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The Electroosmotic Model of Matter Transport through Biological Barriers

JOZEF MICHALOV, ZUZANA PALČÁKOVÁ

The mathematico-physical model presented in the article is basing on the principles of the thermodynamics of irreversible processes, for determining the components (volume flow and streaming current) of longitudinal transport.

The model is available for studying the longitudinal transport, especially for that reason it is not necessary to take into account the structure of the tissue where transport phenomenon is studied.

INTRODUCTION

Working of complicated biological dynamical systems can be analysed from the points of view of interchanging informations, energy and materials. These quantities bound the ones to the others by certain relations, which can be described by help of respective mathematical model.

By finding the solution of this model, it is possible to study exactly complicated problems composed of many reactions. It is, simultaneously, the verification of a given model.

Existence of a process warrants the existence of a general solution. This means, that each mathematico-physical information model is so true as it reflects features of an object, objectively existing in relation to circulating and working up the informations. The above mentioned existence of a general solution is a verification of the feature in question. E.g., in a system, where the transmissible barrier (*membrane, cell wall, segments of roots or branches of higher plants*) divides two electrolytic solutions, not identical with respect to their concentration content, an electrochemical (ΔE) and a concentration (ΔC) difference arises. Changing of the concentration difference causes changes of the electromotoric voltage in the system. This is one of the informations concerning the phenomena taking place in the structure of the barrier itself.

In the case of an experiment, this change of the electromotoric voltage is an answer to the question, given to the system by inducing the concentration difference at the sites of the permeable barrier.

Knowledge of such a system is made possible by giving into relation the question given to the system with the answer given by it, or, in other words, the construction of mathematico-physical model, enables us to better understanding of this system.

An important basic process for the activity of the living object, there is the transport of matter important for life, from the surrounding solutions into the object. This means, the phenomenon of transport, i.e. the transfer of matter and energy to be a basic phenomenon in the process of nutrition of certain biological object. There is necessary: first of all to determine the volume of a matter passing a barrier through a unit area in a unit time, i.e. to determine *the volume flow*. Moreover, it is necessary to determine *the streaming current*, defined by the amount of the particles with electric charge which have passed the barrier through a unit area in a unit time.

Both these quantities, either the volume flow or the streaming current, are very important for the existence and for the functioning of a biological object. It is very useful to know the regularities controlling these kinds of flow, from the point of view of their importance for the correct functioning of a biological object.

Aim of the present work is, to show one mathematical model, also long ago existing in physics, which is derived from the principles of thermodynamics of irreversible processes. It may serve as a tool for studying the regularities controlling volume flow and streaming current in biological objects.

THE THEORETICAL FOUNDATION OF THE MODEL

By affecting an outer electric field upon a system composed of a fluid environment split up by a permeable barrier in two components, the variable phase is set in motion with respect to the solid phase [3], [14]. It is a matter of fact that in the wake of this motion matter and electric energy are transferred [1], [14]. Both these quantities may be measured with the aid of volume flow J and electric current I provided that both fluid phases are of the same composition [2].

It is an electroosmotic process that passes off in the system.

By what this process stands out is that the outer difference of electric potentials $\Delta E \neq 0$ between two phases at a pressure difference of $\Delta P = 0$ gives rise to the flow of matter through the permeable barriers [2], [3], [4], [10], [11]. The size of the generated matter flow may be described by the relation

$$(1) \quad J = \alpha \frac{S}{l} \Delta E; \quad \Delta P = 0,$$

where α is the electroosmotic filtration coefficient, S the surface through which matter flow is passing, l the thickness of the barrier.

If the process is an inverse one, i.e. if pressure difference $\Delta P \neq 0$ between two phases at $\Delta E = 0$, the electric current gives rise to a so called *streaming current* [2], [4], a phenomenon termed pressure current conductivity [3], [4] and it may be described by relation

$$(2) \quad I = \beta \frac{S}{l} \Delta P; \quad \Delta E = 0,$$

where β is the coefficient of pressure current conductivity. Both coefficients α and β may be positive or negative. Their sign is given by the flow direction.

If matter is transferred through the barrier under a pressure difference ΔP and under zero electric voltage $\Delta E = 0$, we speak of filtration [2], [4] and volume flow J may be described by the relation

$$(3) \quad J = \gamma \frac{S}{l} \Delta P; \quad \Delta E = 0,$$

where γ is the filtration coefficient, or, in other words, of hydraulic conductivity. Under a small ΔP , the filtration coefficient depends on temperature T and, to a slight degree, on the mean pressure in the system. Coefficient γ is positive because matter transfer always passes from a phase with higher pressure towards a phase with lower pressure. In the case of a membrane, it is the diameter of the molecules of a matter must be taken in account and together with that of the pores of the corresponding permeable barrier.

If in the circuit battery-fluid phase and back, there is a transflow of electric current I due to connecting voltage $\Delta E \neq 0$ and at $\Delta P = 0$, its size may be expressed by the relation

$$(4) \quad I = \kappa \frac{S}{l} \Delta E; \quad \Delta P = 0,$$

where κ stands for conductivity, depending on temperature, solution composition and negligibly on the mean pressure in the system. It does not depend on area S , length l and voltage ΔE .

