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Josef Böhm; Miroslav Kárný

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SELF-TUNING REGULATORS WITH RESTRICTED INPUTS

JOSEF BÖHM, MIROSLAV KÁRNÝ

A modification of a suboptimal self-tuning control strategy known as "iteration spread in time" [5] is proposed. The simple resulting algorithm respects various restrictions of the digitaly manipulated signals (regulator output) and/or their changes, both for minimum and nonminimum-phase systems. The behaviour of the algorithm is discussed, illustrated on typical simulated examples and its possible extensions are outlined.

1. INTRODUCTION

Self-tuning regulators are attractive for their capability to decrease requirements with respect to prior knowledge of the controlled plant. They try to respect usual conditions of a control design in this way. The fundamental direction in the development of these regulators should be, according to our opinion, the convergence between assumptions under which the synthesis is performed on the one hand and the existing possibilities and restrictions of practice on the other hand.

One of the most frequently encountered discrepancies between an "academical" solution and practical needs seems to be the fact that the synthesis is mostly performed for unbounded inputs and/or their increments. However, restrictions imposed on the input signal are present almost everywhere and are given by the hardware, technology or economy of the considered process [13].

One possibility how to meet the requirements is to saturate the regulator output before the actuator. This often may be a satisfactory solution, especially in the single-input case, but for a multi-input system and sometimes also for single-input ones it leeds to unnecessary increase of losses.

Another possibility is to modify the control law by an adequate change of the design criterion. For the quadratic criterion which is considered here the input restrictions can be respected by an additional input penalty. However, its choice is not a simple task and usually several trials have to be done before the desired behaviour is achieved.

Assuming linear models and one-step-ahead quadratic criterion there is no problem to incorporate any restriction on inputs into the control law design in a consistent way. However, such a criterion is not suitable, in general, for non-minimum phase systems. It leads to the simple algorithm for the control synthesis but unfortunately it cannot guarantee both the stability and good closed loop performance in general case.

The multi- or infinite-step criteria do not suffer from this disadvantage, however, for the price of the increase of the computational burden. Moreover they remove the possibility to find a feasible solution when inputs are to be bounded.

A feasible way how to reduce the computational complexity of the multi-step criteria into well acceptable limits is the use of a suboptimal strategy called "Iterations spread in time" (IST) [5], [1]. This approximation, in more detail recalled below, preserves the simplicity of the one-step-ahead criterion and at the same time is able to stabilize any stabilizable system and to achieve asymptotically the optimality of the infinite-stage criterion.

In this paper an atempt is made to extend the IST strategy so that the linear restriction on inputs and/or their increment becomes a part of the synthesis. The authors believe that the idea used here may also serve as a hint how to approach, at least approximately, similar problems outside the linear-quadratic theory.

2. PROBLEM FORMULATION AND REVIEW OF STRATEGY OF ITERATIONS SPREAD IN TIME (IST)

Let the controlled system be described by the multivariate linear-in-parameters normal regression model with the conditional mean of the output given the past input-output data including input $u_{(t)}$

$$\hat{y}_{(t)} = \sum_{i=1}^{n} A_i y_{(t-i)} + \sum_{i=0}^{n} B_i u_{(t-i)} + c = P^{\mathrm{T}} z_{(t)} = \tag{2.1}$$
$$= [B_0, A_1, B_1, \ldots, A_n, B_n, c] \begin{bmatrix} u_{(t)} \\ \hdashline y_{(t-1)} \\ u_{(t-1)} \\ \vdots \\ 1 \end{bmatrix} = P^{\mathrm{T}} \begin{bmatrix} u_{(t)} \\ \hdashline \\ x_{(t-1)} \\ \\ \end{bmatrix}$$

and with a constant conditional covariance matrix R. In equation (2.1) $u_{(t)}$, $y_{(t)}$ are the system (vector) input and output at the discrete time t; B_i, A_i are (unknown) model coefficients collected into the matrix P and the vectors x, z are composed from the data $d_{(.)} = (u_{(.)}, y_{(.)})$ accordingly.

