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Complex System Evaluating Function

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Given a complex dynamic system (e.g. economical). Its work is described by a large amount of parameters. There are made various demands on them (e.g. minimalization of expenses). In a period of time some values of parameters are able to fulfil our expectations better then the others. All the parameters' values determine the state of the system in the concrete moment. We want to appreciate this state owing to demands made on every parameter. Our aim is to get an object judgement about the whole system and to consider the largest amount of parameters of various aspects of the system's work. The random disturbances of the parameters' values are taken into account too. In this paper there is constructed the method for appreciating of the system's state in the form of the transformation into some numerical interval.

1. GENERAL APPROACH TO THE EVALUATING FUNCTION

In this paper we denote by $\mathcal{N}$ the set of natural numbers, by R some finite subset of the body of rational numbers and by $\mu(\cdot)$ the power of the set "$\cdot$".

Given the set

$$\mathcal{X}(t) = \{X_i(t)\}\,, \quad i \in J_X,\ J_X \subset \mathcal{N}\,,$$

where $X_i(t)$ are parameters describing particular activities of the system and taking the values from the field of real numbers in moments t. We suppose we can measure these values in a finite number of discrete moments. Then we denote:

$$\Xi = \{\mathcal{X}(t_i)\}\,, \quad i \in J_t,\ J_t \subset \mathcal{N}\,.$$

Let us suppose the set Ξ is ordered:

$$\xi = (\Xi, \preccurlyeq)\,.$$

The ordering "$\preccurlyeq$" can be interpreted: "to be less or equal favourable". We denote the poset:

$$\vartheta = (R, \leqq)\,.$$

Then we define evaluating function as homomorphism

$$s : \xi \to \vartheta .$$

As s is not injective, more $\mathscr{X}(t_i)$ will be transformed into the same point $r \in R$. It expresses that some states of the system can be equal favourable in various moments, although the elements $X_i(t) \in \mathscr{X}(t)$ have taken different values in different t.

Transformation s has to be:

a) *evaluating*, i.e. to increase (decrease) its values continually with improving of the system's state,

b) *weighting*, i.e. it has to respect different importance of the parameters $X_i(t)$. We will decompose the set R:

$$\mathscr{R} = \{B_i\}, \quad i \in J_R, \quad J_R \subset \mathscr{N},$$

$$\bigcup_{i \in J_R} B_i = R, \quad B_i \cap B_j = \emptyset, \quad i \neq j .$$

The set $\mathscr{R}$ is ordered:

$$\vartheta' = (\mathscr{R}, <),$$

$$B_i < B_j \Rightarrow \forall_{r_i \in B_i} \forall_{r_j \in B_j} (r_i < r_j), \quad B_i, B_j \in \mathscr{R} .$$

The set of those $\mathscr{X}(t_j)$, for which is true

$$s(\mathscr{X}(t_j)) = r_j \in B_i, B_i \in \mathscr{R},$$

generate the class of equivalence with the equivalence "$\asymp$":

$$\mathscr{X}(t_j) \asymp \mathscr{X}(t_l) \Leftrightarrow r_j, r_l \in B_i.$$

All classes of equivalence generate the factor set $\Xi \mid \asymp$, which has the same number of elements as the set $\mathscr{R}$. Factor set $\Xi \mid \asymp$ is ordered and so we are able to define isomorhism

$$s' : \xi \mid \asymp \to \vartheta',$$

where

$$\xi \mid \asymp = (\Xi \mid \asymp, \prec) .$$

The ordering "$\prec$" and the equivalence "$\asymp$" are generated by isomorphism s'. With it's help we are able to define relatively less number of the system's states, which are ordered by relation "to be less (more) favourable". According to the result we are able to judge about quality of control of the system or to study the system's development after the given sequence of states etc.

