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FUZZY ZERO, ALGEBRAIC EQUIVALENCE: YES OR NO?

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When some algebraic properties of fuzzy numbers and, more generally, fuzzy quantities are investigated then it appears useful to study more deeply the notion of "fuzzy zero" and of the additive equivalence derived from it. This was done, e. g. in [2], [3], [4], [5] and in some other related papers.

In fact, the idea of fuzzy zero, and especially the concept of additive equivalence evidently provoke some questions. Namely, if exactly this concept of fuzzy zero really reflects the intuitive vision of negligibility, connected with the zero. Further, if the equivalence concept based on such fuzzy zero is distinguishing enough, i. e., if elements equivalent in this sense are also similar regarding the common understanding of similarity. In this contribution we briefly recall the necessary notions and discuss some aspects of the questions mentioned above. In this sense, the following paper does not offer new formal results but it summarizes discussion arguments regarding the usefullness and acceptability of a few notions derived to explain the behaviour of fuzzy quantities as algebraic objects.

PART I: CONCEPTS AND TOOLS

1. BASIC CONCEPTS

In the whole paper we denote by R the set of all real numbers.

By *fuzzy quantity* we call any fuzzy subset of R with membership function $\mu_a : R \to [0,1]$ fulfilling

$$\exists\, x_0 \in R : \mu_a(x_0) = 1, \tag{1}$$

$$\exists\, x_1,\, x_2 \in R, \quad x_1 < x_2, \quad \forall\, x \notin [x_1, x_2] : \mu_a(x) = 0. \tag{2}$$

The set of all fuzzy quantities is denoted by $\mathbb{R}$.

If $a \in \mathbb{R}$ then we denote by $-a$ the fuzzy quantity with

$$\mu_{-a}(x) = \mu_a(-x) \tag{3}$$

for all $x \in R$. If $y \in R$ then $\langle y \rangle$ is the degenerated fuzzy quantity with

$$\mu_{\langle y \rangle}(y) = 1, \qquad \mu_{\langle y \rangle}(x) = 0 \quad \text{for } x \neq y.$$

There are two types of fuzzy quantities which deserve special attention. We say that $a \in \mathbb{R}$ is *trapezoidal* iff there exist real numbers $a_1,\ a_0,\ a_0',\ a_2 \in R,\ a_1 \le a_0 \le a_0' \le a_2$ such that

$$\begin{aligned} \mu_a(x) &= (x-a_1)\,/\,(a_0-a_1) & &\text{for } x \in (a_1, a_0], \\ &= 1 & &\text{for } x \in [a_0, a_0'], \\ &= (x-a_2)\,/\,(a_0'-a_2) & &\text{for } x \in [a_0', a_2), \\ &= 0 & &\text{for } x \notin (a_1, a_2), \end{aligned} \tag{4}$$

where equality $a_1 = a_0$ naturally implies $\mu_a(x) = 0$ for $x < a_0$, $\mu_a(a_0) = 1$, and $a_0' = a_2$ implies $\mu_a(a_0') = 1$, $\mu_a(x) = 0$ for $x > a_0'$. Every trapezoidal fuzzy quantity a is fully characterized by the quadruple (a_1, a_0, a_0', a_2).

Trapezoidal fuzzy quantity $a \in \mathbb{R}$ characterized by the quadruple (a_1, a_0, a_0', a_2) is called *triangular* iff $a_0 = a_0'$. Then we also say that a is characterized by the triple (a_1, a_0, a_2).