The control objective is to stabilize the observed output on a fixed (zero) level. This is assumed to be quantified by the expected average loss per step

(2.2) $$\omega = \overline{\lim_{N \to \infty}} \mathsf{E} \left[\frac{1}{N} \sum_{t=1}^{N} q(x_{(t)}) \right]$$

The loss $q(.)$ is assumed to be a quadratic function

(2.3) $$q(x) = x^{\mathrm{T}} Q_x x \,, \quad Q_x \geqq 0^{*})$$

The objective of the output stabilization is reflected by its special form

(2.4) $$q(x_{(t)}) = y_{(t)}^{\mathrm{T}} Q_y y_{(t)} \,, \quad Q_y > 0$$

The minimization of the criteria (2.2) for the system (2.1) is the special type of an optimal control problem for Markovian systems. Under known weak assumptions the following conclusions are valid [7]:

The optimal strategy exists and generates the input $u_{(t)}$ as a deterministic function of the state $x_{(t-1)}$ and the optimal $u_{(t)}$ is the minimizing argument of the functional equation

(2.5) $$\omega + V(x_{(t-1)}) = \min_{u} \mathsf{E}[q(x_{(t)}) + V(x_{(t)}) \mid x_{(t-1)}, u]$$

i.e. the optimal loss (2.2) and the function $V(.)$ (unique up to additive constant) solve the equation (2.5).

For LQ problems (linear system, quadratic criterion) $V(.)$ is the quadratic function with the kernel $S \geqq 0$ and the equation (2.5) takes the form

(2.6) $$V(x_{(t-1)}) = x_{(t-1)}^{\mathrm{T}} S x_{(t-1)} = \min_{u_{(t)}} z_{(t)}^{\mathrm{T}} H z_{(t)}$$

(2.7) $$\omega = \operatorname{tr} \left\{ \bar{S} \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

where

(2.8) $$H = P^{*\mathrm{T}} \bar{S} P^{*}$$

(2.9) $$\bar{S} = S + Q_x$$

(2.10) $$P^{*} = \underbrace{\begin{bmatrix} P^{\mathrm{T}} \\ \hline I \;\; \underbrace{0}_{\dim(d)} \end{bmatrix}}_{\dim(z)} \begin{matrix} \}\dim(y) \\ \end{matrix} \Big\} \dim(x)$$

The kernel S determining the solution of equation (2.6) can be found independently

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* $Q \, (\geqq) \, 0$ is the shorthand notation of a positive (semi) definite matrix Q.

of the value $x_{(t-1)}$. The derived form of the matrix equation is suitable for numerical solution by sucessive approximations. It corresponds to a certain version of the well known Riccati equation.

To describe the IST strategy the equations (2.6)–(2.10) will be rewritten as symbolic relation

$$S = \mathscr{R}(S, Q_x, P) \tag{2.11}$$

Then the kth step of successive approximations is

$$S^k = \mathscr{R}(S^{k-1}, Q_x, P), \quad S^0 \geqq 0 \tag{2.12}$$

The lack of knowledge of the parameters P in (2.11) and (2.12) can be overcome by enforced separation of identification and control which replaces the unknown P by their latest point estimates $\hat{P}_{(t-1)}$.

Thus the sequence of the equations of the type (2.11) should be solved

$$S_{(t)} = \mathscr{R}(S_{(t)}, Q_x, \hat{P}_{(t-1)}) \tag{2.11'}$$

for each time t. Similarly equation (2.12) has its analogy in

$$S^k_{(t)} = \mathscr{R}(S^{k-1}_{(t)}, Q_x, \hat{P}_{(t-1)}), \quad S^0_{(t)} \geqq 0 \tag{2.12'}$$

It is known that the quality of the initial guess $S^0_{(t)}$ is substantial for the convergence of (2.12'). The use of the last estimate of S obtained at the time instant $t - 1$ as the initial one at the time instant t is the main trick of the IST strategy. Just one iteration per time unit can be performed. This simplification will be used in the sequel. Omitting the superfluous superscript $k = 1$ the final certainty equivalence version of the IST strategy takes the simple form.

$$S_{(t)} = \mathscr{R}(S_{(t-1)}, Q_x, \hat{P}_{(t-1)}), \quad S_{(0)} \geqq 0 \tag{2.13}$$

The value $u_{(t)}$ is taken as a minimizing argument in (2.6) with the kernel H (2.8) given by equations (2.8)–(2.10) with $S = S_{(t-1)}$, $P = \hat{P}_{(t-1)}$.

