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Fundamentals of the theory applying to one type of multiparameter non-linear digital control systems are derived in the paper. Some possibilities of its application (seeking of extremum of the multi-variable object function with variable coefficients, adaptive systems for automatic extremum seeking, etc.) are pointed out. This is a continuation of paper [8].

1. INTRODUCTION

At present multi-parameter non linear digital control systems are becoming more and more important for dealing with a number of problems (for example see [1; 2; 3; 4]).

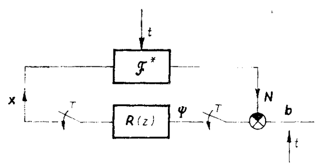

Fig. 1.

In the present paper one type of multi-parameter non-linear digital control systems (further referred to as MS) will be considered, which is described by a matrix block diagram (see Fig. 1), where: t denotes variability in time, $\mathbf{x} = (x_i(n))$ $(i = 1, 2, \ldots, l)$ is the column matrix of output signals related to the square transfer matrix of linear digital controllers with $l \times l$ elements, $\mathscr{F}^* = (\mathscr{F}_{ij})$ is the square matrix of non-linear operators, which can create out of input signals a column functional matrix of output signals $\mathbf{N} = (N_{ii}(x_i, t) + \sum_{j=1, j \neq i}^{l} N_{ij}(x_j, t))$, provided there is no delay, $\mathbf{b} = (b_i(t))$ is the column functional vector of reference imputs, and $\mathbf{\Psi} = (\psi_i(n))$ is the column functional vector of actuating signals. Exceptions to this type of control systems will be mentioned specifically.

We shall suppose further that the following conditions have been fulfilled:

(1.1) $$N_{ii}(0, t) = 0 ,$$

(1.2) $$\sum_{\substack{j=1 \\ j \neq i}}^{l} N_{ij}(0, t) = 0 ,$$

(1.3) $$0 < \partial N_{ii}(x_i, t)/\partial x_i < \infty ,$$

$$0 < \partial N_{ij}(x_j, t)/\partial x_j < \infty , \quad (j \neq i)$$

or

$$-\infty < \partial N_{ij}(x_j, t)/\partial x_j < 0 ,$$

(1.4) $$\lim_{t \to \infty} N_{ii}(x_i, t) = N_{ii}^*(x_i) ,$$

(1.5) $$\lim_{t \to \infty} \sum_{\substack{j=1 \\ j \neq i}}^{l} N_{ij}(x_j, t) = \sum_{\substack{j=1 \\ j \neq i}}^{l} N_{ij}^*(x_j) ,$$

(1.6) $$\lim_{t \to \infty} b_i(t) = b_i^* > 0 .$$

In order to simplify the explanation we shall suppose futher that the matrix equation

(1.7) $$-\boldsymbol{N} + \boldsymbol{b} = \boldsymbol{0}$$

has only one positive solution for a certain form in time t

(1.8) $$\boldsymbol{x}_t^* = (x_{1t}^* > 0, x_{2t}^* > 0, \ldots, x_{lt}^* > 0) ,$$

where x_{it}^* $(i = 1, 2, \ldots, l)$ are constants.
The Z-transformation defined in paper [8] will be used later in the text and some theoretical points will be derived by means of those above Z-transformation.

2. SOME APPLICATION POSSIBILITIES

Many different problems can be solved as the MS. A large group of these problems is formed by problems, which can be solved by means a non-linear equations system (algebraic, exponential, etc.). Coefficients of these equations may assume any value within prescribed intervals. Each time the solution is sought for selected values of these coefficients.

Let us first explain how to transform the solution of a system of l-variable equations into the solution of MS.

Let us solve matrix equation (1.7), the coefficients of which can assume any value within certain prescribed intervals of values. If we assing the vector of actuating signals according to the description of MS on Fig. 1 to the left-hand side of the equation and if we construct the transfer matrix $\mathbf{R}(z)$ of controllers in such a way that the steady-state value of the actuating signals vector equals zero, then it follows from the interconnection of MS that the solution vector $\mathbf{x}^*$ is the steady-state value of vector $\mathbf{x}$.

Let $E(\mathbf{x}, t)$ be a multi-variable object function, the coefficients of which may assume any values within certain prescribed intervals. Each time the extremum is sought for certain values of these coefficients. If the vector of the object function gradient is expressed by

$$\boldsymbol{\Psi} = \partial E(\mathbf{x}, t)/\partial \mathbf{x} = -\mathbf{N} + \mathbf{b} \tag{2.1}$$

and terms $\mathbf{N}$ and $\mathbf{b}$ have been defined in the introductory part of this paper, then we can see that extremum seeking leads to the solution of equation (1.7), i.e. it can be transformed to the solution of MS. The steady-state value of the output signals vector from the transfer matrix of controllers is the solution vector $\mathbf{x}^*$ for certain values of object function coefficients.