Due to the representation principle [1] we define the *sum* of fuzzy quantities $a \oplus b,\ a,\ b \in \mathbb{R}$, as fuzzy quantity with membership function

$$\mu_{a\oplus b}(x) = \sup_{y\in R} \left(\min(\mu_a(y),\ \mu_b(x-y)\right), \quad x \in R. \tag{5}$$

Analogously, the *product* of $a,\ b \in \mathbb{R}$ is a fuzzy quantity $a \odot b$ with membership function

$$\begin{aligned} \mu_{a\odot b}(x) &= \sup_{y\in R,\, y\neq 0} \left(\min(\mu_a(y),\ \mu_b(x/y)\right), \quad x \in R,\ x \neq 0, \\ \mu_{a\odot b}(0) &= \max\left(\mu_a(0),\ \mu_b(0)\right) \end{aligned} \tag{6}$$

If $r \in R,\ a \in \mathbb{R}$ then the *crisp product* of r and a is the fuzzy quantity $r \cdot a = \langle r \rangle \odot a$, i.e. for any $x \in R$

$$\begin{aligned} \mu_{r\cdot a}(x) &= \mu_a(x/r), & &\text{if } r \neq 0, \\ &= \mu_{\langle 0 \rangle}(x), & &\text{if } r = 0. \end{aligned} \tag{7}$$

In the whole paper the equality symbol $a = b$ for $a,\ b \subset \mathbb{R}$ denotes the identity between membership functions, i.e. $\mu_a(x) = \mu_b(x)$ for all $x \in R$. This strict equality does not correspond with the vague nature of fuzzy quantities, and it can be considered for a source of some difficulties. Then some kind of weakening of this equality relation is desirable, and a major part of this contribution is devoted to the discussion and verification of one of such weakenings.

2. PROPERTIES AND PROBLEMS

It is easy to verify (cf. [1], [2], [4] and other works) that for $a,\ b \in \mathbb{R}$ and $r \in R$

$$a \oplus b = b \oplus a, \qquad a \oplus (b \oplus c) = (a \oplus b) \oplus c, \qquad a \oplus \langle 0 \rangle = a, \tag{8}$$

and also

$$r \cdot (a \oplus b) = (r \cdot a) \oplus (r \cdot b). \tag{9}$$

All these properties are useful. Their applicability would be much wider if $\mathbb{R}$ is a group or linear space, it means if

$$a \oplus (-a) = \langle 0 \rangle, \tag{10}$$

$$(r_1 + r_2) \cdot a = (r_1 \cdot a) \oplus (r_2 \cdot a) \tag{11}$$

for $r_1,\ r_2 \in R,\ \ a \in \mathbb{R}$.

The fact that (10) and (11) are not generally fulfilled means a serious complication if some classical procedures with crisp numbers are to be extended to fuzzy numbers or fuzzy quantities.

There are two possible reactions on the fact that $\mathbb{R}$ is neither a group nor a linear space. We can either accept this fact and to respect the essential constraints of the arithmetics of fuzzy quantities. Or, we can re-consider the concepts and to ask if our demands are adequate to the nature of fuzziness. If we conclude that the processing of fuzzy quantities represented by (5) and (7) and reflected in (8) and (9) violates their fuzziness, it could be useful to suggest a weakening of some concepts. Wishing to do so we should study and discuss the relation between fuzziness and classical algebraic concepts. It was done, e. g., in [2], [3] or [4]. Here we briefly remember and re-consider the conclusions.

3. ROOTS OF PROBLEMS AND FUZZY ZERO

There are two equalities (10) and (11), which are not generally valid even if their validity can be desirable. The reasons of these two discrepancies are different.

The situation can be relatively more simple regarding relation (10). It is paradoxal to demand strict equality between left-hand-side fuzzy quantity and the right-hand side crisp number. Hence, some weaker similarity relation can solve the problem. In fact, as the sum of a fuzzy quantity a and its opposite $-a$ should be something like fuzzy zero, it is natural to base this similarity on the fuzzy zero concept.

The interpretation of the fact that (11) is not valid is not as easy as the previous case. As shown in [3] and [5] neither the weaker similarity based on fuzzy zero can generally guarantee the distributivity of that type. It can help only in some special cases. This is very unpleasant as it means, e. g., that even very simple equality $2 \cdot a = a \oplus a$ is not generally fulfilled. Nevertheless, the full distributivity, i. e. the fact that multiple addition of elements is equivalent to the multiplication by a natural number seems to be too connected with determinism or with quite limiting properties of the input elements (see [1]).

