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Kybernetika, Vol. 19 (1983), No. 4, 309--318

Persistent URL: http://dml.cz/dmlcz/125094

KYBERNETIKA — VOLUME 19 (1983), NUMBER 4

BOUNDARIES FOR THE AVERAGE LENGTH OF STRATEGIC TESTS

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A strategic test is a generalization of Wald's sequential probability ratio test enabling a controlled choice of sampled random variables. Results presented in the paper show boundaries for the average length of strategic tests which are independent of the control policy used. A sufficient condition is given representing a situation when the classical Wald's test cannot be improved by any control policy.

INTRODUCTION

There are two widely used methods of sequential decision-making.

First, Wald's sequential probability ratio test ([1], [3]) has become one of the classical methods of mathematical statistics. The second method, less used, is a decision-making by sequential questionnaires ([4], [5], [6], [7], [8], [9]). The two approaches differ not only in the methods used but also in the models they are proposed for.

For the purposes of this paper the most interesting difference is the following one.

Both methods are sequential. At each step in both methods one has to decide whether to stop or not. A decision to stop is accepted when the obtained information is sufficient for taking a final decision. When the information is unsufficient the sequential questionnaire has to determine the elementary test which is to be applied in the next step. This action has no equivalent in the Wald's sequential test where the sequence of elementary tests is supposed to be fixed.

However, one often comes across the need for such generalization of the Wald's sequential test as is shown in the following example of concrete quality test.

There is a file of cubes made of concrete and each block can be examined for its tensile strength or compression strength. Since both tests are destructive no block can be subjected to both examinations.

Let us consider how the Wald's sequential test would be generalized for this example.

Quite naturally the idea occurs to consider such sequential test that having decided not to stop determines at the same time which of the possible examinations is to be performed in the next step.

The situation thus arising is not so complicated, when regular or random (but independent of the preceding results) alternation of possible examinations is used. But, another situation arises when one takes into consideration a controlled (depending on the preceding results) alternation of particular examinations. Naturally, one hopes that a suitable control policy will shorten in the mean the sequential test. However, at the same time, theoretical difficulties accumulate because the results of particular examinations become stochastically dependent and some Wald's results cannot be used any longer.

Some of these problems are treated in this paper.

STRATEGIC FUNCTION

Let $(X, \mathscr{X})$ denote the measurable sample space of all random variables considered throughout the paper. Let N be the set of all positive integers $\{1, 2, 3, \ldots\}$.

In the sequel, by $\mathscr{X}^i$-measurability of a function $f(x_1, x_2, \ldots)$ defined on X^∞ we shall understand its measurability with respect to the σ-algebra $\mathscr{X}^i \times \{\emptyset, X\} \times \times \{\emptyset, X\} \times \ldots$ of the corresponding cylinders. Obviously, such a function depends only on the first i coordinates $x_1, x_2, \ldots, x_i$ of the infinite sequence $x_1, x_2, \ldots$

A definition of a strategic function is introduced first. This function determines which random variable is to be sampled in every individual step. That is why the first argument of strategic function is an integer indicating the serial number of the step.

An important property of the introduced strategic functions is that the value of a strategic function in the ith step depends only on the results of the preceeding $(i - 1)$ steps. Formally:

Definition 1. The function

$$\mu : N \times X^\infty \to \{1, 2\}$$

is a strategic function when it is for every $i \in N$ $\mathscr{X}^{i-1}$-measurable.

Remark. Regarding the above-mentioned condition a shortened notation

$$\mu(i, x_1, \ldots, x_{i-1}, x_i, \ldots) = \mu(i, x_1, \ldots, x_{i-1})$$

will be used throughout the paper.

STRATEGIC TEST

Let H_0 and H_1 be two alternative hypotheses concerning the probability distributions of two (abstract) random variables ξ^1 and ξ^2 (corresponding to two different elementary tests).

Consider a sequence

$$(1) \qquad (\xi_1^1, \xi_1^2), \quad (\xi_2^1, \xi_2^2), \quad (\xi_3^1, \xi_3^2), \ldots$$

of independent repetitions of the pair (ξ^1, ξ^2) and some fixed strategic function μ. Observing the sequence (1) sequentially one may use the strategic function μ to sample only one random variable from each pair. Thus, during the first step x_1 is observed as a realization of the random variable $\xi_1^{\mu(1)}$. Then when x_1 is known, x_2 is observed as a realization of the random variable $\xi_2^{\mu(2,x_1)}$. The process continues in this way, so when the outcomes $x_1, \ldots, x_{i-1}$ of the first $i - 1$ steps are known, the random variable $\xi_i^{\mu(i,x_1,\ldots,x_{i-1})}$ will be sampled in the ith step.