The simultaneous occurrence of all four phenomena [1], [11], [14] (filtration, electric conductivity, electroosmosis and current conductivity) i.e. the case when $\Delta P \neq 0$ and $\Delta E \neq 0$, may be characterized by a system of equations:

$$(5) \quad \begin{aligned} J &= \frac{\alpha}{\kappa} I + \frac{S}{l} \left(\gamma - \frac{\alpha\beta}{\kappa} \right) \Delta P, \\ I &= \frac{\alpha}{\gamma} J + \frac{S}{l} \left(\kappa - \frac{\alpha\beta}{\gamma} \right) \Delta E, \end{aligned}$$

where the first term at the right side of the first equation describes at $\Delta P = 0$ the electroosmotic process and the first term at the right side of the second equation characterized at $\Delta E = 0$ the current conductivity.

Empirically, electrokinetic phenomena may be described with the aid of four coefficients: 1. the electroosmotic filtration coefficient α , 2. the pressure current conductivity coefficient β , 3. the filtration coefficient γ and 4. the electric conductivity coefficient κ .

If measurements are carried out at a constant temperature and in a zero thermal flow, it is possible to derive, on the basis of equations (5), a thermodynamic-phenomenological model [8], [9], [10] describing the electrokinetic phenomena on biological tissues:

$$(6) \quad \begin{aligned} J &= L_{11} \Delta P + L_{12} \Delta E, \\ I &= L_{21} \Delta P + L_{22} \Delta E, \end{aligned}$$

where coefficient L_{11} describes the phenomenon of filtration, L_{12} the phenomenon of electroosmotic filtration, L_{21} the phenomenon of pressure current conductivity and L_{22} that of electric conductivity.

For experimental use it is of importance to introduce, with the aid of Saxén's relation [8], the ratio characterizing electroosmotic efficiency

$$\left(\frac{J}{I} \right)_{\Delta P=0} = \frac{L_{12}}{L_{22}}$$

and, to make coefficients to satisfy the Onsager conditions [8], [9]

$$L_{12} = -L_{21}; \quad L_{11} > 0; \quad L_{22} > 0; \quad L_{22}L_{11} - L_{21}^2 > 0.$$

THE APPLICATION OF THE MODEL

To characterize the transport of the substances and molecules of water with the aid of the electroosmotic model, it is needful, first of all, to establish coefficients L_{11} , L_{12} , L_{21} and L_{22} [3], [4], [10]:

1. Hydraulic conductivity L_{11} pertaining to the area unit of the investigated sample at a zero voltage through sample may be determined with the help of the relation:

$$L_{11} = \left(\frac{J}{\Delta P} \right)_{\Delta E=0} I,$$

where J is the volume flow through the area unit, I is the length of the sample, ΔP the pressure difference. The dimension $[L_{11}]$ is $[m^4 \cdot N^{-1} \cdot s^{-1}]$.

2. The electrokinetic cross coefficient L_{12} with the help of the relation

$$L_{12} = L_{22} \left(\frac{J}{I - I_0} \right)_{\Delta P=0},$$

where I stands for electric current flowing through the circuit at $\Delta P = 0$ and I_0 is the initial current at $\Delta P = 0$. The dimension $[L_{12}]$ is $[m^2 \cdot V^{-1} \cdot s^{-1}]$.

3. The second electrokinetic cross coefficient L_{21} with the aid of the relation:

$$L_{21} = - L_{22} \left(\frac{\Delta E}{\Delta P} \right)_{I=0}.$$

Dimension $[L_{21}]$ amounts to $[m^2 \cdot V^{-1} \cdot s^{-1}]$.

4. Electric conductivity L_{22} with the aid of relation

$$L_{22} = \frac{l}{RS},$$

where R is the ohmic resistance, S the area of the sample cross-section and l is the length of the sample. The dimension $[L_{22}]$ is $[\Omega^{-1} \cdot m^{-1}]$.

The measured values of phenomenological coefficients allow to determine the volume flow and the streaming current through the biological structure from equations (6) under given ΔE and ΔP . This is significant from the point of view of controlling the vital functions of the biological object.

EXAMPLE OF MODEL APPLICATION

With help of the model referred to it was proved that at the longitudinal transport through the root and twig segments of the white birch, the pressure plays the main role, not the electric difference in matter transport. The electric difference is decisive in matter transport to short distances [5], [6], [13].

At present, phenomenological coefficients (Tab. 1) are known for four types of objects enabling us to determine the given object at a given ΔE and ΔP , at a given

Tab. 1.

Coefficient:	Membrane [12]	Root [6]	Maple [13]	Birch [6]
$L_{11}[m^4 \cdot N^{-1} \cdot s^{-1}]$	$1.40 \cdot 10^{-16}$	$23.3 \cdot 10^{-10}$	$64.1 \cdot 10^{-10}$	$3.9 \cdot 10^{-10}$
$L_{12}[m^2 \cdot V^{-1} \cdot s^{-1}]$	$6.88 \cdot 10^{-9}$	$0.09 \cdot 10^{-9}$	$3.2 \cdot 10^{-9}$	$2.16 \cdot 10^{-9}$
$L_{21}[m^2 \cdot V^{-1} \cdot s^{-1}]$	$6.00 \cdot 10^{-9}$	$14.7 \cdot 10^{-8}$	$3.52 \cdot 10^{-9}$	$2.34 \cdot 10^{-9}$
$L_{22}[\Omega^{-1} \cdot m^{-1}]$	$1.20 \cdot 10^{-3}$	$1.42 \cdot 10^{-3}$	$3.9 \cdot 10^{-3}$	$48.7 \cdot 10^{-3}$

temperature the volume flow and streaming current. They are 1. Membrane of the *Nitella* algae, 2. Twig segments of the white maple, 3. Twig segments of white birch, 4. Segments of nodal roots of maize.

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