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DISTURBANCE REJECTION BY PROPORTIONAL AND DERIVATIVE OUTPUT FEEDBACK

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In this paper we take profit of the pole placement procedure using P. D. feedback and the proper approximation schemes of non proper laws (recently developed in [3] and [4]) for considering the rejection of some disturbance signal acting directly on the output of a given (A, B, C) system; it is well known that this problem has no proper solution. Our rejection scheme relies on the use of exact state reconstructor (based on some left inverse of the initial system, see [2]): the idea is to reconstruct the state which would be present if the disturbance signal were not present and then, to use this ideal observer as a compensator in such a way that the actual (disturbed) state exactly matches the previous one; this implies that the corresponding output is unaffected by the disturbance. Simple illustrative examples are given and simulation results show the effect of the approximation of derivators in the control strategy.

1. INTRODUCTION

We have recently introduced in [4] the external reachability concept for implicit (E, A, B) descriptions, $E\dot{x}(t) = Ax(t) + Bu(t)$, which is related to the ability of freely assigning the dynamics with P. D. control laws. We have given geometric conditions which guarantee this P. D. dynamics assignment and we have shown how right inverses can be used to insure some trajectory tracking and also how to assign the closed loop poles.

Introducing the external reachability concept we have given a procedure for freely assigning the dynamics, say changing (E, A, B) into (E_0, A_0, B_0), which essentially consists in the following three steps:

(i) Given any externally reachable (E, A, B) realization, apply a P. D. feedback, $u(t) = F_P x(t) + F_D \dot{x}(t) + r(t)$, in order to obtain a system with no integrators.

(ii) Use, as a compensator, the right inverse of this closed loop system in order to obtain an input-output relation equal to one, independently of the initial conditions.

(iii) Given the identity input-output relation, we can finally apply a second P. D. feedback which contains the new structure E_0, A_0, B_0, whenever $\ker \widehat{N}(\lambda E -$

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$A) \subset \ker \widehat{N}_0(\lambda E_0 - A_0)$, where $\hat{N}$ and $\widehat{N}_0$ are respectively the maximal left anihilators of B and B_0.

Let us note that this synthesis procedure is expressed in an implicit form and in the case of classical state descriptions:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1.1}$$

where $A: \mathcal{X} \to \mathcal{X}$, $B: \mathcal{U} \to \mathcal{X}$, $\ker B = \{0\}$ and $\mathcal{R}^*_{\mathcal{X}} := \operatorname{Im} B + \sum_{i=1}^{n-1} A^i \operatorname{Im} B = \mathcal{X}$, we can find explicit relations in terms of projections.

Indeed, let us first define the following operators:

$$\begin{cases} \widehat{N}: \mathcal{X} \to \mathcal{X}/\operatorname{Im} B & \text{the canonical projection,} \\ P: \mathcal{X} \to \operatorname{Im} B & \text{the natural projection on } \operatorname{Im} B \text{ along } \mathcal{X}_0, \\ Q: \mathcal{X} \to A^{-1}\operatorname{Im} B & \text{the natural projection on } A^{-1}\operatorname{Im} B \text{ along } \mathcal{X}_1, \end{cases} \tag{1.2}$$

where $\mathcal{X} = \operatorname{Im} B \oplus \mathcal{X}_0 = A^{-1}\operatorname{Im} B \oplus \mathcal{X}_1$. And noting that $\mathcal{R}^*_{\mathcal{X}} = \mathcal{X}$ implies that: $A^{-1}\operatorname{Im} B \approx \operatorname{Im} B$ (indeed, $\mathcal{R}^*_{\mathcal{X}} = \mathcal{X} \Longrightarrow \operatorname{Im} A + \operatorname{Im} B = \mathcal{X} \Longrightarrow \dim(A^{-1}\operatorname{Im} B) = \dim \operatorname{Im} B$), we can also define bijections:

$$K: A^{-1}\operatorname{Im} B \longleftrightarrow \operatorname{Im} B; \quad L: \operatorname{Im} B \longleftrightarrow \mathcal{U}. \tag{1.3}$$

Let us then apply to (1.1) the following control law:

$$u(t) = L\zeta(t) + LP(\dot{x}(t) - Ax(t)) - LKQx(t). \tag{1.4}$$

We obtain in this way the following "strictly nonproper" realization (just premultiply the fed back system by the invertible map $\begin{bmatrix} P \\ \widehat{N} \end{bmatrix}$ and note that $PBL = I$):

$$\Sigma_1 : \begin{cases} \begin{bmatrix} 0 \\ \widehat{N} \end{bmatrix} \dot{x}(t) = \begin{bmatrix} KQ \\ \widehat{N}A \end{bmatrix} x(t) + \begin{bmatrix} -I \\ 0 \end{bmatrix} \zeta(t) \\ y(t) = Px(t) \end{cases} \tag{1.5}$$