As mentioned above, the remaining group property (10) and in some special cases also the distributivity (11) can be achieved if we use the specific properties of "fuzzy zero". To do it precisely it is necessary to introduce the following formalism (see [3], [2]).

If $y \in R,\ a \in \mathbb{R}$ then we say that a is *y-symmetric* iff for any $x \in R$

$$\mu_a(y + x) = \mu_a(y - x). \tag{12}$$

The set of all y-symmetric fuzzy quantities is denoted by $\mathbb{S}_y$ and by $\mathbb{S}$ we denote the union

$$\mathbb{S} = \bigcup_{y \in R} \mathbb{S}_y.$$

One of especially significant types of y-symmetry are the 0-symmetric fuzzy quantities. Exactly they will be used to represent our idea of fuzzy zero. The 0-symmetric distribution of possibilities gives balanced chance to both, positive and negative, values of the considered fuzzy quantity. Moreover, the 0-symmetry possesses some other essential properties of the zero element. Namely, for any $a \in \mathbb{R}$,

$$a \oplus (-a) \in \mathbb{S}_0, \tag{13}$$

and for $a \in \mathbb{R},\ s \in \mathbb{S}_0$ also

$$a \odot s \in \mathbb{S}_0. \tag{14}$$

The first one of these two properties tends to some weaker form of the validity of the remaining group property (10). The second property (14) shows that this type of fuzzy zero has not only the additive properties of zero but it also has the strength of zero in the multiplication operation. Obviously, $s = -s$ for $s \in \mathbb{S}_0$, too.

There are infinitely many 0-symmetric fuzzy quantities forming a set closed regarding addition $\oplus$, product $\odot$ and the crisp product. It means that for $s_1,\ s_2 \in \mathbb{S}_0,\ r \in R$

$$s_1 \oplus s_2 \in \mathbb{S}_0, \qquad s_1 \odot s_2 \in \mathbb{S}_0, \qquad r \cdot s_1 \in \mathbb{S}_0, \tag{15}$$

(see [2], [3], [4]).

If we deal with an infinite set of zeros which are in certain sense equivalent in their "zeroness" it could be useful to extend this equivalence to the whole set $\mathbb{R}$ and to part $\mathbb{R}$ into disjoint equivalence classes. This approach was used and discussed, e.g., in [2] and [3] and in some other papers.

4. ALGEBRAIC EQUIVALENCE

If we consider all 0-symmetric fuzzy quantities for being equivalent representatives of fuzzy zero then it is possible to extend this principle to more general fuzzy quantities from $\mathbb{R}$. Namely, we can say that two general fuzzy quantities are equivalent if they differ in 0-symmetric component, i.e. iff their differences can be represented by fuzzy zeros.

More exactly, if $a,\ b \in \mathbb{R}$ then we say that a is *additively equivalent* to b and write $a \sim_\oplus b$ iff there exist $s_1,\ s_2 \in \mathbb{S}_0$ such that

$$a \oplus s_1 = b \oplus s_2. \tag{16}$$

It is not difficult to verify (cf. [3]) that

$$a \sim_\oplus a, \qquad a \sim_\oplus b \Longleftrightarrow b \sim_\oplus a, \qquad a \sim_\oplus b \wedge b \sim_\oplus c \Longrightarrow a \sim_\oplus c, \tag{17}$$

and that, moreover,

$$a \sim_\oplus b \Longleftrightarrow r \cdot a \oplus c \sim_\oplus r \cdot b \oplus c \tag{18}$$

for $r \in R,\ r \neq 0,\ c \in \mathbb{R}$.