2.1. Definition of the basic transformation

Given the set of demands

$$\mathcal{Q} = \{q_i\},$$

which imply from the basic resp. strategic aims of the system. We suppose we are able to determine the degree of the demands' fulfilling. We denote by $\mathcal{T}$ the finite set of time moments:

$$\mathcal{T} = \{t_i\}, \quad i \in J_t, \quad J_t \subset \mathcal{N}$$

and define the set of rational numbers:

$$\mathcal{K} = \{k_i\}, \quad i \in J_K, \quad J_K \subset \mathcal{N}$$

such, that for every k_i, $k_j \in \mathcal{K}$ is true:

$$k_i \equiv k_j (\mathrm{mod}\ d),$$

d is rational number.

Let us define the transformation

$$\sigma : \mathcal{Q} \times \mathcal{Q} \times \mathcal{X} \times T \times \mathcal{X} \times \mathcal{T} \to \mathcal{K}, \quad T \subset \mathcal{T}. \tag{1}$$

This transformation attaches the elements from the $\mathcal{K}$ to the elements from the $\mathcal{X}$ dependently on the demand's $\mathcal{Q}$ fulfilling and on the time T, $\mathcal{T}$. The system has development during the period T, i.e. the elements from $\mathcal{X}(t)$ have taken $\mu(J_t')$ values. We must suppose, that the demands $\mathcal{Q}$ are constant during the time $\mathcal{T}$ and that the elements $X_i(t) \in \mathcal{X}(t)$ are able to fulfil them. This presumption is true, if:

a) the aims of the system and so the demands are defined owing to its abilities,

b) the system is able to develop.

We want to determine the degree of the demands' fulfilling in a moment $t_j \in (\mathcal{T} - T)$. Therefore we will compare it with optimal demands' fulfiling during the period T. We can express this formally:

$$\mathcal{Q} \times \mathcal{X} \times T \to \mathcal{X}^{\mathrm{opt}}. \tag{2}$$

The set $\mathcal{X}^{\mathrm{opt}}(t)$ is not commonly generated by the elements $X_i(t)$ values of them are related to the same moment t, because the parameters $X_i(t)$ can take their optimal values in different moments $t \in T$. Let us suppose the maximal length of the period T is given. We denote it by $T^{\max} \subseteq \mathcal{T}$. The following condition must be fulfiled then:

$$T + t \leqq T^{\max}. \tag{3}$$

If the condition (3) is not fulfilled, we must move the begining of measuring.

Joining the relations (1), (2) we get:

$$\sigma : \mathcal{Q} \times \mathcal{X}^{\mathrm{opt}} \times \mathcal{X} \times \mathcal{T} \to \mathcal{K}. \tag{4}$$

The relation (4) expresses the main idea of the evaluating function construction.

Our situation is complicated, because every parameter $X_i(t) \in \mathscr{X}(t)$ can depend on another parameters and random variables. The compare owing to absolute values without regard to this dependences could drive us to unrealizable demands. Therefore we have to amend the elements from the set $\mathscr{X}^{\text{opt}}(t)$ before and then we are able to compare.

Let us suppose every element $X_i(t) \in \mathscr{X}(t)$ is a function of variables $Z_{il}(t) \in \mathscr{Z}_i(t)$. The dimension of $\mathscr{Z}_i(t)$ depends on subskript i. The variables $Z_{il}(t)$ can be identical with some $X_j(t)$:

$$\exists_{i,j,l} (Z_{il}(t) \equiv X_j(t)) .$$

We can write:

$$X_i(t) = f_i(\mathscr{Z}_i(t)) . \tag{5}$$

We must suppose we cannot know the function (5) exactly. Let the parameter $X_i(t)$ have taken its real optimal value $X_i^{\text{opt}}(t)$ in a moment $t^{\text{opt}} \in T$. There exist theoretical value $\tilde{X}_i^{\text{opt}}(t^{\text{opt}})$ then, which we can determine using (5):

$$\tilde{X}_i^{\text{opt}}(t^{\text{opt}}) = f_i(\mathscr{Z}_i(t^{\text{opt}})) .$$

This value is generally different from $X_i^{\text{opt}}(t^{\text{opt}})$, because in (5) we were not able to find out the influence of unmeasurable random disturbances.