The IST strategy has been tested intensively by experiments and also partially analyzed [1]. It may be useful to summarize the current state of knowledge about this strategy.

The behaviour of IST strategy is determined by the properties of successive approximation for fixed P as well as by the properties of the estimation part.

It is known that the solution of equation (2.11), say S^*, stabilizing the closed loop exists generically. The sequence generated by the recursion (2.12) converges to S^* if and only if the "sufficient rank condition" (SRC) is valid [9]. SRC requires the existence of some finite k_0 for which

$$\mathscr{N}(S^{k_0}) \subset \mathscr{N}(S^*) \tag{2.14}$$

where $\mathscr{N}(S)$ denotes the nullspace of the matrix S. This implies, for fixed P, the validity of (2.14) for all $k \geqq k_0$.

It must be stressed that the condition (2.14) is fundamental for a reliable application of the IST strategy. There is the whole class of penalties Q_x e.g., the penalty (2.4) for which the sequence $\mathcal{N}(S^k)$ may be nondecreasing. In this case the existence of

$$\mathcal{N}(S^{k_0}) \supset \mathcal{N}(S^*), \quad \mathcal{N}(S^{k_0}) \neq \mathcal{N}(S^*) \tag{2.15}$$

must destroy convergence S^k to S^*. To illustrate the influence of this condition on the IST strategy let us suppose the case of the output penalty (2.4) and a non-minimum phase system. If for some time interval the parameter estimates $\hat{P}$ form a minimum phase model, $S_{(t)}$ will converge to the zero matrix. Then (2.15) might appear and $S_{(t)}$ could not converge to S^*, even if the estimates P would achieve the true values of the system parameters.

Two ways of the respecting SRC have been used.

- The cautious version of the IST strategy [5], [1] in which the conditional covariance of the parameters maintains the full rank of $S_{(t)}$.
- The penalty Q_x is selected in such a way that SRC (2.14) holds structurally, e.g. the penalty on inputs and/or their increments is introduced.

The criterion of primary interest (output penalty) grows in some degree in both cases mentioned above. Moreover the proper selection of additional penalties needs some tuning.

The modification proposed in this paper removes these difficulties.

Employing the bayesian approach (cf. [10]) the suitable point estimate of the unknown parameters P is understood as the mathematical expectation of P conditioned on the past observed history.

The straightforward application of the martingale theory [4], [12] shows that the sequence of point estimates converges to some point almost surely. Moreover it is known that

$$\lim_{t\to\infty} \hat{P}_{(t)} = \hat{P} = P \tag{2.16}$$

is valid if the parameters P are asymptotically measurable with respect to the σ-algebra generated by the observed data.

This measurability requirement seems to be the most abstract formulation of conditions of both sufficiently exciting inputs [8] and nonredundancy in the model parametrization.

Experiments indicate that for the IST strategy the identity (2.16) does not need to be fulfilled. In order to achieve the identity (2.16) and in this way also the asymptotic optimality, some dual-control modification [2] is needed. The most promising seems to be a two-step-ahead version of the IST strategy similar to the strategy described in [11]. But the following conjecture follows from intensive experimental studies:

- the IST strategy under SRC generically stabilizes the closed control loop
- transient as well as asymptotic behaviour is not far from the optimal one.

