Kybernetika

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Sampled-Data Controls and the Bilinear Transformation

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By the use of the bilinear transformation, the methods devised for the study of continuous control systems may be applied to sampled-data systems. A simple method facilitating considerably the practical performing of the transformation is presented here.

1. INTRODUCTION

Recently, two methods facilitating the substitution

$$(1) \qquad z = \frac{s + 1}{s - 1}$$

in the polynomial

$$(2) \qquad F(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0$$

have been published in [1], [2]. However, special coefficients for each degree of (2) must be computed in advance in the method described in [1], special (although simple) matrices are needed in the method of [2]. In this article, a very simple method without these inconveniences will be derived.

2. PERFORMING THE BILINEAR TRANSFORMATION

We will make the substitution (1) in two stages

$$(3) \qquad z - 1 = \frac{2}{w},$$

$$(4) \qquad w + 1 = s.$$

First of all, the coefficients b_k $(k = 0, 1, ..., n)$ in the equation

$$(5) \qquad F(z) = b_n(z - 1)^n + ... + b_1(z - 1) + b_0$$

are needed. By successive differentiation on both sides of (5), one gets

$$(6) \qquad b_k = F^{(k)}(1)/k! \,.$$

Then, we substitute from (3) in (5). One gets

$$(7) \qquad F(z) = \frac{1}{w^n} \cdot (b_0 w^n + 2b_1 w^{n-1} + ... + 2^n b_n) = G(w)/w^n \,.$$

Then, we substitute from (4) in $G(w)$.

$$(8) \qquad G(w) = c_n s^n + c_{n-1} s^{n-1} + ... + c_1 s + c_0 \,.$$

By successive differentiation on both sides of (8), one gets

$$(9) \qquad c_k = G^{(k)}(-1)/k! \,.$$

Finally, one gets

$$(10) \qquad F(z) = \frac{1}{(s - 1)^n} \cdot (c_n s^n + ... + c_1 s + c_0) \,.$$

The coefficients in (6) and (9) are easily computed by the known method of Horner schemes.

Thus, the whole procedure can be summarized as follows:

1. Find the coefficients in (6) by the first sequence of Horner schemes.

2. Form the coefficients of $G(w)$ in (7).

3. Find the coefficients in (9) by the second sequence of Horner schemes.

3. ILLUSTRATIVE EXAMPLE

Let (as in [2])

$$F(z) = z^5 + 3z^4 + 4z^3 + 5z^2 + 2z + 4 \,,$$

The steps 1. and 2. of the procedure are

		1		3	4	5	2	4			
				1	4	8	13	15			
1		1		4	8	13	15	19		. 1	19
				1	5	13	26				
1		1		5	13	26	41			. 2	82
				1	6	19					
1		1		6	19	45				. 4	180
				1	7						
1		1		7	26					. 8	208
				1							
1		1		8						.16	128
		1								.32	32

The step 3. of the procedure is

		19		82	180	208	128	32
				−19	−63	−117	−91	−37
−1		19		63	117	91	37	−5
				−19	−44	−73	−18	
−1		19		44	73	18	19	
				−19	−25	−48		
−1		19		25	48	−30		
				−19	−6			
−1		19		6	42			
				−19				
−1		19		−13				
		19						

Thus

$$F(z) = \frac{1}{(s-1)^5} \cdot (19s^5 - 13s^4 + 42s^3 - 30s^2 + 19s - 5).$$

(The last two coefficients are incorrect in [2]. Moreover the term $f_{3,7} = 80$ of the matrix F_1 in [2] and all terms depending theoren are also incorrect. This term should be 60.)

4. CONCLUDING REMARKS

Comparing the described method with the method of [1] (which is less laborious than that of [2]) one sees in the preceding example that 30 additions or subtractions and 6 multiplications are needed here, whereas 30 additions or subtractions and 12 multiplications are needed in [1].

Since no special expressions must be prepared in advance to perform the computations, the described method seems to be useful.

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REFERENCES

[1] Soliman J. I. - Al-Shaikh A.: Sampled-data controls and the bilinear transformation. Automatica 2 (1965), 235—242.
[2] Power H. M.: The mechanics of the bilinear transformation. IEEE Trans. E-10 (1967), 2, 114—116.

VÝTAH

Impulsní regulace a bilineární transformace

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S pomocí bilineární transformace může být k vyšetřování impulsních regulací použito metod, navržených pro regulace spojité. V článku se navrhuje jednoduchá metoda, která podstatně usnadňuje praktické provedení transformace.

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