# Kybernetika

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# Semantic Evaluation of the Components of Ledley and Lusted's Diagnostic Model

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The paper presents a semantic evaluation of the diagnostic model elaborated by Ledley and Lusted. As quantitative measures of the evaluation are introduced: the concept of "diagnostic power", "symptom relevance", "testing relevance" and "diagnostic gain".

## 1. LEDLEY AND LUSTED'S DIAGNOSTIC MODEL AND HEMPEL'S MODEL

To make a diagnosis, i.e. to determine an illness in consideration of the ascertained symptoms, is a complicated process consisting of a complex of procedures, i.e. various operations with data of a not only empirical but also theoretical nature, with data empirically ascertained, as well as with data we dispose of a-priori. Diagnostic processes in technology which lead to ascertaining the causes of technological disorders, to ascertaining the state of the given technical equipment, etc., are of an analogical character. This process of procedures must take into consideration at least the following data:

(1) A survey of all the symptoms available by the given empirical and experimental means.

(2) Symptoms really ascertained. (These are supposed to form a sub-class of the class mentioned sub (1).)

(3) The relevancy of various symptoms (which is supposed to be a relative relevancy with respect to various diagnostic tasks).

(4) A survey of all the diseases that must be taken into account.

(5) A survey of all the knowledge available concerning the interdependence (of a deterministic or statistical nature) among certain symtpoms or complexes of symptoms and certain diseases or complexes of diseases.

(6) A sample of diseases which may be determined on the basis of the data contained in (3), (4) and (5).

Various models of diagnostic procedures that may be regarded as rational reconstructions of actual operations with the data already mentioned, or with some additional ones, are usually based on the fact that, first of all, it is necessary adequately to express the findings concerning the dependences between particular symptoms and particular diseases. This represents the key point of various models of diagnostic procedures. The form of expressing these dependences then corresponds to diverse models of diagnostic procedures. A number of contemporary diagnostic models proceeds from the well-known model of Ledley and Lusted [6]; this operates with logical measures in presenting the complex of symptoms and the complex of diseases and illustrates the interdependence between both these complexes with the aid of either logical or probability measures, actually with the aid of material implication or conditioned probability. It is obvious that the knowledge of these dependences, i.e. the data disposed of a priori in the diagnostic procedure, play a decisive role in determining the diagnosis. This is why these decisions may approximate Bayes decisions to the degree in which the set of a-priori data we are able to dispose of is complete, and to the degree in which we are able to gain complete evidence.

Ledley and Lusted formulated a simple initial scheme of the whole diagnostic procedure which is based on the application of Boole's functions. The arguments of these functions are as follows:

(a) individual symptoms in case of the description of ascertained symptoms,
(b) individual symptoms and individual diseases in case of medical findings,
(c) individual dieseases in case of the diagnosis itself.

These three components correspond to the data sub (2), (5) and (6), while the logical model should supply the possibility to deduce (6) on the basis of (2) and (5).

In formulating the task thus conceived it is necessary to delimit with precision both the vocabulary and the rules of the language $\mathscr{L}$ in which the task is formulated. We will assume that it is the language of the predicate logic of the first order which, besides current logical symbols, comprises the class $\gamma$ of observational predicates and the class $\delta$ of theoretical predicates. It is further expedient to assume that $\gamma$ represents the designations of all the symptoms available by the given empirical and experimental measures, i.e. the data sub (1), and that $\delta$ forms the class of all the designations of diseases coming into consideration, i.e. the data sub (4). This convention in specifying non-logical components is based on the following consideration:

If it were possible to observe the inidivual diseases directly and immediately, i.e. if it were possible to suspend the distinction between $\gamma$ and $\delta$, then all further data and all further operations would be superfluous. This applies primarily to the data sub (5). The entire diagnostic practice hitherto applied, however, seems to confirm the view that designations of diseases are usually lacking the character of observational predicates, i.e. predicates whose statements can be decided directly and immediately on the basis of observation.

In considering the starting-point modified in this way, we find that what could be characterized as the medical theory, i.e. that which corresponds to the data sub (5), involves the following components:

(I) statements operating with elements $\delta$ (i.e. the Campbellian part of the theory that may be considered as a class of postulates or implicit definitions of the elements $\delta$);

(II) statements operating with elements $\gamma$ (i.e. the so-called observational part of the theory that may be considered as a class of empirical generalizations);

(III) correspondence rules of the given theory operating with elements of both the classess.

The fact that the medical theory — designated here as $T(\gamma \cup \delta)$ — involves all the three mentioned components may be demonstrated on the following examples: The knowledge that a given sort of upper-respiratory-tract diseases is a subclass of virus diseases can be expressed with the help of predicates which are elements $\delta$, without being obliged to take elements $\gamma$ into consideration. All the statements that fix the properties and relations of the elements of $\delta$ without using the elements of $\gamma$ belong to the first part of $T(\gamma \cup \delta)$. Statements that fix the properties and relations of the elements of $\gamma$ without using the elements of $\delta$ belong to the second part of $T(\gamma \cup \delta)$. The latter includes e.g. all the statements on the deterministic or statistical dependences of individual symtoms which have been hitherto ascertained and proved in a high degree. Ledley and Lusted's diagnotsic model operates only with the third part of the medical theory, i.e. with correspondence rules, conceiving them as the dependences of the elements of the class $\gamma$ upon the elements of the class $\delta$, presented in terms of logical or probability expressions. In other words, the following dependences are involved: If the complex of diseases $D_i(D_i \in \delta)$ is found in the patient, there is also the complex of symptoms $S_j(S_j \in \gamma)$. Analogically, in case of a probalistic dependence, the probability of the occurrence of the complex of symptoms $S_j$ is reckoned with, on the assumption that the disease $D_j$ has broken out. As Ledley and Lusted point out, this way of reasoning which takes into account the dependence of the occurrence of symptoms upon the occurrence of diseses, and not the other way round, appears in diagnostic procedures more frequently and corresponds to the results of clinical analysis concerning the relation between diseases and symptoms. It may be added that this way of reasoning also corresponds to the causal valuation of the relation between diseases and symptoms where diseases are considered as causes and symptoms as effects in certain generally occurring causal relations.

The basic formula of the diagnosis is intended to express — in the sense of Ledley and Lusted's model — the following intuitive consideration: Given a certain medical theory which can be conceived as a class of statements operating with elements of the classes $\gamma$ and $\delta$ and denoted here as $t(\gamma \cup \delta)$, then, if a certain complex of symptoms — expressed here in the statement $s(\gamma)$ — is ascertained in the diseased person, the occurrence of the complex of diseases — expressed here in the statement $d(\delta)$ —

**538**    can be assumed. This procedure can then be expressed in the following logical scheme:

$$t(\gamma \cup \delta) \rightarrow \left[ s(\gamma) \rightarrow d(\delta) \right].$$

As Ledley and Lusted have pointed out, this logical scheme is equivalent to another scheme:

$$t(\gamma \cup \delta) \rightarrow \left[ \sim d(\delta) \rightarrow \sim s(\gamma) \right],$$

which, in turn, corresponds to the following intuitive consideration: The given medical theory enables us to draw the conclusion that, if a certain disease has ceased, also the symptoms called forth by this disease have disappeared.