($\ker KQ \cap \ker \widehat{N}A = \mathcal{X}_1 \cap A^{-1}\operatorname{Im} B = \{0\}$. And if we take any $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \neq 0 \in \mathcal{X}_0 \oplus \operatorname{Im} B$, we have that $\begin{bmatrix} 0 \\ \widehat{N} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \lambda \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, implies that $\lambda = 0$ since $v_2 \in \operatorname{Im} B$ and thus $\begin{bmatrix} 0 \\ \widehat{N} \end{bmatrix}$ is a nilpotent operator).

Let us note that from the non properness of (1.5) and the fact that it is also a solvable system (this follows from Theorem 1 of [1] since $\ker KQ \cap \ker \widehat{N}A = \{0\}$, see also [10]), we have that (1.5) is right invertible (see [1]). A right inverse of (1.5) is the one proposed in [1] (see also [8]),which is the following system, $\Sigma_2: \operatorname{Im} B \to \operatorname{Im} B$

(obtained after applying the minimization procedure of [5]):

$$\Sigma_2: \begin{cases} \begin{bmatrix} \widehat{N} \\ 0 \end{bmatrix} \dot{\xi}(t) &= \begin{bmatrix} \widehat{N}A \\ P \end{bmatrix} \xi(t) + \begin{bmatrix} 0 \\ -I \end{bmatrix} \bar{y}(t) \\ \zeta(t) &= KQ\xi(t). \end{cases} \tag{1.6}$$

Let us finally apply a control law to the composed system $\Sigma_1(\Sigma_2(\cdot))$ which exactly assigns the desired dynamics. This control law is the following:

$$\bar{y}(t) = PB_0 r(t) + y(t) - P[E_0\dot{x}(t) - A_0 x(t)] \tag{1.7}$$

with the compulsory restriction: $\ker \widehat{N}(\lambda I - A) \subset \ker \widehat{N}_0(\lambda E_0 - A_0)$, where $\widehat{N}_0 : \mathcal{X} \to \mathcal{X}/\mathrm{Im}\, B_0$ is the canonical projection. When applying (1.7), (1.6) and (1.4) to (1.1) we finally have (remember (1.5b)):

$$E_0\dot{x}(t) = A_0 x(t) + B_0 r(t). \tag{1.8}$$

Based on this procedure, we are going to propose a P. D. control, able to reject disturbances acting directly on the output.

Let us finish this introductory section by recalling some useful results [12]:

(i) The supremal (A, B) invariant subspace contained in $\ker C$ and the infimal (C, A) invariant subspace containing $\mathrm{Im}B$, $\mathcal{V}^*$ and $\mathcal{S}_*$, are defined as follows:

$$\begin{cases} \mathcal{V}^* := \sup\{\mathcal{T} \subset \ker C \mid \exists F : \mathcal{X} \to \mathcal{U} \text{ s.t. } (A+BF)\mathcal{T} \subset \mathcal{T}\} \\ \mathcal{S}_* := \inf\{\mathcal{T} \supset \mathrm{Im}B \mid \exists K : \mathcal{Y} \to \mathcal{X} \text{ s.t. } (A+KC)\mathcal{T} \subset \mathcal{T}\} \end{cases}$$

The non unique F and K appearing in these definitions are identified by $F \in \mathbf{F}(A, B; \mathcal{V}^*)$ and $K \in \mathbf{F}(C, A, \mathcal{S}_*)$.

(ii) Let $\mathcal{R}^*$ be the supremal controllability subspace of the pair (A, B) contained in $\ker C$, then:

$$\mathcal{R}^* = \mathcal{V}^* \cap \mathcal{S}_*.$$

Result 1. (Lebret, Loiseau [9]) The finite elementary divisors of the input restricted pencil,

$$\begin{bmatrix} N(\lambda I - A) \\ -C \end{bmatrix},$$

are those of the map $(\mathcal{V}^*/\mathcal{R}^* \| A + BF \| \mathcal{V}^*/\mathcal{R}^*)$, where $N : \mathcal{X} \to \mathcal{X}/\mathrm{Im}\, B$ is the canonical projection and F is any element of $\mathbf{F}(A, B; \mathcal{V}^*)$.