3. MODIFIED STRATEGY OF ITERATIONS SPREAD IN TIME (MIST)

Let us consider the controlled system (2.1), the criterion (2.2), (2.4) and moreover let us assume linearly bounded admissible inputs

$$\bar{u}(x_{(t-1)}) \leqq u_{(t)} \leqq \bar{\bar{u}}(x_{(t-1)}) \tag{3.1}$$

where $\bar{u}$, $\bar{\bar{u}}$ are the given functions. In the multi-input case the inequality is understood entry-wise.

The practical examples of the restriction (3.1) are

(i) The bounds on the admissible input range

$$\bar{u}(.) = \bar{u} = \text{constant} \quad \bar{\bar{u}}(.) = \bar{\bar{u}} = \text{constant} \tag{3.2}$$

(ii) The bounds on admissible speed of the input changes

$$\bar{\Delta} \leqq \Delta u_{(t)} = u_{(t)} - u_{(t-1)} \leqq \bar{\bar{\Delta}} \tag{3.3}$$

in this case

$$\bar{u}(x_{(t-1)}) = \bar{\Delta} + u_{(t-1)}, \quad \bar{\bar{u}}(x_{(t-1)}) = \bar{\bar{u}} + u_{(t-1)} \tag{3.4}$$

(iii) The simultaneous requirements for (i) and (ii) imply

$$\bar{u}(x_{(t-1)}) = \max(\bar{u}, u_{(t-1)} + \bar{\Delta}) \tag{3.5}$$
$$\bar{\bar{u}}(x_{(t-1)}) = \min(\bar{\bar{u}}, u_{(t-1)} + \bar{\bar{\Delta}})$$

The proposed modification of the IST strategy (MIST) taking into account the restrictions (3.1) can be easily explained in the special case of a single input system under the symmetric restriction

$$u^2_{(t)} \leqq \bar{u}^2(x_{(t-1)}) \tag{3.6}$$

Like in the IST strategy the substitution of $\hat{P}$ instead of P is performed and the function $V(.)$ in the functional equation (2.5) is searched as a quadratic form with the kernel $S \geqq 0$.

Performing expectation (integration) on the right-band side of (2.5) we arrive at the minimization problem of the type (2.6) which differs just in the range of admissible inputs. Thus we search

$$V_{(t)}(x_{(t-1)}) = \min_{u^2(t) \leqq \bar{u}^2(x(t-1))} z^{\mathrm{T}}_{(t)} H_{(t-1)} z_{(t)} \tag{3.7}$$

For all values $x_{(t-1)}$ for which the (absolutely) minimizing inputs $u^0_{(t)}$ fall into the interval (3.6) the function $V_{(t)}$ has quadratic form and, hence, the regular step of the IST strategy is performed. In the opposite cases the minimum is achieved at the boundary. The optimizing input minimizes Lagrangian function

$$z^{\mathrm{T}}_{(t)} H_{(t-1)} z_{(t)} + \lambda(x_{(t-1)}, \bar{u}(x_{(t-1)}))\, u^2_{(t)} \tag{3.8}$$

where the multiplier $\lambda \geqq 0$ is determined by the equality

$$u_{(t)}^2 = \bar{u}^2(x_{(t-1)}) \tag{3.9}$$

for the minimizing input. It can be easily found that $\lambda > 0$ when $u_{(t)}^{02} > \bar{u}_{(t)}^2$.

The restriction (3.6) has, in this case, the same influence as a data dependent penalty on inputs.

The proposed extension of the IST strategy preserves the feasible quadratic form of the function $V(.)$, however, at the price of the additional data-dependent input penalty. The penalty is fixed at the value $\lambda(x_{(t-1)}, \bar{u}(x_{(t-1)}))$, which is the minimal one for the measured state $x_{(t-1)}$ (zero for $u_{(t)}^{02} \leqq \bar{u}^2(.)$). More formally

$$x^T S_{(t)} x = \min_u \left\{ \begin{bmatrix} u \\ x \end{bmatrix}^T H_{(t-1)} \begin{bmatrix} u \\ x \end{bmatrix} + \lambda(x_{(t-1)}, \bar{u}(x_{(t-1)}))\, u^2 \right\} \text{ for all } x \tag{3.10}$$

where the weight $\lambda(.)$ is selected in such a way that for $x = x_{(t-1)}$ the minimizing argument of (3.10) minimizes (3.7) in the range (3.6). The kernel $H_{(t-1)}$ has the form

$$H_{(t-1)} = \left[\hat{P}_{(t-1)} \left| \begin{matrix} I \\ \hline 0 \end{matrix} \right. \right] (S_{(t-1)} + Q_x) \left[\begin{matrix} \hat{P}^T_{(t-1)} \\ \hline I \mid 0 \end{matrix} \right] \tag{3.11}$$