Another equivalent logical scheme can be referred to:

$$t(\gamma \cup \delta) \cdot s(\gamma) \rightarrow d(\delta).$$

This corresponds to the following intuitive consideration: If we dispose of the medical theory $t(\gamma \cup \delta)$ and find the occurrence of the complex of symtoms $S_j$ expressed in the statement $s$, then the occurrence of the disease $D_i$ expressed in the statement $d$ can be assumed.

The mentioned logical scheme enables us to deduce $d(\delta)$, i.e. statements operating with predicates that cannot immediately be decided by available empirical and experimental measures, on the basis of data involving both a-priori and a-posteriori components, i.e. on the basis of statements operating with theoretical predicates (designations of diseases) on the one hand, and, on the other hand, with empirical predicates (designations of the symptoms) which can immediately be decided by available empirical and experimental measures. Hence, if we omit the fact that in Ledley and Lusted's model of diagnosis the medical theory is represented only by correspondence rules, we cannot avoid the fact that this model operates with both a-posteriori and a-priori data, with both empirical and theoretical predicates. Thus it is obvious that the logical model of diagnosis elaborated by Ledley and Lusted is connected with some problems the solution of which is by no means easy.

For example, it is possible to show that if we stick to this way of reasoning, taking into account the dependence of the occurrence of symptoms upon the occurrence of diseases, and not vice versa, and expressing this dependence in the current form of correspondence rules worked out with the help of material, implication (as is also indicated by Ledley and Lusted), we are unable to deduce a diagnosis from the premises represented by $t(\gamma \cup \delta)$ and $s(\gamma)$: Let us presume that $t(\gamma \cup \delta)$ is represented by a tetrad of nomological statements which have been sufficiently confirmed:*

(1) $$(\forall x)\,(D_i x \rightarrow S_1 x),$$

(2) $$(\forall x)\,(D_i x \rightarrow S_2 x),$$

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* This is a modification of an example introduced by W. Stegmüller [8, p. 175].

(3) $$(\forall x)\,(D_i x \to S_3 x)\,,$$

(4) $$(\forall x)\,(D_i x \to S_4 x)\,,$$

where $S_1$, $S_2$, $S_3$ and $S_4$ are four different symptoms which can be ascertained in case the disease $D_i$ has broken out. If the doctor finds that the presence of all the mentioned symptoms can be proved in the patient $a$, i.e.

(5) $$S_1 a \,.\, S_2 a \,.\, S_3 a \,.\, S_4 a$$

he is not justified to infer herefrom that the patient a may be supposed to have developed the disease $D_i$. In other words, the statement

(6) $$D_i a$$

may appear plausible but cannot be inferred from a judgement whose premises are formed by the statements (1)−(5). If, moreover, we know that the occurrence of the disease $D_1$ is always accompanied by the occurrence of the symptom $S_5$, the statements (1)−(5) do not allow us to infer that it will also be possible to find $S_5$ in the patient $a$.

For the purposes of demonstrating their logical model of diagnosis, Ledley and Lusted use matrix schemes assigning particular complexes of diseases to the combinatively conceived complex of symptoms. This way of assigning, however, actually conceives $t(\gamma \cup \delta)$ at a set of definitions in which the designations of the complex of diseases are conceived as molecular predicates definable by means of atomic predicates representing the designations of individual symptoms. Hence if we substitute

(7) $$(\forall x)\,(D_i x \equiv S_1 x \,.\, S_2 x \,.\, S_3 x \,.\, S_4 x)\,,$$

for (1)−(4), then (6) can be inferred from (7) and (5) as a conclusion of a deductive inference.

It is obvious that, under these conditions, it is possible to comply with the original requirement of the logical diagnostic model logically to infer $d(\delta)$ from $t(\gamma \cup \delta)$ and $s(\gamma)$; however, $t(\gamma \cup \delta)$ is thus suspended to a set of definitions where the elements of the class $\delta$ are defined by means of the elements of the class $\gamma$. This in turn implies that the differentiation of both the classes, and thus also of the theoretically desidable and the empirically decidable statements, is loosing its substantiation. In defence of such a conception it may be said that this conception partly corresponds to the elementary form of medical knowledge on the basis of which the individual diseases were given their designation and denotation in a predominantly empirical manner. Hence, on this level, medical knowledge was of the nature of empirical generalizations and the differentiation of the three mentioned components of the medical theory was unsubstantiated.

In summarizing the present analysis of what Ledley and Lusted call the logical

base* of their model of diagnosis, we can say that this model may be applied only if the following conditions are fulfilled:

(1) The elements of class $\delta$ must be introduced by explicit definitions in which the definiens operates only with elements of class $\delta$.

(2) The relations between the complexes of class $\delta$ and the complexes of class $\gamma$ with which $t$ operates are symmetrical — in other words: if the complex $S_j$ can be assigned to the complex $D_i$, it also holds good that $D_i$ can be assigned to the complex $S_j$.

The possibility to fulfil these relatively very strict conditions appears to be most very limited, at least for the following reasons: The introduction of class $\delta$ elements with the help of explicit definitions is often problematic due to the fact that it requires the knowledge of all the relevant elements of class $\gamma$ which may be assigned to the given element of class $\delta$. In practice, however, it is impossible to exclude that the given disease will find its expression in further symptoms which, in its definitional presentation, have not been taken into account. Equally problematic is the symmetrical character of the dependence of the complex of diseases and the complex of symptoms $S_j$. As a rule the dependence between the respective complexes of diseases and the complexes of symptoms is, moreover, conditioned by that which could be characterized as testing operations.

The testing operations which we shall denote by means of the predicates $O_1, O_2, \ldots$ $\ldots, O_n$ may also be regarded as stimuli; the single symptoms $S_1, S_2, \ldots, S_n$ are repercussions of these stimuli under specific conditions. This implies that the class $\gamma$ disintegrates into two sub-classes — the class of testing operations and the class of symptoms. Then the dependence of the individual elements of the classes $\gamma$ and $\delta$ or of their complex may be expressed by the so-called symptom statements.* Symptom statements may appear in two different forms:

(a) If any object of the diagnosis is afflicted by the disease $D$, then, in case the testing operation $O_1$ is applied, he reacts by the symptom $S_2$, etc., i.e.

$$(\forall x) \left[ Dx \to (O_i x \to S_i x) \right] \quad (i = 1, 2, \ldots, n).$$

(b) If the testing operation $O_1$ is applied, then, in case it calls forth the symptom $S_1$, the object of the diagnosis is afflicted with the disease $D$; if the testing operation

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* As concerns the probabilistic version of this model, the scheme with conditioned probability is substituted for the scheme with material implication. This presupposes the applicability of Bayes decision principles, including rigorous finitism and the so-called principle of total evidence. It is beyond doubt that a real diagnostic process may only more or less approach the abstract scheme of Bayes decisions.