2. OUTPUT DISTURBANCE REJECTION

In this section, we are going to adapt the procedure recalled in the Introduction in order to reject disturbances which act directly on the output. For this, let us consider the following state description:

$$\dot{x}(t) = Ax(t) + Bu(t); \quad y(t) = Cx(t) + e(t), \tag{2.1}$$

where $A: \mathcal{X} \to \mathcal{X}$, $B: \mathcal{U} \to \mathcal{X}$ and $C: \mathcal{X} \to \mathcal{Y}$. We will assume, as usually, that the inputs and outputs are independent, i.e.,

$$\ker B = \{0\} \quad \text{and} \quad \operatorname{Im} C = \mathcal{Y}. \tag{2.2}$$

We shall here restrict our attention to square and invertible (A, B, C) operators, i.e.:

$$\mathcal{U} \approx \mathcal{Y} \quad \text{and} \quad \mathcal{V}^* + \mathcal{S}_* = \mathcal{X} \tag{2.3}$$

or equivalently:

$$\mathcal{R}^*_{\mathcal{X}} = \{0\}. \tag{2.4}$$

We will also assume that the disturbance, $e(t)$, is bounded and at least n-times differentiable, i.e.,

$$\|e(t)\| < \infty \quad \text{and} \quad e(t) \in C^n. \tag{2.5}$$

We shall first consider minimum phase systems, i.e., systems such that

$$\mathit{spectrum}\{\mathcal{V}^* | A + BF | \mathcal{V}^*\} \subset \mathbb{C}^- \tag{2.6}$$

for any friend F of $\mathcal{V}^*$ (F$\in \mathbf{F}(\mathcal{V}^*)$), and $\mathbb{C}^- := \{s \in \mathbb{C} \,|\Re e\ s < 0\}$ and later non minimum phase ones. In both cases we will assume that system (2.1) has been already stabilized, i.e.,

$$\mathit{spectrum}\{A\} \subset \mathbb{C}^-. \tag{2.7}$$

2.1. Minimum Phase Systems

In terms of the operators (1.2) and (1.3) let us define the control law (cf. (1.4)):

$$u(t) = LP\left(Br(t) - (\dot{z}(t) - Az(t)) + (\dot{x}(t) - Ax(t))\right), \tag{2.8}$$

where $z(t)$ is the output of the following filter:

$$\begin{bmatrix} \widehat{N} \\ 0 \end{bmatrix} \dot{z}(t) = \begin{bmatrix} \widehat{N}A \\ C \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ -I \end{bmatrix} y(t). \tag{2.9}$$

The following fact is proved in the appendix:

Fact 3.1. The pencil

$$\begin{bmatrix} \widehat{N}(\lambda I - A) \\ -C \end{bmatrix}$$

is regular and:

$$\left\{\lambda \middle| \det \begin{bmatrix} \lambda\widehat{N} - \widehat{N}A \\ C \end{bmatrix} = 0\right\} = spectrum\{\mathcal{V}^*|A + BF|\mathcal{V}^*\}; \quad \mathrm{F} \in \mathbf{F}(\mathcal{V}^*). \tag{2.10}$$

Applying (2.8) to (2.1), we obtain:

$$(I - BLP)\,\dot{x}(t) = (I - BLP)\,Ax(t) + BLPBr(t) - BLP(\dot{z}(t) - Az(t)). \tag{2.11}$$

Applying P to (2.11) we have (remember that: $PBL = I$):

$$P\dot{z}(t) = PAz(t) + PBr(t). \tag{2.12}$$

From (2.12) and (2.9), we have the following state description:

$$\dot{z}(t) = Az(t) + Br(t); \quad y(t) = Cz(t). \tag{2.13}$$

We see from (2.13) that we have isolated the disturbance $e(t)$ from the output, by using the filter (2.9) as a compensator. Now, taking into account that $\dot{x}(t) - Ax(t) = Bu(t)$ and that $LPB = I$, we can write (2.8) as

$$u(t) = u(t) + r(t) - LP(\dot{z}(t) - Az(t)). \tag{2.14}$$

From (2.14) we realize that we have an algebraic loop and up to now numerical computers are not able to implement it (this is not the case for analog computers). But this is easily surmounted using the following approximation (see Figure 1):

$$\varepsilon u(t) = r(t) - LP(\dot{z}(t) - Az(t)). \tag{2.15}$$

From (2.15) and (2.1), equation (2.12) becomes:

$$P\dot{z}(t) = PAz(t) + PBr(t) - \varepsilon P(\dot{x}(t) - Ax(t)) \tag{2.16}$$

and from (2.16) and (2.9), we have in place of (2.13) (since: $\widehat{N}\dot{x}(t) = \widehat{N}Ax(t)$):