Similarly for a moment $t \in (\mathscr{T} - T)$:

$$\tilde{X}_i(t) = f_i(\mathscr{Z}_i(t)) .$$

We count the difference

$$\Delta_i(t) = \tilde{X}_i(t) - \tilde{X}_i^{\text{opt}}(t^{\text{opt}}) .$$

The $\Delta_i(t)$ expresses theoretically necessary increase (decrease) of $X_i(t)$ owing to $\mathscr{Z}_i(t)$. Now we can determine the amended $'X_i^{\text{opt}}(t)$:

$$'X_i^{\text{opt}}(t) = X_i^{\text{opt}}(t^{\text{opt}}) + \Delta_i(t) .$$

We will compare the real value of $X_i(t)$ in a moment $t \in (\mathscr{T} - T)$ with amended value of $'X_i^{\text{opt}}(t)$. If we make this process for all $i \in J_X$, we get the amended set $'\mathscr{X}^{\text{opt}}(t)$. Then we can rewrite the relation (4):

$$\sigma : \mathscr{Q} \times {}'\mathscr{X}^{\text{opt}} \times \mathscr{X} \times \mathscr{T} \to \mathscr{K} . \tag{6}$$

2.2. The realization of proper evaluating function

The relation (6) and the condition 1b) imply the general form of the evaluating function (it is denoted by s):

$$s : \mathscr{K} \times \mathscr{W} \to R . \tag{7}$$

The set $\mathscr{W} = \{w_i\}$, $i \in J_X$ is the set of the parameters' weights. We suppose the weights were subjoined to the parameters $X_i(t) \in \mathscr{X}(t)$ using a group of empirically determined rules.

2.2.1. *The analysis of the parameters' set*

We define the equivalence ε in the set $\mathscr{X}(t) = \{X_i(t)\}$, $i \in J_X$:

$$X_i(t)\, \varepsilon\, X_j(t) \Rightarrow w_i = w_j\,,$$

where w_i, w_j are weights of parameters $X_i(t), X_j(t)$. Using the equivalence ε we can define the factor set

$$\mathscr{X}(t) \mid \varepsilon = \{p_\varepsilon X_i(t) : X_j(t) \in p_\varepsilon X_i(t) \Rightarrow X_i(t)\, \varepsilon\, X_j(t)\,; \\ X_i(t), X_j(t) \in \mathscr{X}(t)\}\,.$$

From the definition of equivalence ε follows the set $\mathscr{X}(t) \mid \varepsilon$ is ordered:

$$p_\varepsilon X_i(t) \lhd p_\varepsilon X_j(t) \Rightarrow w_g < w_h\,,$$

where w_g, w_h are the weights of the elements $X_g(t) \in p_\varepsilon X_i(t)$ resp. $X_h(t) \in p_\varepsilon X_j(t)$ Let us subjoin the superscript $l \in J_\varepsilon$, $J_\varepsilon \subset \mathscr{N}$ to every class of equivalence:

a) the smalest element from J_ε is unit,

b) for every l, $m \in J_\varepsilon$ is true

$$l \equiv m (\mathrm{mod}\ d')\,, \quad d' \in \mathscr{N}\,.$$

We'll write this superscript in such way:

$${}^{(l)}p_\varepsilon X_i(t)\,.$$

Then we subjoin to all elements of every class of equivalence the weight, which is equal to the superscript of this class:

$$(X_j(t) \in {}^{(l)}p_\varepsilon X_i(t)) \Rightarrow (X_j(t) \mapsto w_j = l = w_i)\,.$$