** [3, p. 460], [8, p, 123]. The problems of symptom statements, as may be seen in the following comments, conform with the traditional problems of the so-called reduction sentences and disposition predicates.

$O_2$ is applied, then, in case it calls forth the symptom $S_2$, the object of the diagnosis is afflicted with the disease $D$, etc., i.e.

$$(\forall x)\left[O_1 \to (S_i \to D)\right] \quad (i = 1, 2, \ldots, n)\,.$$

Both the forms of symptom statements may be conceived as a sequence of partial symptom statements, each of which represents a partial criterion for the applicability of $D$. It is also possible to conceive a probabilistic analogy of both the forms of symptom statements which, in the case of form (a), can be expressed in the following way: If the object of the diagnosis $x$ is afflicted by the disease $D$, then, if the testing operation $O_1$ is applied, he reacts — with the probability $q_1$ — by the symptom $S_1$; if the testing operation $O_2$ is applied, $x$ reacts — with the probability $q_2$ — by the symptom $S_2$, etc. In the case of form (b), the probabilistic analogy can be expressed in the following way: If the testing operation $O_1$ is applied, then, if the object $x$ reacts by the symptom $S_1$, he is afflicted with the disease $D$ with the probability $q_1$; if the testing operation $O_2$ is applied, then, if the object $x$ reacts by the symptom $S_2$, he is afflicted with the disease $D$ with the probability $q_2$, etc.

The set of symptom statements operating with a particular complex of testing operations and with a corresponding complex of symptoms as reactions to these operations under specific conditions which we characterize with the help of $D$ always presents — in contrast to the correspondence rules — an incomplete characteristic of $D$. The symptom statements, of course, also have the function of correspondence rules — but rules of quite a different nature than explicit definitions. These rules facilitate a certain delimitation of the elements of class $\delta$ by means of the elements of class $\gamma$, i.e. by means of testing operations and symptoms; this, however, is only a partial interpretation of elements $\delta$ by means of elements $\gamma$.

If we now return to both the forms of symptom statements, we cay say that the form (b) serves directly to determine the respective disease. If we know that

$$(\forall x)\left(O_1 x \,.\, S_1 x \,.\, O_2 x \,.\, S_2 x \,.\, \ldots \,.\, O_n x \,.\, S_n x \to D x\right)$$

and if we find in the patient $a$ that $O_1 a \,.\, S_1 a \,.\, O_2 a \,.\, S_2 a \ldots$, etc., then we can deductively draw the inference that $Da$, i.e. that the person $a$ is afflicted with the disease $D$. In principle, this scheme is quite isomorphous to Hempel's classical deductively nomological model of explanation and prediction.

Neither is the form (a) lacking in significance. If, on the basis of several selected testing operations and positive reactions to these testing operations, the possibility of inferring the disease $D$ is ascertained and if symptom statements of the type (a) are known to us for $D$, it is evident that the further selection of testing operations is far from being accidental but depends on the knowledge of the respective symptom statements of the type (a). This means that, first of all, the working hypothesis $D$ is chosen, which is followed by selecting the complex of test operations for confirming or refuting the chosen working hypothesis. From this point of view, diagnostic

procedures can be divided into two parts which, of course, are mutually complementary and interrelated:

1. the actual determination of the complex of diseases which can be realized on the basis of deductive-nomological models isomorphous with Hempel's model of explanation and prediction, or on the basis of probabilistic analogies of these models;

2. the confirmation of the obtained diagnosis by further testing operations and symptoms connected with these.

If we now return to that which forms the medical theory as a starting-point of diagnostic procedures in the sense mentioned above, we may distinguish the following components:

(I) Statements operating with theoretical concepts, including concepts of individual diseases, i.e. exclusively with $\delta$-class elements e.g. the ascertainment that the given class of diseases is a sub-class of diseases of another kind, that certain $\delta$-class elements are mutually incompatible, etc.

(II) Statements operating with concepts forming the class $\gamma$, i.e. the empirical and experimental component of the given area, i.e. primarily the concepts of testing operations and symptoms — e.g. statements on the dependence of individual symptoms, on the dependences of individual testing operations, etc.

(III) Statements fixing the dependence of the elements of the classes $\gamma$ and $\delta$, i.e. correspondence rules. The character of these rules may be diverse — e.g. that of explicit definitions (in the sense implied by Ledley and Lusted), that of the so-called symptom statements.

As concerns the statements sub (I)—(III), they must be considered as general nomological statements which have been sufficiently confirmed. It is also necessary to point to the fact that the differentiation of the single $\delta$-class elements we have characterized as theoretical (non-observational or explanatory concepts) is not only in the interest of the theory, but also is, as a rule, associated with certain therapeutic procedures. This is why the medical theory contains a further component which assigns individual complexes of therapeutic measured to individual complexes of diseases. This then implies that, besides (I)—(III), there are

(IV) therapeutic statements which may be conceived in the following way: if $\beta$ denotes the class of all therapeutic measures, then therapeutic statements characterize the efficiency of the elements or complexes of elements of class $\beta$ with respect to the elements or complexes of class $\delta$.

Consequently, the medical theory — denoted here as $t(\beta \cup \gamma \cup \delta)$ — is a class of nomological statements operating with the elements of the classes $\beta$, $\gamma$ and $\delta$ and relating to all the objects of the given universe, i.e. to all the individuals composing the given population or all mankind. This theory includes nomological statements of the type (I)—(IV), i.e. Campbellian part of the theory, its empirical-experimental

part (i.e. statements operating with testing operations and symptoms), correspondence
rules, and the therapeutic part of the medical theory.

## 2. STATEMENT COMPONENTS OF THE DIAGNOSTIC PROCEDURE

So far we have been considering only the components that must necessarily be
included in the medical theory, if it is to serve as a starting-point in fixing the diagnosis
concerning a given person, or as a starting-point in determining suitable therapeutic
procedures. However, for determining an adequate diagnosis or, eventually, an ade-
quate therapy, we must have at our disposal — besides the medical theory — also
further statements which, in contrast to the nomological statements forming $t(\beta \cup$
$\cup \gamma \cup \delta)$, have the character of singular statements, i.e. statements relating to single
objects of the given universe, i.e. to individual persons or to concretely determine
groups of persons.

Thus the diagnostic procedure operates with the following kinds of statements
all of which operate with the elements $\beta$, $\gamma$ and $\delta$ (in other words, the classes $\beta$, $\gamma$ and $\delta$
form the predicate components of the vocabulary applied):

(1) Nomological statements of the type (I)—(IV) whose aggregate forms the
medical theory; let us denote them as $h_1, h_2, ..., h_n$.

