and T. Kailath: Impulsive behaviour in dynamical systems: Structure and significance. In: Proc. 4th Internat. Symp. Mathematical Theory of Networks and Systems, Delft 1979, The Netherlands, pp. 162–168.

[17] L. Gohberg, P. Lancaster and L. Rodman: Matrix Polynomials. Academic Press, New York 1982.

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