In other words, a strategic function μ is used to transform each sequence (which is a sequence of realizations of random variables (1))

$$(x_1^1, x_1^2), \quad (x_2^1, x_2^2), \quad (x_3^1, x_3^2), \ldots$$

into a single sequence $x_1, x_2, \ldots$ The transformation proceeds according to the recursive relation:

$$(2) \qquad x_i = x_i^{\mu(i,x_1,\ldots,x_{i-1})}.$$

Definition 2. As in [10], by a strategic sequential test (A, B, μ) (μ – strategic function; A, B – constants, $0 < B < 1 < A < \infty$) applied to the sequence (1) we understand a Wald's sequential probability ratio test with boundaries (A, B) which utilizes only the mixed sequence $x_1, x_2, \ldots$ (obtained according to the procedure (2)) for computation of the likelihood ratio. Thus, the strategic sequential test (A, B, μ) proceeds as follows.

In the first step, observe the random variable $\xi_1^{\mu(1)}$. When the result x_1 of the first observation is known, observe the random variable $\xi_2^{\mu(2,x_1)}$, then $\xi_3^{\mu(3,x_1,x_2)}$ and so on. At each step m compute the corresponding likelihood ratio λ_m,

$$\lambda_m = \frac{p(x_1, \ldots, x_m \mid H_1)}{p(x_1, \ldots, x_m \mid H_0)},$$

and compare it with boundaries A, B. If $\lambda_m \leqq B$, stop observation and accept H_0; if $\lambda_m \geqq A$, stop observation and accept H_1; otherwise continue and observe $\xi_{m+1}^{\mu(m+1,x_1,\ldots,x_m)}$ in the next step.

BASIC PROPERTIES OF STRATEGIC TESTS

Let us repeat some known properties of a strategic test (A, B, μ) of strength (α, β), i.e.

$$\mathsf{P}(\textit{test accepts } H_1 \mid H_0) = \alpha,$$
$$\mathsf{P}(\textit{test accepts } H_0 \mid H_1) = \beta.$$

In [10] the following theorem was proved.

Assertion 1. Under condition (4) (cf. next paragraph) a strategic sequential test (A, B, μ) applied to a sequence of *i.i.d.* pairs of random variables (1) terminates with probability one under both hypotheses H_0 and H_1.

The importance of this assertion lies in the fact that the following Wald's theorems (cf. [1]) remain valid also for strategic tests. Indeed, these theorems do not assume any independence or identity of the distributions of the random variables implied so that they may be extended directly to the case of strategic tests.

Assertion 2. If the strategic sequential test (A, B, μ) of strength (α, β) terminates with probability one (under both hypotheses H_0 and H_1), then

(i) $$A \leqq \frac{1-\beta}{\alpha} \quad \text{and} \quad B \geqq \frac{\beta}{1-\alpha}$$

and

(ii) $$\mathrm{E}(S \mid H_0) \doteq (1-\alpha) \log B + \alpha \log A\,,$$
$$\mathrm{E}(S \mid H_1) \doteq (1-\beta) \log A + \beta \log B\,,$$

where S denotes the logarithm of the likelihood ratio on termination of the test.

Assertion 3. If the strategic sequential test $((1-\beta)/\alpha, \beta/(1-\alpha), \mu)$ terminates with probability one (under both hypotheses) and is of strength (α', β') then

(i) $$\alpha' \leqq \frac{\alpha}{1-\beta} \quad \text{and} \quad \beta' \leqq \frac{\beta}{1-\alpha}\,,$$

(ii) $$(\alpha' + \beta') \leqq (\alpha + \beta)\,,$$

(iii) $$\mathrm{E}(S \mid H_0) \doteq (1-\alpha) \log \frac{\beta}{1-\alpha} + \alpha \log \frac{1-\beta}{\alpha}\,,$$
$$\mathrm{E}(S \mid H_1) \doteq (1-\beta) \log \frac{1-\beta}{\alpha} + \beta \log \frac{\beta}{1-\alpha}\,.$$

Remark. Note that all Assertions hold for all strategic functions.