It is apparent that:

(i) The proposed extension does not increase computation burden of the IST strategy substantially.

(ii) The sufficient rank condition (SRC) (2.14) is automatically respected also for the loss (2.4) with the minimal additional penalty preserving the range of inputs.

(iii) Other one-dimensional problems (nonsymmetric restrictions) are easily transformed into the symmetric form (3.6) by the data-dependent linear transformation of input, the kernels are then modified by a one-rank positive semidefinite matrix.

(iv) The general multi-input case appears to be a simple problem of mathematical programming: the minimization of a positive semidefinite quadratic form with linear inequality constraints.
The algorithm [3] or its numerically more stable modification [6] seem to be appropriate for this purpose. Details will not be described here. The extension is rather straightforward but more technically involved.

To close the section let us mention that even in the minimum-phase multi-input case and for one-step-ahead optimization it is reasonable to perform the exact conditional minimization. The increase of the quality may be rather significant in comparison with the usual practice of the input saturation.

4. ILLUSTRATIVE EXAMPLES

In this section some examples will be given which compare the behaviour of the proposed modification with the standard technique.

It is known that stable minimum-phase systems are optimally stabilized when one-step-ahead criterion is used. In the case of the known parameters it means that no danger arises when inputs (or its increments) are saturated and that any additional penalty on $u_{(t)}$ or $\Delta u_{(t)}$ causes an increase of the output criterion (2.4). However, in the unknown-parameter case, when input sequence is substantially saturated, the excitation of the system may be insufficient. This may result in poor identification and in poor closed loop behaviour. The last observation applies, of course, also to nonminimum-phase systems. However, in the latter case the output behaviour is more dependent on the whole sequence of $u_{(t)}$ and a change of one term, due to the limits, can cause the significant deterioration of the control performance for some time interval.

The proposed modification will change adequately the control law and not only the current input $u_{(t)}$, unlike the simple saturation. The behaviour of the modification can be shortly summarized in the following way. If the control variable moves in the admissible limits no correction is applied. If it reaches the bounds, the correction takes place. The increase of the rank of S by $Q_u(Q_\Delta)$ and its influence in the Riccati equation iteration causes a change in the control law. The additional penalty is applied until the generated $u_{(t)}(\Delta u_{(t)})$ is within the admissible limits. The finite number of such steps is always sufficient in case of stable systems. Then the control system works without an additional penalty until the bounds are reached again. When $u_{(t)}(\Delta u_{(t)})$ reaches the limits frequently the system behaves approximately like an optimal one with an additional input-penalty in the criterion.

Using the IST strategy a slow convergence rate of successive approximation (2.12) may deteriorate the closed loop behaviour. It can be shown that additional penalization of $u(\Delta u)$ accelerates the convergence, especially when the current controller is far from the steady state one.

Two different systems, both of nonminimum-phase, were chosen to demonstrate the properties of the algorithm.

System I: $\hat{y}_{(t)} = 0{\cdot}98y_{(t-1)} - 0{\cdot}08u_{(t)} + 0{\cdot}12u_{(t-1)}, \; R = 1$

System II: $\hat{y}_{(t)} = 0{\cdot}98y_{(t-1)} - 0{\cdot}48u_{(t)} + 0{\cdot}52u_{(t-1)}), \; R = 1$

System I has a zero far enough from the unit circle while the zero of the other is rather close to the unit circle. The limits on $u_{(t)}$ were imposed and chosen to be $\bar{u} = 10$, $\underline{u} = -10$. This range is just sufficient for the optimal unrestricted control with known parameters.