(2) Singular statements on the realization of testing operations and their results
with respect to individual persons; let us denote them as $s_1, s_2, ..., s_n$.

(3) Singular statements on the presupposed diagnosis concerning individual per-
sons, i.e. singular statements operating with elements of class $\delta$; let us denote them
as $d_1, d_2, ..., d_0$.

(4) Singular statements on an adequate therapy relating to individual persons;
let us denote them as $e_1, e_2, ..., e_p$.

The diagnostic task therefore consists in determining a singular statement con-
cerning the presupposed diagnosis on the basis of the statements sub (1) and (2),
or in determining a singular statement on an adequate therapy on the basis of the
statements sub (1), (2) and (3). In case of the applicability of the deductive-nomological
model of diagnosis, the problem lies in finding a $d$ where

$$h . s \underset{L}{\rightarrow} d ,$$

where $h$ is a nomological statement or a class of nomological statements of the type
(I)—(IV) and $s$ is a singular statement or a class of singular statements concerning
the realization of testing operation and their results. In case of the applicability of the
probabilistic analogy of this model, the problem lies in discovering the degree of
probability with which $d$ can be inferred if we dispose of statements (or classes of
statements) $h$ and $s$.

The pair of statements $\langle h, s \rangle$ can then be characterized as the diagnostic base with regard to $d$. The concept "diagnostic base" thus corresponds to that which, in models of explanation and prediction, is usually characterized as the explanans. In contrast to this, the actual determination of the diagnosis, i.e. the singular statement on the presupposed diagnosis concerning certain persons, corresponds to that which is characterized as the explanandum in the models of explanation and as a prognostic statement in models of prediction.

In our further explications we will refer to some possibilities of the semantic evaluation of the statement components of the diagnostic procedures which take advantage of some results of the semantic theory of information. The semantic theory of information usually operates with the means of inductive logics, i.e. it operates with probabilistic measures assigned to individual statements, or with measures assigned to that which the single statements relate to — i.e. the extensional or denotative version of this semantic conception. In our further explanation we will keep to the denotative version; this also implies that all the measures of semantic evaluation mentioned below are semantic metaliguistic characteristics relating to the actual statement components of the diagnostic procedure.

First of all, the semantic evaluation of the given diagnostic base with regard to a certain statement on the presupposed diagnosis may be considered. Of course, this evaluation is always relativized to a certain statement $d$. In the following comments we will, therefore, introduce the concept of the "diagnostic power" of the pair $\langle h, s \rangle$ with regard to the statement $d$. Further, it is possible to introduce the concept of "likelihood" of a particular statement on the presupposed diagnosis with respect to $h \cdot s$. This signifies that even the concept of likelihood of $d$ is always relativized to medical knowledge and to statements on the realization of testing operations and their results. A further concept that can be used in evaluating the statement components of the diagnostic procedures is the concept of "symptom relevance". Symptom relevance is a concept for evaluating the realized testing operations and their results with respect to medical knowledge $h$ and with respect to $d$. (As may be evident from the foregoing intuitive considerations, $h$ should be considered as a conjunction of all the nomological statements which are substantial for the determined diagnosis, i.e. with regard to $d$.)

It is possible to consider some further possibilities of evaluating the statement components of the diagnostic procedure, e.g. the means assessing $e_1, e_2, \ldots$ etc. with regard to the ascertainments concerning $s_1, s_2, \ldots$ etc., i.e. evaluating the separate therapeutic steps, etc. (The means assessing $e_1, e_2, \ldots$ etc. can eventually be relativized to further components, e.g. also to $d_1, d_2, \ldots$ etc.) Since the aim of this study is the reconstruction of the abstract scheme of the actual diagnostic procedure, we will leave these further possibilities aside.

## 3. EVALUATION OF THE STATEMENT COMPONENTS OF THE DIAGNOSTIC PROCEDURE

### a) Diagnostic base and diagnostic power

The pair $\langle h, s \rangle$ which has been characterized here as the diagnostic base with regard to $d$ is aimed at reducing the uncertainty with respect to that to which $d$ related. In other words, the diagnostic base with regard to $d$ should ensure the maximum certainty as concerns the presupposed diagnosis. If $d$ can be determined with unambiguous precision on the basis of $h$ and $s$, e.g. since it holds good that

$$h \cdot s \overset{\rightarrow}{L} d$$

(this corresponds to the so-called deductive-nomological model of explanation, prediction, or other analogical procedures of the same type), it is evident that the diagnostic power of the pair represented by a nomological statement or a class of such statements and a class of statements concerning the realized testing operations and their results is maximum.

From the intuitive point of view it is desirable for the diagnostic power of the pair $\langle h, s \rangle$ to be the greater, the more reduced is the initial uncertainty connected with that to which $d$ relates. If we are able to express the initial uncertainty connected with that to which $d$ relates, as well as the reduction of this initial uncertainty given by our disposing of an adequate diagnostic base — i.e. the pair $\langle h, s \rangle$ — we may say that the measure of the diagnostic power of the pair $\langle h, s \rangle$ with regard to $d$ — which we will denote as $\Delta(h \cdot s/d)$ — is the greater, the greater is the difference between the initial and the conditioned uncertainty.* It is therefore, also desirable for the diagnostic power to increase in case the difference between the initial uncertainty of that to which $d$ relates and the conditioned uncertainty with regard to $d$ is increasing — if we also dispose of the pair $\langle h, s \rangle$ —, and to decrease in case this difference is decreasing. If we dispose of two different diagnostic bases with regard to $d$, e.g. $\langle h_j, s \rangle$ and $\langle h_k, s \rangle$, then the diagnostic power of that diagnostic base which is capable of reducing the initial uncertainty to a larger extent is greater.

These considerations naturally take for granted that — if, moreover, we dispose of an adequate diagnostic base — we are able to determine both the initial uncertainty and the shift of this initial uncertainty. Consequently, the determination of an adequate measure of the initial and the conditioned uncertainty is the basic precondition of the quantifications $\Delta(h \cdot s/d)$. There are, evidently, various possibilities of quantitatively determining the initial and the conditioned uncertainty. Four of these possibilities which, in principle, are based on the measure of semantic information

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* It is evident that if the concept "diagnostic base" corresponds to the concept "potential explanans", then the concept "diagnostic power" corresponds to the concept "explanatory power" or "systematic power".

introduced by Y. Bar-Hillel and R. Carnap in 1952, were pointed out by J. Pietarinen [1].