2.2.2 *The concrete form of the evaluating function.*

We will consider the function s in the form:

$$s(t) = \sum_{i=1}^{M} k_i(t)\, w_i\,, \tag{8}$$

where $M = \mu(J_X)$ is the number of the elements of the set $\mathscr{X}(t)$. The number of the functional values of the function $s(t)$ is finite and it is equal to the number

$$H = \mu(\mathscr{K})^M\,. \tag{9}$$

There are some $s(t_i) = s(t_j)$. As the set $\mathscr{K}$ is ordered and finite, it has maximal and minimal elements. We denote them $k^{\max}$, $k^{\min}$. Then we can write:

$$k_i(t) = k^{\min} + \alpha_i(t)\, d\,, \quad i \in J_k\,.$$

Here the $\alpha_i(t)$ takes integer values from the interval $0 \leqq \alpha_i(t) \leqq \Lambda$, where

$$\Lambda = \frac{k^{\max} - k^{\min}}{d}\,.$$

Let us denote

$$s^{\min} = k^{\min} \,.\, \sum_{i=1}^{M} w_i\,.$$

Then

(10) $$s(t) = s^{\min} + d \sum_{i=1}^{M} \alpha_i(t)\, w_i\,.$$

Lemma 1. For arbitrary moments t_b, t_c is true:

$$s(t_b) \equiv s(t_c) \,(\mathrm{mod}\; d)\,.$$

Using the definition of the congruence and the relation (10) we can write:

$$\sum_{i=1}^{M} \alpha_i(t_b)\, w_i - \sum_{i=1}^{M} \alpha_i(t_c)\, w_i = C\,.$$

It is allways true, because $\alpha_i(t)$, w_i and C are integer numbers.

Following condition must be fulfilled to be able to prove Theorem 1: the power of the set R must be less or equal to the number H (see (9)):

$$\frac{s^{\max} - s^{\min}}{d} + 1 \leqq H\,,$$

where $s^{\max} = s^{\min} + \Lambda d \sum_{i=1}^{M} w_i$. As $\Lambda = \mu(\mathscr{K}) - 1$, we get:

(11) $$\mu(\mathscr{K}) \leqq \frac{\mu(\mathscr{K})^M - 1}{\sum_{i=1}^{M} w_i} + 1\,.$$

Theorem 1. If the condition (11) is fulfiled, then all values $s(t)$, which are different one from another, fill up uniformly the interval $\langle s^{\min}; s^{\max}\rangle$ in such way, that every two neighbouring values differ for d.

Proof: a) We must find $s(t)$ such, that

$$\frac{s(t) - s^{\min}}{d} = 1$$

Then $\sum_{i=1}^{M} \alpha_i(t)\, w_i = 1$. As $\alpha_i(t) = 0, 1, \ldots, \Lambda$ and one $w_i = 1$ at least, it is true.

b) Let us suppose there is given the selection of $\alpha_i(t_b)$ such, that

$$\sum_{i=1}^{M} \alpha_i(t_b)\, w_i = n\,, \quad n < \Lambda \sum_{i=1}^{M} w_i\,.$$

If the condition (11) is fulfiled, we must find the selection of $\alpha_i(t_c)$ such, that

$$\sum_{i=1}^{M} \alpha_i(t_c)\, w_i - \sum_{i=1}^{M} \alpha_i(t_b)\, w_i = 1\,.$$

As at least one $w_i = 1$, we can choose the $\alpha_j(t_c)$ multiplying the $w_i = 1$ higher by unit then corresponding $\alpha_j(t_b)$. If that $\alpha_j(t_b) = \Lambda$, we'll put it equal to zero. Keeping the rest of $\alpha_i(t_b)$, $i \neq j$, we get:

$$\sum_{i=1}^{M-1} \alpha_i(t_b)\, w_i = n - \Lambda\,.$$

Then there must exist such combination of $\alpha_i(t_c)$, $i \neq j$, which corresponds with simultaneous increasing and decreasing of some $\alpha_i(t_b)$. Denoting this increasing and decreasing by τ_i, we get the condition:

$$0 < \tau_1 w_1 + \ldots + \tau_l w_l - \tau_{l+1} w_{l+1} - \ldots - \tau_{M-1} w_{M-1} \leqq \Lambda + 1\,. \tag{12}$$

As we can write every w_i in the form

$$w_i = 1 + (n - 1)\, d'\,, \quad n = 1, 2, \ldots,$$

we rewrite the inequality (12):

$$0 < p_1 - p_2 \,.\, d' \leqq \Lambda + 1$$

As there exist finite arithmetic progression among the numbers w_i and the condition (11) is fulfiled, we are able to choose the numbers τ_i to be $p_1 = 1$ and $p_2 = 0$. This finishes the proof.