In Fig. 1 the output and input of the system can be seen when the IST strategy was used, Fig. 2 represents the same system when MIST was in action. There is no significant difference between IST and MIST strategy for this system.

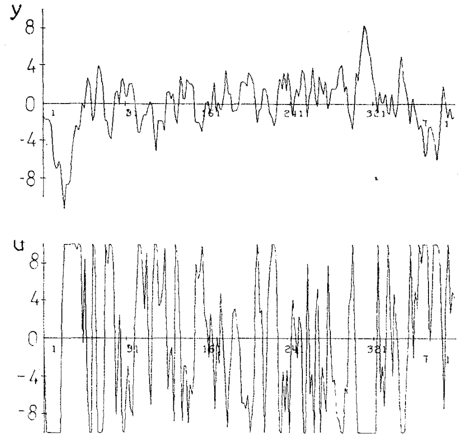

Fig. 1. System I, IST strategy. Output var. = 8·39, input var. = 52·4.

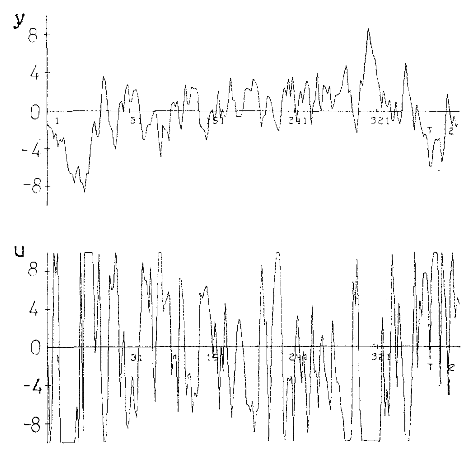

Fig. 2. System I, MIST strategy. Output var. = 8·59, input var. = 44·2.

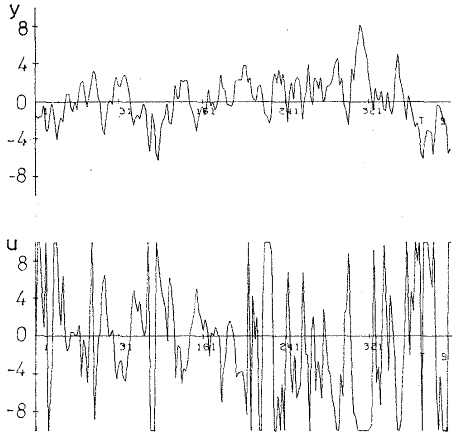

Fig. 3. System I, 10 step strategy. Output var. = 5·94, input var. = 37·1.

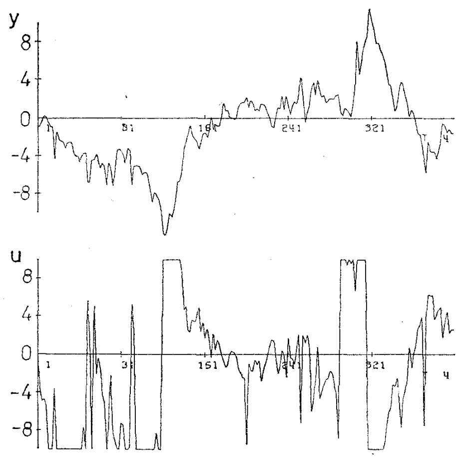

Fig. 4. System II, IST strategy. Output var. = 75·9, input var. = 42·7.

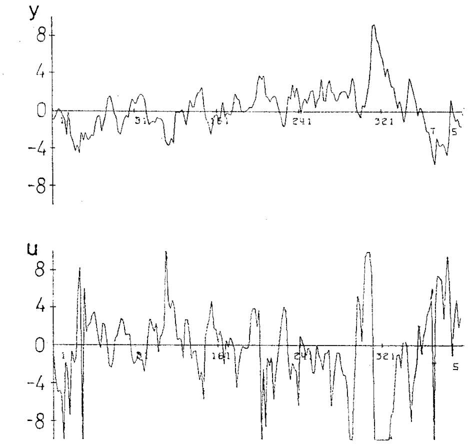

Fig. 5. System II, MIST strategy. Output var. = 22·8, input var. = 17·8.