For the time being, let us leave aside the concrete determination of the measure of uncertainty. From the intuitive point of view, the uncertainty connected with the given statement is the greater, the more alternatives or alternative possibilities are excluded by the given statement. If the given statement does not exclude any alternative possibility whatsoever, e.g. because it logically follows from a confirmed nomological statement, its uncertainty is minimum, i.e. zero. If we denote the uncertainty connected with the statement $d$ as $U(d)$ and the uncertainty connected with $d$, if $h$ is given, as $U(d/h)$ — which may also be characterized as a conditioned uncertainty —, it is possible to formulate the following postulates which must be fulfilled by a satisfactory measure of diagnostic power of the pair $\langle h, s \rangle$ with regard to $d$ (these postulates correspond to the postulates for the quantitative measure of the power of explanation or prediction, introduced by J. Pietarinen [7]):

(R1) $\Delta(h \cdot s/d) = f[U(d), U(d/h \cdot s)]$,

(R2) $\Delta(h \cdot s/d) \gtreqless 0$ iff $U(d/h \cdot s) \lesseqgtr U(d)$,

(R3) $\Delta(h \cdot s/d) = \max \Delta = 1$ iff $U(d/h \cdot s) = \min U = 0$,

(R4) $\Delta(h_j \cdot s/d) \geq \Delta(h_k \cdot s/d)$ iff $U(d/h_j \cdot s) \leq U(d/h_k \cdot s)$,

(R5) $\Delta(h \cdot s_j/d) \geq \Delta(h \cdot s_k/d)$ iff $U(d/h \cdot s_j) \leq U(d/h \cdot s_k)$,

(R6) $f$ is the linear function of its second argument.

(It is evident that, on the basis of (R5) and (R6), the values of this function are increasing in case the values of its second argument are decreasing.)

(R7) $\Delta(h \cdot s/d) = \min \Delta = 0$ iff $U(d/h \cdot s) = U(d)$.

The intuitive sense of the above postulates is evident: the measure of the diagnostic power is the linear function of two arguments, i.e. the initial uncertainty and the conditioned uncertainty. The greater or the smaller is the difference between the initial uncertainty and the conditioned uncertainty, the greater or the smaller is the measure of the diagnostic power of the given diagnostic base with regard to the given $d$. If the conditioned uncertainty $U(d/h \cdot s)$ is minimum, i.e. equal to zero, or, in other words, if the given diagnostic base does not leave anything uncertain with regard to $d$, then $\Delta(h \cdot s/d) = \max \Delta = 1$. On the basis of (R4) and (R5) it is obvious that a higher diagnostic power with regard to the given $d$ can be obtained either by acquiring new medical knowledge or by realizing further testing operations which are relevant with regard to the given $d$.

On the basis of the mentioned postulates, the measure of the diagnostic power can be defined as follows:

(D1) $\Delta(h \cdot s/d) =_{df} \dfrac{U(d) - U(d/h \cdot s)}{U(d)}$.

It is evident that the quantification of this measure of the diagnostic power depends on the extent to which it is possible to quantity $U(d)$ and $U(d/h \cdot s)$. If uncertainty is considered as the measure which is decreasing together with the growing number of excluded alternative possibilities, then it holds that

$$\text{if} \quad h \cdot s \underset{L}{\overset{\rightarrow}{\phantom{.}}} d, \quad \text{then} \quad U(d/h \cdot s) = \min U = 0.$$

In all the other cases, $U(d/h \cdot s) > 0$. It is, moreover, evident that the conditioned uncertainty equals the initial uncertainty $U(d)$, if $h \cdot s$ is a tautological statement. Further, it is useful to choose the equality of the initial and the conditioned uncertainty in case the statement $d$ logically results from the negation of $h \cdot s$, i.e.

$$\text{if} \quad \sim (h \cdot s) \underset{L}{\overset{\rightarrow}{\phantom{.}}} d, \quad \text{then} \quad U(d/h \cdot s) = U(d).$$

This postulate is in connection with the intuitive consideration that the diagnostic basis $\langle h, s \rangle$ is not capable of reducing the initial uncertainty connected with $d$, if $d$ logically follows from $\sim(h \cdot s)$.

If it is possible to assign to the individual statement components the probabilistic measure $p$ fulfilling the current axioms of the theory of probability, it holds that

$$U(d) = 1 - p(d)$$

and*

$$U(d/h \cdot s) = p[(h \cdot s) \lor d] - p(d) = p(h \cdot s) - p(h \cdot s \cdot d).$$

Let us now pay heed to some properties of the measure of the diagnostic power thus introduced: If the given diagnostic base $\langle h, s \rangle$ reduces the initial uncertainty $U(d)$ to zero, i.e. if $h \cdot s \underset{L}{\overset{\rightarrow}{\phantom{.}}} d$, then $\Delta(h \cdot s/d) = \max \Delta = 1$. In this case, which corresponds to the Hempel's deductive-nomological model, the given diagnostic base does not leave any uncertainty with regard to $d$ or, in other words, it facilitates an unambiguous determination of $d$. If, on the contrary, the given diagnostic base is not capable of reducing the initial uncertainty $U(d)$, then it is irrelevant with regard to $d$.

It may be presumed that what represents medical knowledge in the diagnostic procedure, can be expressed by the final class if nomological statements $\{h_1, h_2, \ldots, h_n\}$. The concrete diagnostic procedure usually requires to take account of a greater number of nomological statements. In other words, it is necessary to aggregate

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* The simple uncertainty corresponds to the content measure introduced by Bar-Hillel and Carnap [1]. If, however, we should choose also for $U(d/h \cdot s) = \text{cont } (d/h \cdot s)$, i.e. the conditioned content measure $d$ with regard to $h \cdot s$, the above-mentioned postulate that the maximum uncertainty $U(d/h \cdot s)$ should equal the initial uncertainty $U(d)$ would be impaired. A certain embarrassment might arise from assigning probabilistic measures $p$ to nomological statements. In this case it is advantageous to consider the measure $p$ as the measure of confirmation by all the evidence available so far.

548 more nomological statements expressing medical knowledge. In this connection there arises the problem of the additivity of the above-mentioned measure of the diagnostic power. If the conditions of additivity are relativized to an aggregation of nomological statements requiring the conjunctive connection of individual nomological statements, it holds true that, in view of the same testing operations and on the assumption that the single nomological statements are logically disjunct,*

$$\Delta(h_1 . h_2 \ldots h_n . s/d) = \Delta(h_1 . s/d) + \Delta(h_2 . s/d) + \ldots + \Delta(h_n . s/d).$$

On the basis of the mentioned condition for the additivity of the quantitative measure of the diagnostic power, two theoretically significant tasks can be solved: (a) the determination of the optimum diagnostic base and (b) the rules for reducing nomological statements as components of the diagnostic base.

(a) If we dispose of a certain ultimate class of nomological statements representing medical knowledge and if we are able to realize a certain set of testing operations, then $\langle h_1, h_2, \ldots, h_i, s \rangle$ is to be regarded as the optimum diagnostic base with respect to $d$ and on the assumption of the realization of $s$, assuming that

(1) $h_1, h_2, \ldots, h_i$ is a minimum sub-class of all the available nomological statements,

(2) $\Delta(h_1, h_2, \ldots, h_i . s/d)$ is max $\Delta$.