Due to (17), $\sim_\oplus$ is a regular equivalence relation which parts $\mathbb{R}$ into disjoint equivalence classes. If $y \in R$ and $a \in \mathbb{S}$ then evidently

$$\langle y \rangle \in \mathbb{S}_y \quad \text{and} \quad a \sim_\oplus \langle y \rangle \Longleftrightarrow a \in \mathbb{S}_y. \tag{19}$$

It is easy to verify that every trapezoidal fuzzy quantity $a \in \mathbb{R}$ characterized by the quadruple (a_1, a_0, a_0', a_2) is equivalent to the triangular fuzzy quantity b characterized by (b_1, b_0, b_2) where

$$\begin{aligned} b_0 &= (a_0 + a_0')/2, \\ b_1 &= a_1 + (a_0' - a_0)/2, \\ b_2 &= a_2 - (a_0' - a_0)/2, \end{aligned}$$

as $a = b \oplus s$, where s is 0-symmetric trapezoidal fuzzy quantity characterized by (s_1, s_0, s_0', s_2) fulfilling

$$\begin{aligned} s_1 = s_0 &= (a_0 - a_0')/2, \\ s_2 = s_0' &= (a_0' - a_0)/2, \end{aligned}$$

(i.e. s is a crisp interval $[s_1, s_2]$). This procedure can continue, and b is generally equivalent to triangular $c \in \mathbb{R}$ characterized by (c_1, c_0, c_2) where

$$b = c \oplus \overline{s},$$

$\overline{s} \in \mathbb{S}_0$ is triangular, characterized by $(\overline{s}_1, \overline{s}_0, \overline{s}_2)$,

$$\overline{s}_0 = 0, \quad \overline{s}_1 = -\min(b_0 - b_1,\, b_2 - b_0), \quad \overline{s}_2 = \min(b_0 - b_1,\, b_2 - b_0),$$

$$c_0 = b_0, \quad c_1 = b_1 + \overline{s}_2, \quad c_2 = b_2 - \overline{s}_2.$$

The pair $(c, s \oplus \overline{s})$ represents the maximal reduction of a to symmetric and "irregular" component (see [6]).

It is discutable if this kind of equivalence is natural, i.e., if it reflects the intuitive feeling of similarity between fuzzy quantities. In the remaining parts of this paper we briefly deal with this question.

PART II: ARGUMENTS AND DISCUSSIONS

5. WHAT IS WRONG WITH EQUIVALENCES?

The objections against the idea of additive equivalence can be grouped into three principal arguments.

The first one is based on the intuitive idea of similarity between fuzzy quantities which really need not correspond with the one expressed by the relation $\sim_\oplus$. Indeed, fuzzy quantities $a,\ b \in \mathbb{R}$ such that

$$\begin{aligned}
\mu_a(x) &= 1 && \text{for } x \in [-999,\ 1001], \\
&= x + 1000 && \text{for } x \in [-1000,\ -999], \\
&= -x + 1002 && \text{for } x \in [1001,\ 1002], \\
&= 0 && \text{for } x \notin [-1000,\ 1002],
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mu_b(x) &= x && \text{for } x \in [0, 1], \\
&= -x + 2 && \text{for } x \in [1, 2], \\
&= 0 && \text{for } x \notin [0, 2]
\end{aligned}$$

are equivalent, and both of them are also equivalent to $\langle 1 \rangle$. The same is true for $c,\ d \in \mathbb{R}$ such that

$$\begin{aligned}
\mu_c(x) &= -x + 1 && \text{for } x \in [0, 1], \\
&= x - 1 && \text{for } x \in [1, 2], \\
&= 0 && \text{for } x \notin [0, 2],
\end{aligned}$$

$$\begin{aligned}
\mu_d(x) &= |\sin(x - 1)| && \text{for } x \in [-\pi + 1, \pi + 1], \\
&= 0 && \text{for } x \notin [-\pi + 1, \pi + 1].
\end{aligned}$$

Evidently $a \sim_{\oplus} \langle 1 \rangle$ or $b \sim_{\oplus} c$ and $b \sim_{\oplus} d$ however different they are. But, on the other side, none of them is equivalent to $e \in \mathbb{R}$,

$$\begin{aligned}
\mu_e(x) &= 1 && \text{for } x \in [-999,\ 1001], \\
&= \alpha x + \beta && \text{for } x \in [-1000.01,\ -999], \\
&= -x + 1002 && \text{for } x \in [1001,\ 1002], \\
&= 0 && \text{for } x \notin [-1000.01,\ 1002],
\end{aligned}$$

with $\alpha = (1/1.01)$, $\beta = (1000.01/1.01)$, in spite of the fact that real difference between a and e is purely negligible. Hence, the equivalence $\sim_{\oplus}$ on one side forms too wide equivalence classed and, on the other side, ignores some very close similarities.