Let us mention that seeking of relative extremum of an object function with constraints (see e.g. [5; 7]) can also be solved as the described basic problem, i.e. the seeking of object function extremum without constraints.

So we can transform the automatic seeking of the object function extremum $E(\mathbf{x}, t)$ into MS solving and we can construct this MS by means of a digital computer or special controllers (the latter being very simple if the simplest possible transfer matrix of controllers is used), and obtain an adaptive system for seeking the extremum of the given object function.

When transforming the seeking of extremum of a multi-variable object function into MS solving, then an optimum transfer matrix of controllers, stability and quality (defined in an appropriate way) of this MS corresponds to an optimum computing algorithm, convergence, and speed of computation convergence.

Fundamental importance of the MS theory follows from the previous text already. By means of the MS theory we can solve a large number of various problems in a uniform way, with many advantages, and very clearly.

Fundamentals of the MS theory will be described in the following text.

3. THE TRANSFER MATRIX OF CONTROLLERS FOR A ZERO VECTOR OF STEADY-STATE ACTUATING SIGNALS

According to Fig. 1 we can denote in MS

$$\mathbf{F}_{\mathbf{x},t}(z) = (F_{ij,x_j,t}(z)), \tag{3.1}$$

where

$$F_{ij,x_j,t}(z) = Z[N_{ij}(n)]/Z[x_j(n)]. \tag{3.2}$$

The ratio of transforms (3.2) in Z-transformation (similar to paper [8]) can be advantegeously utilized in some theoretical considerations, where the control process is not actually calculated because the above mentioned ratio depends on the input signal in non-linear systems and the input signal is not known beforehand.

A transform of the actuating signals vector can be written in the form

$$\boldsymbol{\Psi}(z) = (\boldsymbol{E} + \boldsymbol{F}_{x.t}(z)\,\boldsymbol{R}(z))^{-1}\,\boldsymbol{b}(z)\,, \tag{3.}$$

where $\boldsymbol{E}$ is a unit matrix and $\boldsymbol{b}(z) = (b_i(z)) = (Z[b_i(n)])$ is transform of the column vector of reference inputs.

The transform of the reference inputs vector satisfying condition (1.6) is given by

$$\boldsymbol{b}(z) = \boldsymbol{b}_s(z)/(z - 1)\,, \tag{3.4}$$

where $\boldsymbol{b}_s(z)$ is the functional column vector whose elements are fractional rational functions with no poles and zeros for $z = 1$.

If the steady-state actuating signals vector is to be zero then the following equation must be valid for a stable MS:

$$\lim_{z\to 1} \boldsymbol{\Psi}(z)\,(z - 1) = \lim_{z\to 1} (\boldsymbol{E} + \boldsymbol{F}_{x,t}(z)\,\boldsymbol{R}(z))^{-1}\,\boldsymbol{b}_s(z) = \boldsymbol{0}\,. \tag{3.5}$$

Under the assumption that

$$\lim_{z\to 1} \boldsymbol{b}_s(z) \neq \boldsymbol{0}, \quad \text{(i.e. steady-state value of signal } \boldsymbol{b} \neq \boldsymbol{0}) \tag{3.6}$$

$$\lim_{z\to 1} \boldsymbol{F}_{x,t}(z) \neq \boldsymbol{0}\,, \quad \lim_{z\to 1} \boldsymbol{F}_{x.t}(z) \neq \infty\,,$$

it follows from the condition (3.5) that the controllers transfer matrix $\boldsymbol{R}(z) = (R_{ij}(z))$ for a zero vector of steady-state actuating signals must be such that each diagonal element $R_{ii}(z)$ has a pole for $z = 1$ and all elements can be realized.

The simplest structure of the transfer matrix of controllers $\boldsymbol{R}(z)$ will be obtained, when we choose

$$\boldsymbol{R}(z) = (R_{ii}(z)) = (c_{ii})/(z - 1) \tag{3.7}$$

where c_{ii} are properly chosen constants, i.e. values of non-zero elements in a diagonal matrix (c_{ii}).

The MS with the transfer matrix of controllers expressed by (3.7) will be considered next. Simple results, which may be easily applied in practice, can be obtained when using this type of matrix.

According to designation in Fig. 1 the following expression can be written for the transfer matrix of controllers expressed by (3.7)

$$\mathbf{x}(z) = [(c_{ii})/(z - 1)]\, \boldsymbol{\Psi}(z)\,. \tag{4.1}$$

The i-th element of vector $\mathbf{x}(z)$, which can be obtained after performing the multiplication, is expressed in the form

$$x_i(z) = [c_{ii}/(z - 1)]\, \psi_i(z)\,. \tag{4.2}$$

It follows from the above that when utilizing the transfer matrix of controllers in the form of (3.7), the calculation of the control process can be expressed as a simultaneous calculation of l fictitious synchronously operating one-parameter non-linear digital control systems which influence one another. This influencing is expressed by the matrix $\mathscr{F}^*$ (see elements of actuating signals vector $\boldsymbol{\Psi}$).