(b) From the intuitive standpoint it is evident that all the statement components which are incapable of increasing the diagnostic power with regard to $d$ may be excluded from the diagnostic base. If we pressume the realizations of $s$, then all the nomological statements for which it holds good that

$$\Delta(h_j . s/d) = 0$$

may be excluded from the diagnostic base.

Let us declare the pair $\langle h_j, s \rangle$ for which the mentioned relation with regard to $d$ holds true as diagnostically irrelevant with regard to $d$. From this point of view, the following rule for reducing the diagnostic base can be formulated:

(Red 1) All the statement components which are diagnostically irrelevant with regard to $d$ may be excluded from the diagnostic base with regard to $d$.

It is also impossible to exclude situations where the present diagnostic base represented by certain nomological statements and certain statements concerning realized testing operations and their results will appear unsatisfactory with regard to $d$. This

* This relatively simple condition of additivity, i.e. the postulate that individual nomological statements be logically disjunct, implies the application of the above-mentioned conception of conditioned uncertainty. In case cont $(d/h . s)$ is chosen for the conditioned uncertainty $U(d/h . s)$, the determination of additivity is more complicated and less advantageous.

situation corresponds to those cases in medical practice when further experts are
asked in for consultations. From the point of view of our model, this signifies that

$$\varDelta(h_1, h_2, ..., h_i . s/d) \leqq \varepsilon,$$

where $\varepsilon$ is a conventionally agreed value for the minimum admissible measure of the diagnostic power with regard to $d$. In this case it is desirable to extend the diagnostic base by further statement components, e.g. $h_j$, $h_k$, so that

$$\varDelta(h_1 . h_2 ... h_i . h_j . h_k . s/d) \geqq \varepsilon.$$

On the basis of these considerations, the concept of a "sufficient diagnostic base" may be introduced: $\langle h_1, h_2 ... h_n, s \rangle$ is a sufficient diagnostic base with regard to $d$, if it holds that

$$\varDelta(h_1 . h_2 ... h_n . s/d) \geqq \pi,$$

where $\pi$ is a conventionally agreed value for the measure of the sufficient diagnostic power with regard to $d$. The sufficient diagnostic base may be connected with a lesser diagnostic power than the optimum diagnostic base. If $\langle h_1, h_2 ... h_n, s \rangle$ is a sufficient diagnostic base with regard to d, it holds good that

$$\max \varDelta \geqq \varDelta(h_1 . h_2 ... h_n . s/d) \geqq \pi.$$

In determining the optimum diagnostic base with regard to $d$ or in determining the sufficient diagnostic base, it is expedient to formulate the following postulate (which is analogical to the well-known postulate of total evidence for inductive and probabilistic logics): In determining the optimum diagnostic base (or the sufficient diagnostic basis) with regard to $d$ and on the assumption of the realization of $s$,* it is necessary to take into account all the statement components which are not diagnostically irrelevant with regard to $d$.

### b) Symptom relevance and testing relevance

So far, the present analysis of the diagnostic base and the possibilities of the semantic evaluation of the diagnostic base was principally concerned with evaluating nomological statements as components of the diagnostic basis. In the following part we shall discuss the possibilities of the semantic evaluation of further components of the diagnostic base with regard to $d$, i.e. statements concerning testing operations and their results. The concept of "symptom relevance" as a measure of evaluating the testing operations and their results will be introduced at first as a comparative concept and then as a quantitative concept.

* As a rule, the realization of a certain set of testing operations depends on a certain limited group of means standing at our disposal (e.g., a certain technical equipment of the clinic, etc.).

Let us assume that we are able to realize two complexes of testing operations, including the registration of their results. We will denote these two complexes as $s_i$ and $s_j$. If we are to determine $d$ and if we dispose of medical knowledge represented by the conjunction of nomological statements $h$, then $s_i$ is symptomatically more relevant than $s_j$ with respect to $h$ and $d$, if

$$\Delta(h . s_i/d) > \Delta(h . s_j/d)$$

or if

$$U(d/h . s_i) < U(d/h . s_j) .$$

Since $0 \leq \Delta(h . s/d) \leq 1$, while $0 \leq U(d/h . s) \leq U(d)$, it is more advantageous to consider the measures of the diagnostic power as the basis of the quantification concept of "symptom relevance".

The complex of the testing operations $s_i$ is then positively relevant with regard to $\langle h, s_j . d \rangle$ if

$$\Delta(h . s_i/d) > \Delta(h . s_j/d) ,$$

it is negatively relevant with regard to $\langle h, s_j, d \rangle$ if

$$\Delta(h . s_i/d) < \Delta(h . s_j/d) ,$$

and, finally, it is equally relevant with regard to $\langle h, s_j, d \rangle$ if

$$\Delta(h . s_i/d) = \Delta(h . s_j/d) .$$

Besides the comparative concept of "symptom relevance" characterized in this way there is the possibility of introducing the quantitative concept of the measure of the symptom relevance $s_i$ relating to $s_j$ with regard to $h$ and $d$, denoted here as

$$\Sigma(s_i, s_j/h . d) .$$

There are two possibilities of quantifying the concept of "system relevance"; the first of these corresponds to Carnap's concept of the relevance quotient [2, p. 356] and the second to Carnap's concept of the "relevance measure" [2, p. 360]. The first possibility assumes that quantification relies on the quotient of both the measures of the diagnostic power, so that in case of a positive relevance of $s_i$ with regard to $\langle h, s_j, d \rangle$ the quotient of symptom relevance is greater than 1, in case of a negative relevance it is smaller than 1, and in case of an equal relevance it is equal to 1. In our definition of the measure of the symptom relevance presented below we proceed from the second possibility, i.e. we operate with the difference of both the diagnostic powers. This second method offers the possibility that, in case $0 \leq \Delta(h . s/d) \leq 1$, it holds good that $-1 \leq \Sigma(s_i, s_j/h . d) \leq 1$. The measure of symptom relevance fulfilling these requirements is then defined as follows:

(D2) $\quad \Sigma(s_i, s_j/h . d) =_{df} \Delta(h . s_i/d) - \Delta(h . s_j/d) = \dfrac{U(d/h . s_j) - U(d/h . s_i)}{U(d)} .$

Consequently, the measure of symptom relevance thus defined determines the extent to which the diagnostic power with regard to $d$ increases or decreases if, in the diagnostic base $\langle h, s_j \rangle$, the complex $s_j$ is substituted by the complex $s_i$. It is evident that this evaluation of the measure of symptom relevance is only relative. This implies, e.g., that in case the diagnostic base $\langle h, s_j \rangle$ with regard to $d$ functions on the basis of the deductive-nomological model, i.e. if $h \cdot s_j \underset{L}{\rightarrow} d$, then any $s_i$ is negatively relevant or, in the extreme case, equally relevant with regard to $\langle h, s_j, d \rangle$. This is why the measure of symptom relevance characterized in this way may serve for comparing the extent to which two different complexes of testing operations and their results with regard to the same $h$ and to the same $d$ affect the higher or lower diagnostic power.