The second objection which can be given against $\sim_{\oplus}$ is that even symmetric and 0-symmetric noise is a significant characteristic of fuzziness. It describes its extent (or dispersion) and in this sense ignoring this component of a fuzzy number we neglect an important information about it. Really, equivalence between $a \in \mathbb{R}$ given by (20) and $\langle 1 \rangle$ means that one remarkable property was not taken into consideration.

The third objection is of rather different sort. The universality of $\sim_{\oplus}$ is not as wide as it could seem to be if we know (18). In fact, the additive equivalence is completely useless if we have to multiply fuzzy quantities. It is not preserved by multiplication (except the very special case of multiplication by 0-symmetric fuzzy quantities – cf. (14)), and it does not guarantee the group properties of multiplication (see [3]) or, generally, the distributivity (see [3] and [4]).

6. WHAT IS EQUIVALENCE GOOD FOR?

In spite of the objections mentioned above there are also a few reasons which motivate the suggestion of the additive equivalence $\sim_{\oplus}$. Let us remember them.

First, as already mentioned above, the crisp zero $\langle 0\rangle$ as result or component of calculations with fuzzy numbers is not adequate to their character. It does not allow to rely on the group property of $a \oplus (-a)$ being zero-valued but, what is worse, it implements the crisp view and crisp demands into the processing of fuzzy quantities. The additive equivalence $\sim_{\oplus}$ is a logical consequence of interpreting 0-symmetric quantities as fuzzy zero, and of the fact that for any $a \in \mathbb{R}$ the sum $a \oplus (-a)$ is 0-symmetric.

Consequently, and it is the second argument for introducing $\sim_{\oplus}$, the fact that $a = b$ implies $a \sim_{\oplus} b$, together with (8) and (13), means that the set $\mathbb{R}$ with operation $\oplus$ forms an additive group where the equivalence $\sim_{\oplus}$ is considered instead of the equality. Group properties are useful and widely exploited in many algorithms. Their, even weakened, validity has attractive consequences for the effective processing of fuzzy data.

Moreover, due to (14), also the product of any $a \in \mathbb{R}$ and fuzzy zero is fuzzy zero, i.e. it is equivalent to $\langle 0\rangle$.

The additive equivalence does not generally guarantee the validity of the distributivity (11) in a weaker form

$$(r_1 + r_2)\cdot a \sim_{\oplus} (r_1 \cdot a) \oplus (r_2 \cdot a), \quad r_1, r_2 \in R,\ a \in \mathbb{R}. \tag{21}$$

Nevertheless, (21) is fulfilled for some classes of fuzzy quantities. The fact that it is true for $a \in \mathbb{S}$ (i.e., for a being y-symmetric for some $y \in R$) is not very significant. Evidently, if we ignore the 0-symmetric components of fuzzy quantities, and if $a \in \mathbb{S}_y$, i.e. $a = \langle y\rangle \oplus s$, for some $y \in R$, $s \in \mathbb{S}_0$ then (21) reflects the distributivity for crisp $\langle y\rangle$, only.

It is much more significant to note that (21) is fulfilled for trapezoidal a (see [5]). The class of trapezoidal fuzzy quantities is relatively important by itself, but the fact mentioned in the previous sentence, combined with (18) means that the distributivity (21) is fulfilled also for any $a \in \mathbb{R}$ which is additively equivalent to some trapezoidal $b \in \mathbb{R}$. Indeed, if $a \sim_{\oplus} b$ for $a, b \in \mathbb{R}$, b trapezoidal then $a \oplus s_1 = b \oplus s_2$ for some $s_1, s_2 \in \mathbb{S}_0$, and