AVERAGE LENGTH OF STRATEGIC TESTS

We shall now deal with estimates of the average length of a strategic sequential test. For that purpose an analogous technique to the technique used by Wald ([1]) will be used.

Let $P_{j|k}$ (for $j = 1, 2$; $k = 0, 1$) be the probability measure induced by ξ^j (corresponding to the elementary test j) under the hypothesis H_k on $(X, \mathscr{X})$, and P be some probability dominanting all four $P_{j|k}$ (i.e. $P_{j|k}$ are absolute continuous with respect to P). Let us denote the Radon-Nikodym density $\mathrm{d}P_{j|k}/\mathrm{d}P$ by $p_j(x \mid H_k)$. Further, for both $j = 1, 2$ let

(3)
$$z_j(x) = \log \frac{p_j(x \mid H_1)}{p_j(x \mid H_0)}$$

and for $j = 1, 2$ and $k = 0, 1$

$$\mathsf{E}(z_j \mid H_k) = \int z_j(x) \, \mathrm{d}P_{j|k} = \int z_j(x) \, p_j(x \mid H_k) \, \mathrm{d}P \, .$$

For $k = 1$ it can be expressed

$$\mathsf{E}(z_j \mid H_1) = \int z_j(x) \, \mathrm{d}P_{j|1} = \int \frac{p_j(x \mid H_1)}{p_j(x \mid H_0)} \, z_j(x) \, \mathrm{d}P_{j|0} = \mathbf{H}(P_{j|1}, P_{j|0}) \, ,$$

and for $k = 0$

$$\mathsf{E}(z_j \mid H_0) = - \int \log \frac{p_j(x \mid H_0)}{p_j(x \mid H_1)} \, \mathrm{d}P_{j|0} =$$

$$= - \int \frac{p_j(x \mid H_0)}{p_j(x \mid H_1)} \log \frac{p_j(x \mid H_0)}{p_j(x \mid H_1)} \, \mathrm{d}P_{j|1} = - \mathbf{H}(P_{j|0}, P_{j|1})$$

where by $\mathbf{H}(Q, Q')$ we denote the well-known generalized entropy of the probability measure Q with respect to the probability measure Q'.

From [2] it is known that

$$\mathbf{H}(Q, Q') \geqq 0 \, ,$$

where the equality holds *iff* $Q = Q'$.

From the obvious technical reasons, let us suppose throughout the paper that for all $j = 1, 2$; $k = 0, 1$

(4)
$$0 < |\mathsf{E}(z_j \mid H_k)| < \infty \, .$$

Let us remark, that this is a sufficient condition under which strategic tests terminate with probability one (cf. Assertion 1).

The probability density (with respect to P^i) concerning the first i variables of the mixed sequence $x_1, x_2, \ldots$ (cf. (2)) under the hypothesis H_k will be denoted by $p(x_1, \ldots, x_i \mid H_k)$. Similarly, $p(x_i \mid x_1, \ldots, x_{i-1}, H_k)$ will denote the conditional probability density (with respect to P) of the ith variable (of the sequence $x_1, x_2, \ldots$) under the hypothesis H_k and given that the values of the first $i - 1$ variables have been $x_1, \ldots, x_{i-1}$.

As it has been told above, the random variable $\xi^{\mu(1)}$ is observed at the first step. Thus

$$p(x_1 \mid H_k) = p_{\mu(1)}(x_1 \mid H_k) \, .$$

Similarly, when the outcomes $x_1, \ldots, x_{i-1}$ of the first $i - 1$ steps are known, the random variable $\xi^{\mu(i,x_1,\ldots,x_{i-1})}$ is observed at the ith step, and therefore

$$(5) \qquad p(x_i \mid x_1, \ldots, x_{i-1}, H_k) = p_{\mu(i,x_1,\ldots,x_{i-1})}(x_i \mid H_k)\,.$$