In diagnostic practice, considerations about the symptom relevance which assume that a further complex $s_i$ has been added to the present complex $s_j$ — while we assume that the same $h$ and the same $d$ are involved — are apparently more frequent and also more important. This means that we are interested in the extent to which the diagnostic base $\langle h, s_j \rangle$ with regard to $d$ decreases or increases, if we pass over to the diagnostic base $\langle h, s_i \cdot s_j \rangle$. It seems evident that, from the inductive — logical point of view, this intuitive consideration corresponds to that which Carnap characterized as a problem of the relevance measure of two observations and their relations [2, p. 365]. It might appear that the diagnostic power always increases or, at least, remains the same if we extend the complex of testing operations and their results by further testing operations. In other words, it might seem that

$$\Delta(h \cdot s_i \cdot s_j / d) \geqq (h \cdot s_j / d).$$

In reality, however, further testing operations and their results also may, more or less, throw doubt upon the results of previous testing operations. Hence, the extreme case is a situation wherein further testing operations and their results utterly negate the preceding results — e.g. because it holds that $s_i \equiv \sim s_j$. In this case the diagnostic power is reduced to the lowest value, i.e. to zero. Sometimes, however, another approach is chosen in diagnostic practice: It is assumed that further testing operations — or testing operations carried out later on — and their results have a higher likelihood than the previously performed testing operations, provided that further results suspend previous results. (Naturally, this assumption does not refer to such situations in which the time sequence of certain symptoms is, on the contrary, the attendant phenomenon of a certain course of the disease. In this case, further testing operations and their results extend the given diagnostic basis and increase its power.) If we proceed from this assumption, which is analogical to the principle "actio posterior deroget priori", then the solution of these problems is reduced to the above-mentioned comparison of the powers of two different diagnostic bases.

From the measure of symptom relevance defined above, it is necessary to differentiate another concept which may be useful for judging the adequacy of the respective

testing operations with regard to the hypotheses known so far. In other words, we are concerned with the question to what extent a certain testing information and its results may offer information with respect to the facts reckoned with in the given nomological statements. In this connection we shall refer to the test relevance of the given complex of testing operations $s$ with regard to the complex of nomological statement $h$. The measure of the testing relevance, denoted here as $\tau(s/h)$, may be considered as a special case of the measure of the transmitted information, i.e. the measure of the extent to which the given complex of testing operations is relevant with regard to that to what the complex $h$ relates. If it is possible to consider $\tau(s/h)$ as the measure corresponding to the measure of the transmitted information (see [5]) or to the measure of the information obtained by observation with a view to the given hypothesis (see [4]), then $\tau(s/h)$ may be defined in the following way:

(D3) $$\tau(s/h) =_{df} \frac{\text{cont } (s \lor h)}{\text{cont } (h)},$$

while we proceed — similarly as in the above-mentioned definitions from the concept if "content measure".

If the content measure may be determined by the probabilistic measure $p$, it holds that

$$\tau(s/h) = \frac{1 - p(s \lor h)}{1 - p(h)} = p(\sim s/\sim h).$$

To all appearance, the concept of the "measure of testing relevance" is not adequately applicable directly in the diagnostic procedures but in the clinical and experimental procedures preceding the diagnosis itself. This concept is especially suitable for judging the adequate selection of those testing operations which — with a view to the given complex of nomological statements $h$ — it is expedient to choose with the aim of making it possible to infer the diseases to the occurrence of which $h$ relates.

The measure of testing relevance is the maximum measure in cases where the performed testing operations and their results are so strong that $h$ logically follows from them, i.e.

$$\text{if } s \underset{L}{\to} h, \quad \text{then} \quad \tau(s/h) = \max \tau = 1 .$$

The conditions of additivity are important for the measure of the testing relevance. The measure of the testing relevance is additive for two complexes of testing operations and their results $s_i$ and $s_j$, if both the complexes fo not offer (communicate) any common information with regard to $h$, i.e., logically expressed, if $s_i$ and $s_j$ are logically disjunct. Then it holds that

$$\tau(s_i . s_j/h) = \tau(s_i/h) + \tau(s_j/h) .$$

Unless the condition that $s_i$ and $s_j$ are logically disjunct is fulfilled, it holds that

$$\tau(s_i \cdot s_j/h) = \tau(s_i/h) + \tau(s_j/h) - \tau(s_i \vee s_j/h) \,.$$

The minimum value of the testing relevance measure is obtained either when (a) $s$ is a tautological statement, i.e. when it is incapable of presenting any information with respect to $h$, or (b) when the performed testing operations and their results are so weak that $h$ logically ensues from their negation. We thus obtain:

if $s$ is a tautological statement of if $\sim s \underset{L}{\rightarrow} h,$ then $\tau(s/h) = \min \tau = 0$

It holds, therefore, that

$$0 \leqq \tau(s/h) \leqq 1 \,.$$

### c) Likelihood of the diagnosis

The concept of the "likelihood of the diagnosis" must also be relativized, i.e. relativized to the diagnostic base $\langle h, s \rangle$. Since the diagnostic statement $d$ with respect to the diagnostic base $\langle h, s \rangle$ is the more reliable, the lesser is the conditioned uncertainty $U(d/h \cdot s)$, following definition may be used — in agreement with the above-mentioned method of quantification — for the measure of the likelihood of the statement $d$ with regard to the diagnostic base $\langle h, s \rangle$, noted here as $\Pi(d/h \cdot s)$

(D4) $\qquad\qquad\qquad \Pi(d/h \cdot s) =_{\mathrm{df}} p(d/h \cdot s) \,.$

Then, in agreement with the above-mentioned conditions, we obtain the following marginal values for the measure of the likelihood of $d$ with regard to $h \cdot s$

If $h \cdot s \underset{L}{\rightarrow} d$, then $p(d/h \cdot s) = 1$ and thus

$$\Pi(d/h \cdot s) = \max \Pi = 1 \,,$$

If $h \cdot s \underset{L}{\rightarrow} \sim d$, then $p(d/h \cdot s) = 0$ and thus

$$\Pi(d/h \cdot s) = 0 \,.$$

It holds, therefore, that

$$0 \leqq \Pi(d/h \cdot s) \leqq 1 \,.$$

Thus it is evident that the likelihood of $d$ with regard to the diagnostic base $\langle h, s \rangle$ is minimum when the negation of $d$ logically ensues from this base.