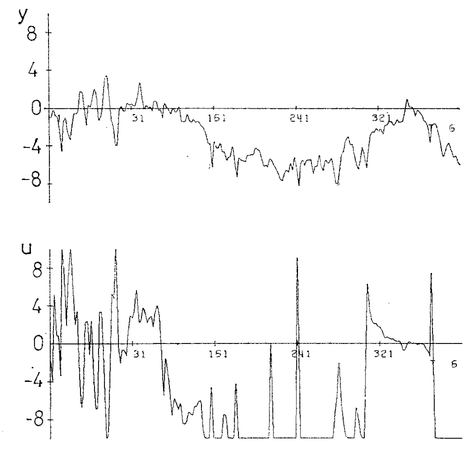

Fig. 6. System II, 10 step strategy. Output var. = 61·3, input var. = 52·5.

To compare the IST strategy with the usual multistep one the additional run was performed also with limit $u_{(t)}$. The output and input of the system, when 10 iterations were made in every control step, starting always from given standard initial conditions, can be seen in Fig. 3. As it could be expected it behaves better at the beginning but no difference can be seen later. However, it spends approximately ten times more computation time.

The very similar trials were made with the system II. Figures 4, 5, 6 represent runs with IST, MIST and 10 iterations per control step, algorithms. In this case the results are much more favourable for the MIST strategy.

The additional run with IST, $Q_u = 2$ and without limits on $u_{(t)}$ is shown in Fig. 7 to compare the behaviour of the closed loop with the fixed penalization of $u_{(t)}$. The output of the system might be satisfactory but even with this rather severe Q_u it is difficult to get $u_{(t)}$ into the desired limits.

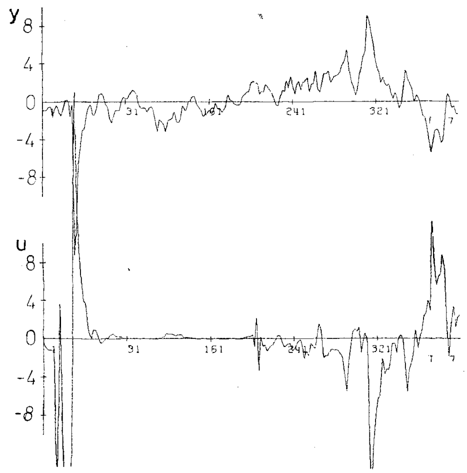

Fig. 7. System II, IST strategy. $Q_u = 2$, output var. = 26·4 input var. = 31·0.

The last series demonstrates the influence of the limit on $\Delta u_{(t)}$. The system I was simulated and limit $\overline{\Delta}^2 = 2^2$ was applied as the most representative. Fig. 8 shows the case with the IST strategy while much better results are obtained by MIST strategy as can be seen in Fig. 9. The properly chosen fixed penalization of $\Delta u_{(t)}$ (in this case $Q_\Delta = 1$) gives a very similar result which can be seen in Fig. 10. It may be worthy to note that the choice of adequate Q_Δ needs several trials. This by-hand

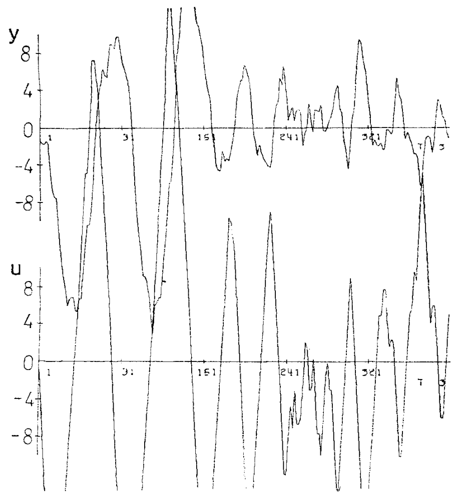

Fig. 8. System I, IST strategy. $-2 < \Delta u < 2$, output var. = 65·4 input var. = 225.