$$\begin{aligned}
&(r_1 + r_2)\cdot a \sim_{\oplus} (r_1 + r_2)\cdot a \oplus (r_1 + r_2)\cdot s_1 = (r_1 + r_2)\cdot(a \oplus s_1)\\
&= (r_1 + r_2)\cdot(b \oplus s_2) \sim_{\oplus} (r_1 + r_2)\cdot b \sim_{\oplus} r_1\cdot b \oplus r_2\cdot b \sim_{\oplus} r_1\cdot b \oplus r_1\cdot s_2 \oplus r_2\cdot b \oplus r_2\cdot s_2\\
&= r_1(b + s_2) \oplus r_2(b + s_2) = r_1\cdot(a \oplus s_1) \oplus r_2\cdot(a \oplus s_1) \sim_{\oplus} r_1\cdot a \oplus r_2\cdot a,
\end{aligned}$$

where (18) and (9) were used. The set of fuzzy quantities which are equivalent to a trapezoidal one is quite rich, and it can cover very large class of eventual applied problems. On the theoretical level the previous conclusion means that the class of fuzzy quantities $\mathbb{R}^T \subset \mathbb{R}$ which are equivalent to trapezoidal fuzzy quantities forms a *linear space* with the additivity operation $\oplus$, with the multiplication by real number represented by the crisp product, and with the additive equivalence $\sim_{\oplus}$

used instead of equality. This formal but for theoretical purity essential conclusion represents an important argument for considering the equivalence $\sim_{\oplus}$.

Omitting symmetric component of fuzzy quantity can be interpreted and in some cases justified by the need to reduce the extent of fuzziness cummulated during its processing. It is evident and well known that the addition of fuzzy quantities generally increases the support of their membership functions and after a few additions of fuzzy quantities the extent of possible values of the output quantity may be too large. Partition of such quantity on 0-symmetric and non-symmetric component (cf., e. g., [6]), and omitting the 0-symmetric one can limit the support of the membership function to an acceptable and manageable extent. This procedure is especially effective in the case of trapezoidal fuzzy quantities or fuzzy quantities from $\mathbb{R}^T$ which can be reduced to triangular core, as mentioned in Section 4.

The last consequence of the additive equivalence, namely the reduction of fuzzy quantities to their non-symmetric "core" (cf. [6]) can be especially useful for the defuzzyfication of results of some complex procedures. For the trapezoidal quantities this reduction is quite effective.

7. CONCLUSIVE REMARKS

It is not easy to weight the arguments summarized in the previous two sections in order to formulate some balanced evaluation of the additive equivalence concept. Each equivalence which is not the strict identity stresses some features of the considered objects, and suppresses some others. It is the matter of actual application or of specific theoretical problem being solved, to decide whether the given type of equivalence reflects the very type of similar features or not. The formally very different but in its proper nature analogous problem appears in the L_2 functional spaces regarding the equivalence of their elements through the almost everywhere equality.

In our case the suggested type of additive equivalence stresses the formal algebraic properties and, consequently, it is especially adequate to the formulation and solution of general theoretical problems. In practical applications, may be, the similarities illustrated in Section 5 by fuzzy quantities a and e can appear more important. But, in such case, it is necessary to accept some serious theoretical difficulties whenever classical algebraic methods should be used.

Analogous approach can be accepted regarding the decomposition of some fuzzy quantities into 0-symmetric and irregular component. Generally, every $a \in \mathbb{R}$ can be parted into a sum

$$a = b \oplus s, \quad s \in \mathbb{S}_0, \ b \in \mathbb{R}. \tag{22}$$

Where, in some extremal cases, may be $s = \langle 0 \rangle$ or $b = \langle y \rangle$, $y \in R$ (cf. [6]). It is possible to interpret s as a regular, symmetric or balanced component of a, and b as its irregular, unbalanced "core". Each of them represents another type of fuzziness, and it depends on the actual application or problem, which one of them is considered for being significant. Generally, such decomposition decreases the extent of the supports of the membership functions as mentioned in Section 6. Independently of the importance of particular components of a in (22), the possibility of such decomposition opens an interesting view on the structure of fuzziness.

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