In judging the decision about $d$ on the basis of the diagnostic base $\langle h, s \rangle$ we may choose some further means relying on the previously introduced concepts. Assuming that the decision-making takes place in a situation where all the data are certain and fully reliable, there is no need to calculate with any risk. If, however, this is not the case, it is expedient to consider what will be gained or what will be lost by our choosing $d$ and not $\sim d$. In this connection, the concept of "diagnostic gain" may be

considered. This gain is the greater, the greater is the likelihood of the diagnosis $d$ on the basis of the base $\langle h, s \rangle$, and the greater, the greater is the diagnostic power of this base with regard to $d$. Simultaneously it may be said that this gain is the smaller, the greater is the likelihood $\sim d$ on the basis of the base $\langle h, s \rangle$ and the smaller, the greater is the diagnostic power of this base with respect to $\sim d$. On the basis of this consideration we may then introduce the following quantitative measure of diagnostic gain which we will denote as $\Gamma(d/h \cdot s)$ and which may be defined in the following way:

(D5) $\qquad \Gamma(d/h \cdot s) =_{df} \Pi(d/h \cdot s) \, \Delta(h \cdot s/d) - \Pi(\sim d/h \cdot s) \, \Delta(h \cdot s/\sim d)$ .

Therefore, on the basis D5 and with a view to the previous definitions, it holds that

$$-1 \leqq \Gamma(d/h \cdot s) \leqq 1 .$$

This means that $\Gamma(d/h \cdot s)$ may also acquire negative value, i.e. that $\Gamma(d/h \cdot s)$ may be conceived as a measure of diagnostic gain or diagnostic loss.

Let us now examine more closely the marginal situations, i.e. situations where $\Gamma(d/h \cdot s) = 1$ and where $\Gamma(d/h \cdot s) = -1$.

In case it holds good that $h \cdot s \underset{L}{\rightarrow} d$ and we have decided for $d$ and not for $\sim d$, it holds that

$$\Pi(d/h \cdot s) \quad = 1 ,$$
$$\Delta(h \cdot s/d) \quad = 1 ,$$
$$\Pi(\sim d/h \cdot s) = 0 .$$

Even if the value $\Delta(h \cdot s/\sim d)$ may differ from 0 we get that

$$\Gamma(d/h \cdot s) = \max \Gamma = 1 .$$

In case it holds true that $h \cdot s \underset{L}{\rightarrow} d$ and we have decided for $\sim d$, we analogically get that

$$\Gamma(\sim d/h \cdot s) = -1 .$$

It may be seen that marginal values $\Gamma(d/h \cdot s)$ are conceivable only in case the deductive-homological model assuming deterministic dependences is applicable. In all the other cases, $\Gamma(d/h \cdot s)$ acquires values between 1 and $-1$.

The discussed method of alternative decisions concerned with the decisions between $d$ and $\sim d$ on the basis of the diagnostic base $\langle h, s \rangle$, naturally implies considerable simplification taking into account merely the possibility of deciding in favour of $d$ and the opposite of this possibility.

REFERENCES

[1] Y. Bar-Hillel, R. Carnap: An Outline of a Theory of Semantic Information. Technical Report No. 247, MIT 1952. Repr. in: Y. Bar-Hillel: Language and Information, Addison-Wesley, Reading, Mass. 1964.

[2] R. Carnap: Logical Foundations of Probability. Univ. of Chicago Press, Chicago 1950.

[3] C. G. Hempel: Aspects of Scientific Explanation. The Free Press, New York 1965.

[4] Risto Hilpinen: On the Information Provided by Observations. In: J. Hintikka, P. Suppes: Information and Inference. D. Reidel Publ. Comp., Dordrecht 1970.

[5] Jaakko Hintikka: The Varieties of Information and Scientific Explanation. In: Rootselaar, Steal: Logic, Methodology and Philosophy of Sciences. North-Holland Publ. Comp., Amsterdam 1968.

[6] R. S. Ledley, L. B. Lusted: Reasoning Foundations of Medical Diagnosis. Science *130*, (1959), 3366.

[7] Juhani Pietarinen: Tools for Evaluating Scientific Systemizations. In: J. Hintikka, P. Suppes: Information and Inference. D. Reidel Publ. Comp., Dordrecht 1970.

[8] Wolfgang Stegmüller: Wissenschaftliche Erklärung und Begründung. Springer, Berlin—Heidelberg—New York 1969.

VÝTAH

## K problému sémantického ocenění komponent diagnostického modelu Ledleye a Lusteda

LADISLAV TONDL

Práce předkládá kritický rozbor tzv. logické báze diagnostického modelu Ledleye a Lusteda, jehož výsledkem je zjištění, že aplikabilita tohoto modelu je závislá na obtížně splnitelných podmínkách (zavedení výrazů pro onemocnění na základě výrazů pro symptomy na základě explicitních definic, symetričnost vztahů obou výrazů aj.) Je předložena modifikace logické báze diagnostického modelu, která specifikuje pojem „lékařské teorie", zavádí pojmy „symptomových výpovědí", „testových operací" aj.

Druhá část předkládá několik možností sémantického ocenění výpovědních komponent diagnostického modelu pomocí prostředků sémantické teorie informace. Lékařská teorie je formalizována jakožto konečná třída nomologických výpovědí $\{h_1, ..., h_n\}$, která zahrnuje Campbellovu složku teorie, empiricko-experimentální složku a korespondenční pravidla. Dalšími komponentami diagnostického modelu jsou singulární výpovědi o realizaci testových operací a jejich výsledků $\{s_1, ..., s_m\}$ a singulární výpovědi o předpokládané diagnóze $\{d_1, ..., d_0\}$. Jsou pak zavedeny tyto kvantitativní pojmy: pojem diagnostické mohutnosti $\Delta(h . s/d)$, který představuje sémanticko-informační ocenění dvojice $\langle h, s \rangle$ vzhledem k určitému $d$, pojem

symptomové relevance $\Sigma(s_i, s_j/h . d)$, který umožňuje komparaci dvojice $\langle s_i, s_j \rangle$ vzhledem k $h$ a $d$, pojem testové relevance $\tau(s/h)$, který oceňuje relevanci testových operací vzhledem k danému souboru nomologických výpovědí, a pojem diagnostického zisku $\Gamma(d/h . s)$, který umožňuje ocenění rozhodnutí pro $d$ vzhledem k $h$ a $s$. Jsou zavedeny podmínky pro additivitu těchto pojmů, jakož i podmínky pro

$$0 \leq \Delta(h . s/d) \leq 1 ,$$
$$-1 \leq \Sigma(s_i, s_j/h . d) \leq 1 ,$$
$$0 \leq \tau(s/h) \leq 1 ,$$
$$-1 \leq \Gamma(d/h . s) \leq 1 .$$

Dále jsou naznačeny některé možnosti redukce s ohledem na možnosti sémanticko-informačních prostředků.

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