$$\dot{z}(t) = Az(t) + Br(t) - \varepsilon(\dot{x}(t) - Ax(t)). \tag{2.17}$$

Defining now: $\zeta(t) = z(t) + \varepsilon x(t)$, we have from (2.17), (2.1b) and (2.9):

$$\left\{ \begin{array}{rcl} \dot{\zeta}(t) & = & A\zeta(t) + Br(t) \\ (1+\varepsilon)\,y(t) - \varepsilon e(t) & = & C\zeta(t) \end{array} \right. \tag{2.18}$$

which implies (denoting $p = \mathrm{d}/\mathrm{d}t$):

$$\left\{ \begin{array}{rcl} F(p)\,y(t) & = & \frac{1}{1+\varepsilon}G(p)\,r(t) + \frac{\varepsilon}{1+\varepsilon}F(p)\,e(t) \\ G(p)\,u(t) & = & \frac{1}{1+\varepsilon}G(p)\,r(t) - \frac{1}{1+\varepsilon}F(p)\,e(t), \end{array} \right. \tag{2.19}$$

where $C[\lambda I - A]^{-1}B = F^{-1}(\lambda)\,G(\lambda)$.

Let us now consider the internal stability. Let $\tilde{z}(t) = x(t) - z(t)$, we have from (2.1) and (2.9):

$$\begin{bmatrix} \widehat{N} \\ 0 \end{bmatrix} \dot{\tilde{z}}(t) = \begin{bmatrix} \widehat{N}A \\ C \end{bmatrix} \tilde{z}(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} e(t) \tag{2.20}$$

which dynamics is given by (2.10). From the minimum phase assumption (2.6), the dynamics of the state error $\tilde{z}(t)$ is thus exponentially stable; and from the fact that e(t) is bounded and C^n (see (2.5)), we conclude that $\tilde{z}(t)$ is bounded and so $x(t)$ too (see also: (2.9), (2.19), (2.10), (2.6) and (2.7)).

We can make the following important observations:

(i) With this control strategy the dynamics of the original system is not changed, and the control signal remains bounded with variation dictated by the disturbance itself since $e(t)$ is bounded and $G(p)$ is a Hurwitz polynomial (see (2.19) and remember (2.5) and (2.6)).

(ii) In view of the boundedness of the disturbance, we then have that for a sufficiently small ε the disturbance will be practically neglectible on the output (assuming, of course, that the sytem has been already stabilized).

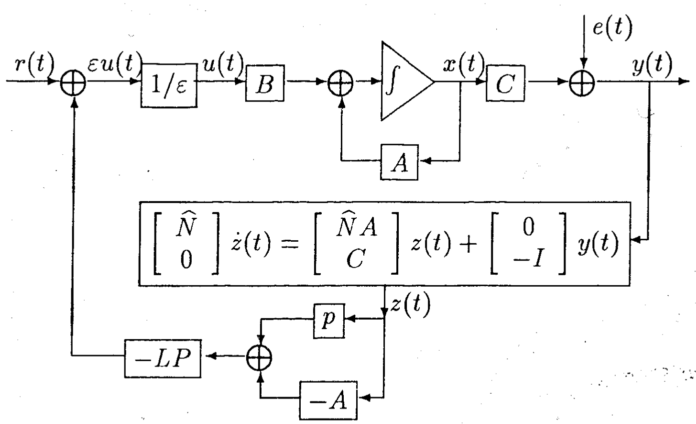


Fig. 1.

Let us finish this subsection with the following example which shows how to use suitable approximations of derivators in the practical implementation of our control strategy:

Example 1. Let us consider the following stabilized system:

$$
\left\{
\begin{array}{rcl}
\dot{x}(t) & = & \begin{array}{c} \overleftrightarrow{A^{-1}\operatorname{Im} B} \quad \overleftrightarrow{\quad \mathcal{X}_1 \quad} \\ \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{array}\right] \\ \underleftrightarrow{\quad \mathcal{X}_0 \quad} \quad \underleftrightarrow{\operatorname{Im} B} \end{array} x(t) + \left[\begin{array}{c} 0 \\ 0 \\ b_0 \end{array}\right] u(t) \\
y(t) & = & \left[\begin{array}{ccc} \beta & 1 & 0 \end{array}\right] x(t) + e(t).
\end{array}
\right.
\tag{2.21}
$$

The operators $\widehat{N}, P, \widehat{N}A, Q, K$ and L are:

$$
\left\{
\begin{array}{l}
\widehat{N} = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]; \quad P = \left[\begin{array}{ccc} 0 & 0 & 1 \end{array}\right]; \quad \widehat{N}A = \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \\
Q = \left[\begin{array}{ccc} 1 & 0 & 0 \end{array}\right]; \quad K = 1; \qquad L = 1/b_0.
\end{array}
\right.
\tag{2.22}
$$