Let us remark that as usually for $i = 1$

$$p(x_i \mid x_1, \ldots, x_{i-1}, H_k) = p(x_1 \mid H_k)\,,$$

and

$$\mu(i, x_1, \ldots, x_{i-1}) = \mu(1)\,.$$

The strategic sequential test is based on computing the likelihood ratio at every step. If $x_1, \ldots, x_n$ have been observed at the first n steps then the likelihood ratio λ_n at the nth step is:

$$\lambda_n(x_1, \ldots, x_n) = \frac{p(x_1, \ldots, x_n \mid H_1)}{p(x_1, \ldots, x_n \mid H_0)} = \prod_{i=1}^{n} \frac{p(x_i \mid x_1, \ldots, x_{i-1}, H_1)}{p(x_i \mid x_1, \ldots, x_{i-1}, H_0)}\,.$$

According to (5) and (3), we can further write

$$(6) \qquad s_n(x_1, \ldots, x_n) = \log \lambda_n(x_1, \ldots, x_n) =$$

$$= \log \prod_{i=1}^{n} \frac{p_{\mu(i,x_1,\ldots,x_{i-1})}(x_i \mid H_1)}{p_{\mu(i,x_1,\ldots,x_{i-1})}(x_i \mid H_0)} = \sum_{i=1}^{n} z_{\mu(i,x_1,\ldots,x_{i-1})}(x_i)\,.$$

Let us define for $i = 2, 3, 4, \ldots$ the functions

$$\eta_i(x_1, \ldots, x_{i-1}) = 1 \quad \textit{iff} \quad (\forall k = 1, \ldots, i-1)\; s_k(x_1, \ldots, x_k) \in (\log B, \log A)$$

$$\eta_i(x_1, \ldots, x_{i-1}) = 0 \quad \textit{iff} \quad (\exists k = 1, \ldots, i-1)\; s_k(x_1, \ldots, x_k) \notin (\log B, \log A)$$

and

$$\eta_1 = 1\,.$$

The functions η_i describe the stopping rule of the strategic test (A, B, μ). The function η_i is equal to 1 for those $(x_1, \ldots, x_{i-1}) \in X^{i-1}$ for which the strategic test (A, B, μ) does not terminate during or after the first $i - 1$ steps.

Using these functions one can define the random variable

$$(7) \qquad S_\mu(x_1, x_2, \ldots) = \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1})\, z_{\mu(i,x_1,\ldots,x_{i-1})}(x_i)$$

which is for every sequence $(x_1, x_2, \ldots) \in X^\infty$ equal to the value of logarithm of the likelihood ratio on termination of the test (A, B, μ) (cf. expression (6)).

Introducing two new functions μ_1 and μ_2 in a shortened form

$$\mu_1(i, x_1, \ldots, x_{i-1}) = 2 - \mu(i, x_1, \ldots, x_{i-1})$$

$$\mu_2(i, x_1, \ldots, x_{i-1}) = \mu(i, x_1, \ldots, x_{i-1}) - 1$$

it is possible to modify the expression (7).

$$S_\mu(x_1, x_2, \ldots) =$$

$$= \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1}) \left[\mu_1(i, x_1, \ldots, x_{i-1}) z_1(x_i) + \mu_2(i, x_1, \ldots, x_{i-1}) z_2(x_i)\right] =$$

$$= \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1}) \left[\mu_1(i, x_1, \ldots, x_{i-1}) z_1(x_i^1) + \mu_2(i, x_1, \ldots, x_{i-1}) z_2(x_i^2)\right] =$$

$$= \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1}) z_1(x_i^1) +$$

$$+ \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1}) \mu_2(i, x_1, \ldots, x_{i-1}) \left[z_2(x_i^2) - z_1(x_i^1)\right].$$

During this modification the notation introduced above has been used (cf. (1) and (2)). (x_1^1, x_1^2), (x_2^1, x_2^2), ... denote a sequence of realizations of the random variables (1) and $x_1, x_2, \ldots$ denote the sequence obtained according to the procedure (2).

Replacing simultaneously z_1 with z_2 and μ_1 with μ_2, an analogous expression is obtained

$$S_\mu(x_1, x_2, \ldots) = \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1}) z_2(x_i^2) + \tag{8}$$

$$+ \sum_{i=1}^{\infty} \eta_i(x_1, \ldots, x_{i-1}) \mu_1(x_1, \ldots, x_{i-1}) \left[z_1(x_i^1) - z_2(x_i^2)\right].$$

Since the sequence (1) is supposed to be a sequence of independent repetitions of the pair (ξ^1, ξ^2), the following relations are valid.