The number of different values of the function $s(t)$ is:

$$H_r = \frac{s^{\max} - s^{\min}}{d} + 1\,.$$

After modification we get:

$$H_r = (\mu(\mathscr{K}) - 1) \sum_{i=1}^{M} w_i + 1\,. \tag{13}$$

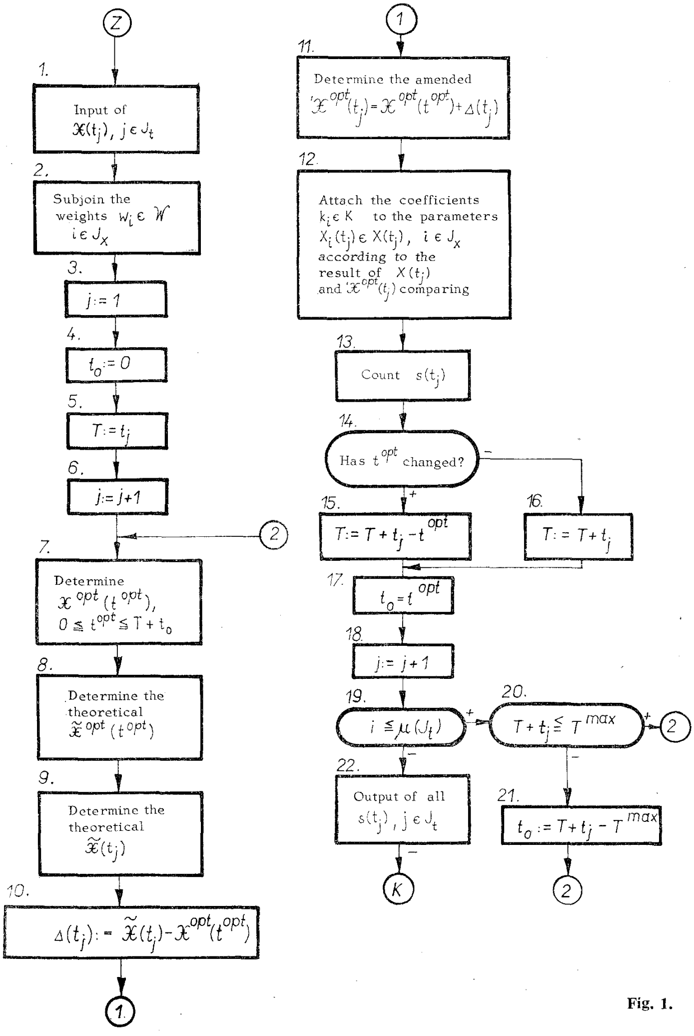
Z
1.
Input of
$\mathcal{X}(t_j)$, $j \in J_t$
2.
Subjoin the weights $w_i \in \mathcal{W}$ $i \in J_X$
3.
$j := 1$
4.
$t_0 := 0$
5.
$T := t_j$
6.
$j := j+1$
2
7.
Determine $\mathcal{X}^{opt}(t^{opt})$, $0 \leq t^{opt} \leq T + t_0$
8.
Determine the theoretical $\tilde{\mathcal{X}}^{opt}(t^{opt})$
9.
Determine the theoretical $\tilde{\mathcal{X}}(t_j)$
10.
$\Delta(t_j) := \tilde{\mathcal{X}}(t_j) - \mathcal{X}^{opt}(t^{opt})$
1
1
11.
Determine the amended $'\mathcal{X}^{opt}(t_j) = \mathcal{X}^{opt}(t^{opt}) + \Delta(t_j)$
12.
Attach the coefficients $k_i \in K$ to the parameters $X_i(t_j) \in X(t_j)$, $i \in J_X$ according to the result of $X(t_j)$ and $'\mathcal{X}^{opt}(t_j)$ comparing
13.
Count $s(t_j)$
14.
Has t^{opt} changed?
+
−
15.
$T := T + t_j - t^{opt}$
16.
$T := T + t_j$
17.
$t_0 = t^{opt}$
18.
$j := j+1$
19.
$i \leq \mu(J_t)$
+
−
20.
$T + t_j \leq T^{max}$
+
2
−
22.
Output of all $s(t_j)$, $j \in J_t$
−
K
21.
$t_0 := T + t_j - T^{max}$
2

Fig. 1.