The filter (2.9) is:

$$
\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ - & - & - \\ 0 & 0 & 0 \end{array}\right] \dot{z}(t) = \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ - & - & - \\ \beta & 1 & 0 \end{array}\right] z(t) + \left[\begin{array}{c} 0 \\ 0 \\ -- \\ -1 \end{array}\right] y(t)
\tag{2.23}
$$

and the control law (2.15) is:

$$
\varepsilon u(t) = -\frac{1}{b_0}\left(\dot{z}_3(t) + \left[\begin{array}{ccc} a_3 & a_2 & a_1 \end{array}\right] z(t)\right) + r(t).
\tag{2.24}
$$

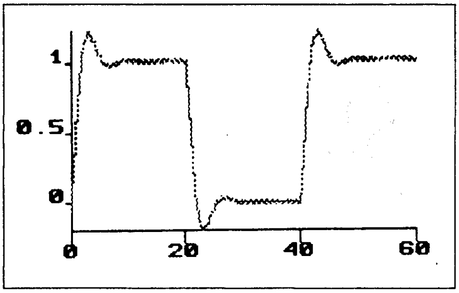
Note from (2.21),(2.23) and (2.24) that (cf. (2.19)):

$$
y^{(3)}+a_1\ddot{y}(t)+a_2\dot{y}(t)+a_3y(t) = \frac{b_0}{1+\varepsilon}[\beta r(t)+\dot{r}(t)]+\frac{\varepsilon}{1+\varepsilon}[e^{(3)}+a_1\ddot{e}(t)+a_2\dot{e}(t)+a_3e(t)].
\tag{2.25}
$$

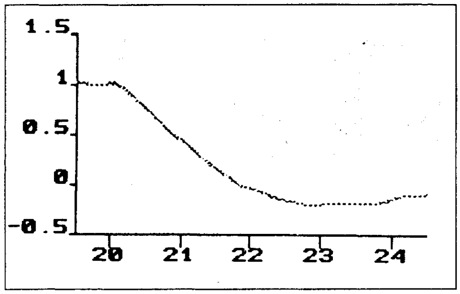
Now since system (2.21) is stable, we can apply the approximation, for state reconstructors, given in [3] (see also [6]), and obtain:

$$
\left\{
\begin{array}{rcl}
\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \dot{\bar{z}}(t) & = & \left[\begin{array}{cccc} 1 & 1/\beta & 0 & \varepsilon_0 \\ 0 & 1 & 0 & \varepsilon_0\alpha_1 \\ 0 & 0 & 1 & \varepsilon_0\alpha_2 \\ -a_3 & -a_2 & -a_1 & 1 \end{array}\right] \bar{z}(t) + \left[\begin{array}{c} -1/\beta \\ 0 \\ 0 \\ 0 \end{array}\right] y(t) \\
\varepsilon u(t) & = & r(t) - \left[\begin{array}{cccc} 0 & 0 & 0 & 1/b_0 \end{array}\right] \bar{z}(t).
\end{array}
\right.
\tag{2.26}
$$

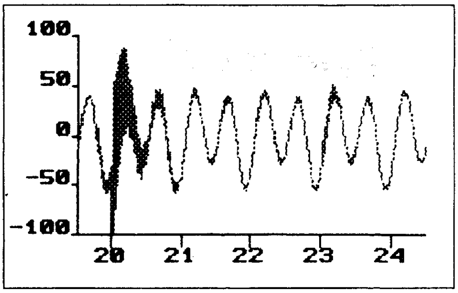
In Figure 2, we show some simulations.