$$\mathsf{E}(S_\mu \mid H_k) = \sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \mid H_k)\, \mathsf{E}(z_1 \mid H_k) +$$

$$+ \sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \mu_2(i, x_1, \ldots, x_{i-1}) \mid H_k) \left(\mathsf{E}(z_2 \mid H_k) - \mathsf{E}(z_1 \mid H_k)\right) =$$

$$= \mathsf{E}(z_1 \mid H_k)\, L(H_k, \mu) +$$

$$+ \left(\mathsf{E}(z_2 \mid H_k) - \mathsf{E}(z_1 \mid H_k)\right) \sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \mu_2(i, x_1, \ldots, x_{i-1}) \mid H_k).$$

In the last expression $L(H_k, \mu)$ denotes the average length (average number of steps) of the test utilizing the strategic function μ under the hypothesis H_k.

Analogically, from the expression (8) one can obtain

$$\mathsf{E}(S_\mu \mid H_k) = \mathsf{E}(z_2 \mid H_k)\, L(H_k, \mu) +$$

$$+ \left(\mathsf{E}(z_1 \mid H_k) - \mathsf{E}(z_2 \mid H_k)\right) \sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \mu_1(i, x_1, \ldots, x_{i-1}) \mid H_k).$$

Since our aim is to study the average length of the test, the last two expressions are rewritten into a more proper form.

$$L(H_k, \mu) = \frac{\mathsf{E}(S_\mu \mid H_k)}{\mathsf{E}(z_1 \mid H_k)} + \tag{9}$$

$$+ \frac{\mathsf{E}(z_1 \mid H_k) - \mathsf{E}(z_2 \mid H_k)}{\mathsf{E}(z_1 \mid H_k)} \sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \, \mu_2(i, x_1, \ldots, x_{i-1}) \mid H_k)$$

$$L(H_k, \mu) = \frac{\mathsf{E}(S_\mu \mid H_k)}{\mathsf{E}(z_2 \mid H_k)} + \tag{10}$$

$$+ \frac{\mathsf{E}(z_2 \mid H_k) - \mathsf{E}(z_1 \mid H_k)}{\mathsf{E}(z_2 \mid H_k)} \sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \, \mu_1(i, x_1, \ldots, x_{i-1}) \mid H_k) \, .$$

To take advantage of the equations (9) and (10) let us recollect some known results. It has been mentioned that unless $z_j(x) = 0$ a.e. (which happens to contradict the assumption (4)) in [2] it was shown that $\mathbf{H}(P_{j|k}, P_{j|1-k}) > 0$ and therefore

$$\mathsf{E}(z_j \mid H_0) < 0$$

and

$$\mathsf{E}(z_j \mid H_1) > 0 \, .$$

Further, it should be noticed that the strategic sequential test with constant strategic function $\mu \equiv j$ is equivalent to the Wald's sequential test utilizing the random variable ξ^j only. Therefore, the classical Wald's result ([1]) can be used to express the average length $L(H_k, \mu \equiv j)$ of the strategic test with constant strategic function $\mu \equiv j$.

Accordingly,

$$L(H_k, \mu \equiv j) = \frac{\mathsf{E}(S_{\mu \equiv j} \mid H_k)}{\mathsf{E}(z_j \mid H_k)} \, . \tag{11}$$

Let us note that according to the Assertions (2) and (3) the value $\mathsf{E}(S_\mu \mid H_k)$ is for different strategic functions μ approximately constant, not depending on the strategic function, depending only on the boundaries (A, B) and probabilities of error (α, β). However, we should be aware that the use of this approximation renders the validity of next inequalities (12), (13), (14) and (15) approximate only.

Now, let us return our attention to the expressions (9) and (10).

It is obvious that

$$\sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \, \mu_j(i, x_1, \ldots, x_{i-1}) \mid H_k) \geqq 0$$

since

$$\eta_i(x_1, \ldots, x_{i-1}) \geqq 0$$

and also

$$\mu_j(i, x_1, \ldots, x_{i-1}) \geqq 0 \, .$$

Moreover, it is clear that if $\mu \neq j$ for all arguments then $\mu_j \equiv 0$ and therefore also

$$\sum_{i=1}^{\infty} \mathsf{E}(\eta_i(x_1, \ldots, x_{i-1}) \, \mu_j(i, x_1, \ldots, x_{i-1}) \mid H_k) = 0 .$$

Four different cases will be distinguished.