The decomposition of the interval R must be made in such way, that in every subinterval B_i, $i \in J_R$ there will be the same number of different values of the function $s(t)$. Therefore the number of intervals B_i (the power $\mu(J_R)$) has to divide the H_r:

$$\frac{H_r}{\mu(J_R)} = C .$$

The integer number C means the number of different values of the function $s(t)$ being in every subinterval B_i. The intervals B_i are the values' field of the isomorphism s'.

The algorithm of the evaluating function is described in the form of the flowing diagram on Fig. 1. In the blocks 1, 7, 8, 9, 10, 11, 12 there are marked the sets. We must of course, make all steps of the algorithm with every element of the concrete set. The t_0 denotes a moment of the last change of the $\mathscr{X}^{\text{opt}}(t)$. It must be changed, when $\mathscr{X}^{\text{opt}}(t)$ changes or if $T^{\max}$ is surpassed. We must recount t^{opt} then.

3. USING OF THE EVALUATING FUNCTION

Evaluating function was verified by application of concrete values of the selected technical-economic parameters in a mining establishment (it were e.g. gain, capacity of the mining, thickness etc.). There were 21 of such parameters. The modulus d' of the weights was chosen $d' = 1$. The set $\mathscr{K}$ of the evaluating coefficients:

$$\mathscr{K} = \{0; 0{\cdot}5; 1{\cdot}0; 1{\cdot}5\} .$$

Then using (9) we get the number $H = 4^{21}$. Using (13) we get the number $H_r = 520$ of the different values of the evaluating function. Using (14) we determined the number $\mu(J_R) = 10$ of the intervals B_i. There are $C = 52$ of the different values of the evaluating function in every interval B_i and every two neighbouring values differ for 0·5. The function $s(t)$ can attain values between $s^{\min} = 0$ $s^{\max} = 259{\cdot}5$. If there are, for example, the values of $s(t)$ in the interval $B_{10} = \langle 234{\cdot}0; 259{\cdot}5 \rangle$, we say, that the state of the system is very favourable.

We counted all cross regression dependences among all parameters. For all parameters were made demands for minimalization resp. maximalization and the coefficients $k_i \in \mathscr{K}$ were attached from the point of view

decrease,
equality,
increase,
more increase

(the proportion between $X_i(t)$ and $'X_i^{\text{opt}}(t)$ should be understood). With aid of significance level of the regression dependences the parameters were divided into 12 types. With dependence on this types were the coefficients $k_i \in \mathscr{K}$ attached to every parameter in moments t. The length of the period $T^{\max}$ when $\mathscr{X}^{\text{opt}}(t)$ must be changed, was 12 months.

We used for calculating the computer Odra 1204. Nineteen month's values of the parameters was evaluated. We received 18 different values (!) of the function $s(t)$. They have fallen into five different intervals B_i ($i = 1, 2, \ldots, 10$), i.e. during this very short period has interchanged five discrete states of the system. The evaluating function has reacted on the change of the conditions during the year, i.e. on holidays time in July and September was the state of the system worse, then in other months.

The above control example we can consider as a proof, that the evaluating function is able to describe totaly the behaviour of the system during the time. We get the pregression of the numbers, from which every is bearer of the information about quality of the system's state (owing to the aisms the system is to reach). This is leading us to choose the control interventions, to see the developing of the system and similarly. For application of the evaluating function the cooperation of more experts is needed. Only in such a case the objectivity of attaching of the weights to the parameters and of the coefficients k_i is ensured.

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