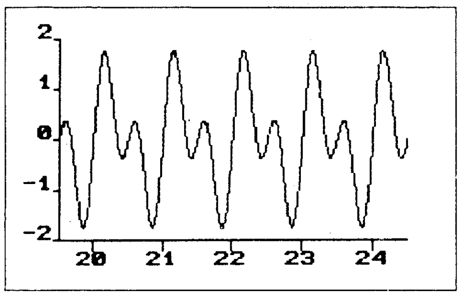
$y(t)$



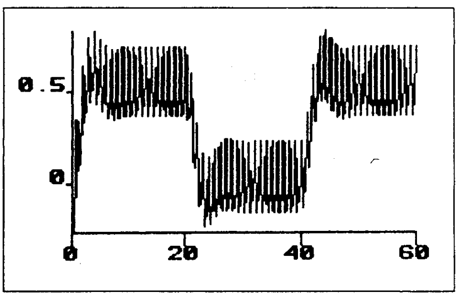
$y(t)$



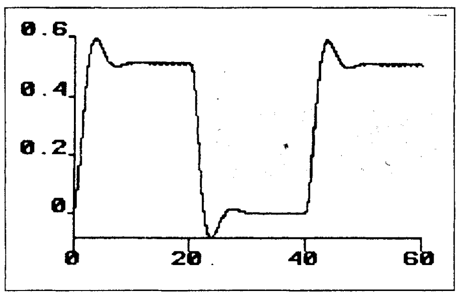
$u(t)$



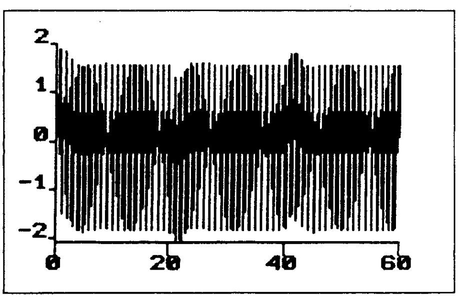
$e(t)$



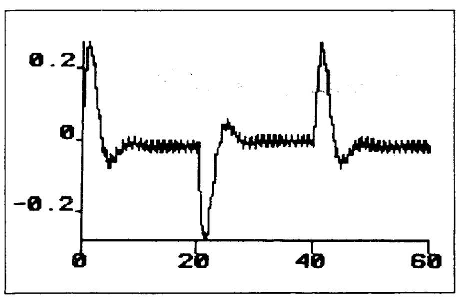
$x_1(t)$



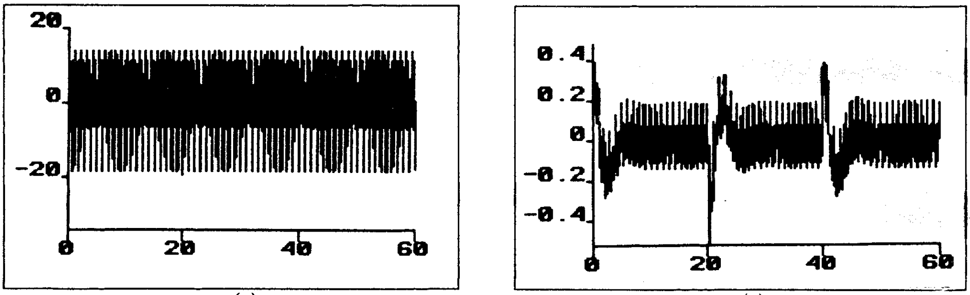
$\bar{z}_1(t)$



$x_2(t)$



$\bar{z}_2(t)$



$x_3(t)$ $\bar{z}_3(t)$

Fig. 2. $r(t) = \begin{cases} 1 & 40(k-1) \le t \le 20(2k-1) \\ 0 & 20(2k-1) < t < 40k \end{cases}$ with $k = 1, 2, \ldots$, $e(t) = \sin 4\pi t + \sin 2\pi t$, $(a_1, a_2, a_3; \beta; b_0) = (11, 11, 10; 2; 5)$, and $(\varepsilon; \varepsilon_0, \alpha_1, \alpha_2) = (0.01; 0.001, 3, 3\varepsilon_0)$.

2.2. Non Minimum Phase Systems

In this subsection, we restrict ourselves to SISO systems described by the following polynomial description (denoting $p = \mathrm{d}/\mathrm{d}t$):

$$\begin{cases} f(p)\,\bar{y}(t) &= g(p)\,u(t) \\ y(t) &= \bar{y}(t) + e(t), \end{cases} \tag{2.27}$$

where the disturbance is generated in the following way:

$$h(p)\,e(t) = 0. \tag{2.28}$$

We assume that the polynomials $\mathrm{h}(\lambda)$ and $\mathrm{g}(\lambda)$ are coprime.

The control law proposed here is described in the following three steps:

(i) We are going to synthesize the filter:

$$\sigma(t) = -f(p)\,y(t) + g(p)\,u(t) \tag{2.29}$$

which aim is to "measure" the disturbance. Indeed, $\sigma(t) = -f(p)\,e(t)$, noting again that $h(p)\,\sigma(t) = 0$,

(ii) Now, we solve the Bezout equation:

$$1 = g(p)\,r(p) + h(p)\,s(p). \tag{2.30}$$

(iii) And finally, the control law is (see Figure 3):

$$u(t) = r(p)\,\sigma(t). \tag{2.31}$$

We can verify from (2.27)–(2.31) that:

$$\begin{aligned} f(p)\,y(t) &= g(t)\,u(t) + f(p)\,e(t) \\ &= f(p)\,[1 - g(p)\,r(p)]\,e(t) = 0. \end{aligned} \tag{2.32}$$

And since we have assumed that the system has been already stabilized and that the disturbance is bounded (2.28), the system is internally stable.