I. Let $\mathsf{E}(z_1 \mid H_0) \leqq \mathsf{E}(z_2 \mid H_0)$. Then

$$\frac{\mathsf{E}(z_1 \mid H_0) - \mathsf{E}(z_2 \mid H_0)}{\mathsf{E}(z_1 \mid H_0)} \geqq 0$$

and according to (9) and (11)

$$L(H_0, \mu) \geqq \frac{\mathsf{E}(S \mid H_0)}{\mathsf{E}(z_1 \mid H_0)} = L(H_0, \mu \equiv 1) . \tag{12}$$

II. Let $\mathsf{E}(z_1 \mid H_0) \geqq \mathsf{E}(z_2 \mid H_0)$. Then

$$\frac{\mathsf{E}(z_2 \mid H_0) - \mathsf{E}(z_1 \mid H_0)}{\mathsf{E}(z_2 \mid H_0)} \geqq 0$$

and according to (10) and (11)

$$L(H_0, \mu) \geqq \frac{\mathsf{E}(S \mid H_0)}{\mathsf{E}(z_2 \mid H_0)} = L(H_0, \mu \equiv 2) . \tag{13}$$

III. Let $\mathsf{E}(z_1 \mid H_1) \leqq \mathsf{E}(z_2 \mid H_1)$. Then

$$\frac{\mathsf{E}(z_2 \mid H_1) - \mathsf{E}(z_1 \mid H_1)}{\mathsf{E}(z_2 \mid H_1)} \geqq 0$$

and according to (10) and (11)

$$L(H_1, \mu) \geqq \frac{\mathsf{E}(S \mid H_1)}{\mathsf{E}(z_2 \mid H_1)} = L(H_1, \mu \equiv 2) . \tag{14}$$

IV. Eventually, when $\mathsf{E}(z_1 \mid H_1) \geqq \mathsf{E}(z_2 \mid H_1)$ then

$$L(H_1, \mu) \geqq \frac{\mathsf{E}(S \mid H_1)}{\mathsf{E}(z_1 \mid H_1)} = L(H_1, \mu \equiv 1) . \tag{15}$$

These four partial results may be summarized into the following theorems.

Theorem 1. If $\mathsf{E}(z_1 \mid H_0) \leqq \mathsf{E}(z_2 \mid H_0)$ & $\mathsf{E}(z_1 \mid H_1) \geqq \mathsf{E}(z_2 \mid H_1)$ $(\mathsf{E}(z_1 \mid H_0) \geqq \mathsf{E}(z_2 \mid H_0)$ & $\mathsf{E}(z_1 \mid H_1) \leqq \mathsf{E}(z_2 \mid H_1))$ then the strategic sequential test (A, B, μ) with constant strategic function $\mu \equiv 1$ $(\mu \equiv 2)$ has almost the shortest average length among all strategic tests with the same strength (α, B).

Theorem 2. Let $\mathsf{E}(z_1 \mid H_0) < \mathsf{E}(z_2 \mid H_0) < 0 < \mathsf{E}(z_1 \mid H_1) < \mathsf{E}(z_2 \mid H_1)$ then for the average length $L(H_k, \mu)$ of an arbitrary strategic test (A, B, μ) it holds

$$L(H_0, \mu \equiv 1) \leqq L(H_0, \mu) \leqq L(H_0, \mu \equiv 2)$$

and

$$L(H_1, \mu \equiv 2) \leqq L(H_1, \mu) \leqq L(H_1, \mu \equiv 1) .$$

Theorem 1 shows a sufficient condition representing the situation when the almost best strategic function is constant. Theorem 2 gives boundaries which cannot be exceeded by the average length of any strategic test regardless of the choice of the strategic function. Then, if

$$\mathsf{E}(z_1 \mid H_0) + \varepsilon_0 \geqq \mathsf{E}(z_2 \mid H_0) \geqq \mathsf{E}(z_1 \mid H_0)$$

and

$$\mathsf{E}(z_1 \mid H_1) + \varepsilon_1 \geqq \mathsf{E}(z_2 \mid H_1) \geqq \mathsf{E}(z_1 \mid H_1)$$

for small positive ε_0 and ε_1, there is no sense in finding out sophisticated strategic functions because according to Assertion 2 the boundaries given by Theorem 2 cannot be far each from the other. For that reason different strategic functions give strategic tests of nearly the same average length.

(Received September 19, 1982.)

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