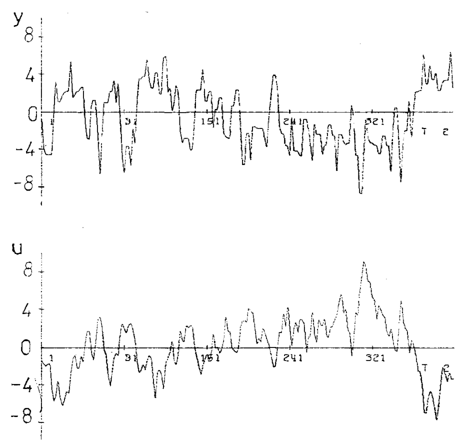

Fig. 9. System I, MIST strategy. $-2 < \Delta u < 2$, output var. = 9·03 input var. = 10·9.

tuning can be well done in simulations but may be rather impractical in real situations.

The given examples more or less confirm what could be expected. We believe that even better results can be obtained in more realistic situations when the system

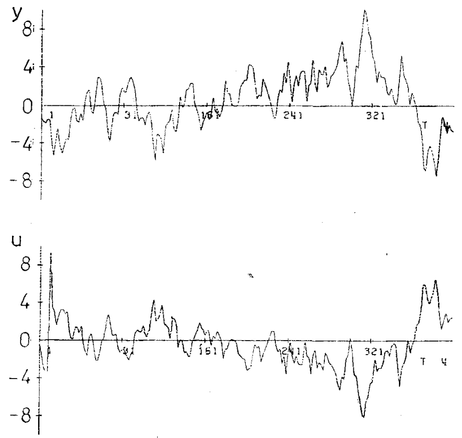

Fig. 10. System I, IST strategy. $Q_{\Delta u} = 1$, output var. = 9·58 input var. = 6·64.

parameters do vary in time and a small forgetting factor has to be used in identification and when there are substantial differences in the solutions to which the Riccati equation should converge.

5. CONCLUSIONS

The MIST suboptimal strategy for self-tuning control described in the paper is able to handle any controllable multivariate system which can be described by a normal linear regression model. While saving the computational simplicity of one-stage-ahead control, it does not rely on the minimum-phase property of the system. Moreover it is able to respect linear restrictions imposed on inputs and/or their changes.

The presented idea of the automatic generation of the data-dependent weights brings the following advantages

- convergence of the IST strategy cannot be destroyed through nonrespecting sufficient rank condition
- number of artificial manually tuned parameters is decreased — penalties on inputs

are replaced by the range restrictions which are more closely connected with technological requirements

- higher quality of closed loop behaviour can be achieved because additional penalties are close to the minimal ones needed in order to respect the restrictions

There are some straightforward till unexploited extensions of the MIST strategy. As an example we can formulate a dead input zone as the set of linear restrictions

$$\Delta u_{(t)} = 0 \quad \text{or} \quad 0 < \bar{\Delta} \leqq |\Delta u_{(t)}| \leqq \bar{\bar{\Delta}}$$

The lower bound $\bar{\Delta}$ here describes the range of input changes undistinguishable from zero.

It can be easily seen that this problem can be solved in the same manner. In a similar way one can generate data-dependent output weight in order to respect that a whole range of outputs is (in some cases) equivalent from the point of view of a user.

The above extensions are rather apparent but we believe that there is a possibility to extend the idea of using simple forms of the function $V(.)$ to some nonlinear systems and/or non-quadratic criteria. A strong connection with the Lyapunov stability theory and the theory of dynamic programming can be felt and some attempts have been done in this direction.

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Ing. Josef Böhm, CSc., Ing. Miroslav Kárný, CSc,. Ústav teorie informace a automatizace ČSAV (Institute of Information Theory and Automation — Czechoslovak Academy of Sciences), Pod vodárenskou věží 4, 182 08 Praha 8. Czechoslovakia.