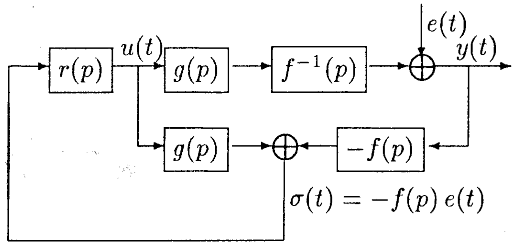


Fig. 3.

Let us finish this section with the following example:

Example 2. Let us consider the following system:

$$\begin{cases} [p^2 + 2ap + a^2]\,\bar{y}(t) &= (p-\beta)\,u(t) \\ y(t) &= \bar{y}(t) + e(t), \end{cases} \tag{2.33}$$

where

$$[p^2 + p]\,e(t) = 0 \tag{2.34}$$

from (2.30) we have that the polynomial $r(p)$ is (with $g(p) = p - \beta$ and $h(p) = p^2 + p$):

$$r(p) = -\frac{1}{\beta} - \frac{1}{\beta(1+\beta)}\,p. \tag{2.35}$$

The following filter gives a state approximation of (2.29):

$$\begin{cases} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \dot{\bar{z}}(t) &= \begin{bmatrix} 1 & 0 & \varepsilon \\ 0 & 1 & \varepsilon\alpha \\ 0 & 0 & 1 \end{bmatrix} \bar{z}(t) + \begin{bmatrix} 0 & 1 \\ -1 & 2a \\ \beta & a^2 \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \\ \bar{\sigma}(t) &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \bar{z}(t). \end{cases} \tag{2.36}$$

Note that, although we have used a stable approximation of derivation ($\bar{\sigma} \to \sigma(t)$ when $\varepsilon \to 0$), the signal $\bar{\sigma}$(t) is only given by e(t) and thus is independent on u(t).

The control law is:

$$\begin{cases} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \dot{\bar{\xi}}(t) &= \begin{bmatrix} 1 & \varepsilon \\ 0 & 1 \end{bmatrix} \bar{\xi}(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \bar{\sigma}(t) \\ u(t) &= \begin{bmatrix} \frac{-1}{\beta} & \frac{-1}{\beta(1+\beta)} \end{bmatrix} \bar{\xi}(t). \end{cases} \tag{2.37}$$

From (2.37), (2.36), (2.34) and (2.33) we realize that:

$$\begin{cases} \phi_1(p)\,\phi_2(p)\,[p^2 + 2ap + a^2]\,y(t) &= \varepsilon[p(p+\alpha_1) + p\phi_1(p)]\,e(t) \\ \phi_2(p)\,u(t) &= -[\frac{1}{\beta} - \frac{1}{\beta(1+\beta)}p]\,[p^2 + 2ap + a^2]\,e(t), \end{cases} \tag{2.38}$$

where $\phi_1(p) = \varepsilon p^2 + \varepsilon \alpha p + 1$, $\phi_2(p) = 1 + \varepsilon p$.

In Figure 4 we show some simulations.

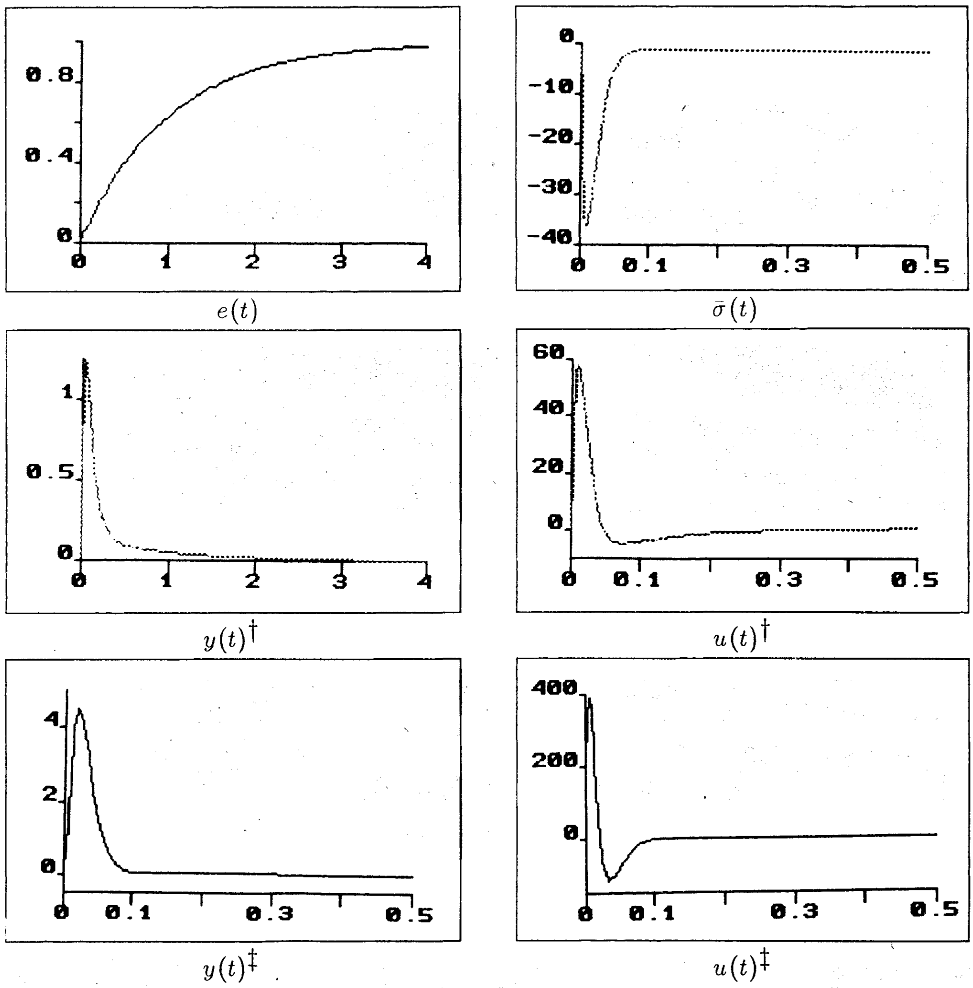


Fig. 4. $(a;\beta) = (1;2)$, $\varepsilon_0 = 0.01$, $e(0) = 0$ and $\dot{e}(0) = 1$. $\dagger : \varepsilon = 0.1$. $\ddagger : \varepsilon = 0.01$.

Let us note that the smaler ε is, the faster the disturbance is rejected, but the bigger the required energy is.

2.3. Concluding Remarks

The aim of this paper has been to show that it is possible to use in a practical way derivative actions for the solution of disturbance rejection. This is feasible thanks to the recent approximations of non proper compensators by some proper ones (see [3] and [6]): the control laws rely on a family of proper compensators, parametrized by some ε, which tends in a stable way towards the theoretical non proper one as ε tends to zero. This contribution may be thought as a new starting point for designing controllers on the basis of implicit systems theory (using derivators): we can, not only revisit some classical control problems and simplify the control design but also consider some new control problems for which no proper solution exist (like the rejection of disturbances acting directly on the output).

APPENDIX

Proof of Fact 3.1:

Let us prove first that the pencil is regular [7]. First note that since $\mathcal{U} \approx \mathcal{Y}$, this pencil is square. Let us suppose that it is not monic, i.e., there exists $\mathcal{T} \subset \ker \widehat{N}$ with $\dim \mathcal{T} > 0$ such that:

$$\begin{bmatrix} \lambda \widehat{N} - \widehat{N} A \\ -C \end{bmatrix} \mathcal{T} = \{0\} , \quad \forall \lambda \in \mathbb{C} \tag{A.1}$$

namely, $\mathcal{T} \subset \ker C$ and $\mathcal{T} \subset A^{-1}\mathrm{Im}\ B$ (remember that $\ker \widehat{N} = \mathrm{Im}\ B$), i.e., $\mathcal{T} \subset \mathcal{V}^*$. On the other hand, $\mathcal{T}$ is the limit of the following algorithm (see Lemma 5.2 of [12]):

$$\mathcal{S}^0 = \{0\}; \quad \mathcal{S}^\mu = \mathcal{T} \cap (A\mathcal{S}^{\mu-1} + \mathrm{Im}\, B). \tag{A.2}$$

Indeed, since $\mathcal{T} \subset A^{-1}\mathrm{Im}\ B$ we have that $\mathcal{S}^1 = \mathcal{S}^2 = \mathcal{T}$.

We then have from Theorem 5.3. of Wonham [12] that $\mathcal{T}$ is a controllability subspace contained in $\ker C$ which implies that (see Lemma 1.1 of Morse [11]): $\mathcal{T} \subset \mathcal{V}^* \cap \mathcal{S}_*$ and thus $\mathcal{T} = \{0\}$, since $\mathcal{V}^* \cap \mathcal{S}_* = 0$, from our invertibility assumption. Finally (2.10) directly comes from Result 1. □

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