The following difference equations can be easily derived from the expression (4.2)

$$x_i(n + 1) = x_i(n) + c_{ii}\psi_i(n) \tag{4.3}$$

and utilized for numerical calculation of signals in MS.

When graphically investigating the control process in MS, then according to [8] we must grafically analyze the control process in l fictitious, synchronously operating one-parameter non-linear digital control systems which influence one another. Non-linear transfer characteristic of the i-th fictitious one-parameter system is $N_{ii}(x_i, t)$ and a fictitious reference input is

$$b_{if}(x_j, t) = b_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^{l} N_{ij}(x_j, t)\,. \tag{4.4}$$

Graphical analysis of the control process in a t-invariant two-parameter MS, with functions $N_{ii}(x_i, t)$ and $N_{ij}(x_j, t)$ defined graphically with certain values of gain factor c_{ii}, is in Fig. 2. Changes of fictitious reference inputs $b_{1f}(n) = b_1 - N_{12}[x_2(n)]$ and $b_{2f}(n) = b_2 - N_{21}[x_1(n)]$ are caused only by changes in variables x_j in this case. The values of actuating signals $\psi_i(n)$ are defined by lengths of abscisae $\overline{A_{in}B_{in}}$ ($i = 1, 2;\ n = 0, 1, 2, \ldots$). The control process of t-variant MS with any initial conditions $x_i(0)$ can be analysed in a similar way, i.e. graphically too.

The methods of analysing control process described until now can also be utilized for investigating some problems of stability and quality of MS.

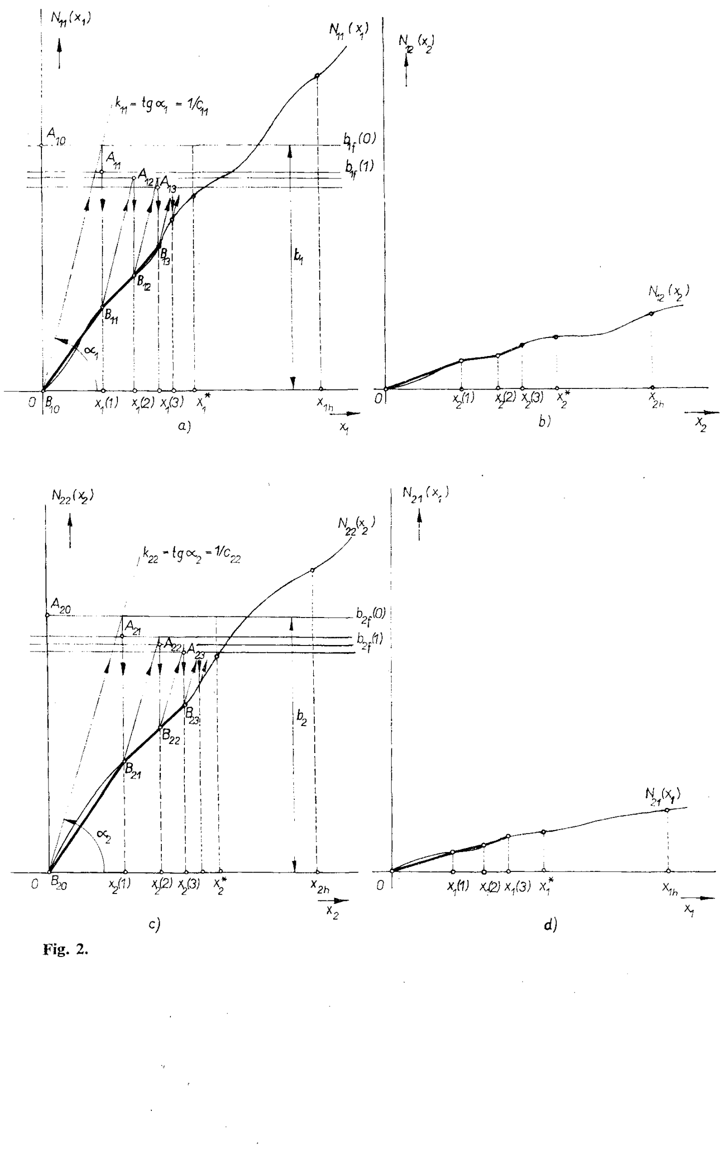

Fig. 2.

Let us first consider a special t-invariant MS with a linear matrix of operators $\mathscr{F}^* = (a_{ij})$, where $a_{ii} > 0$ and where the vector of reference inputs is $\boldsymbol{b} = (b_i)$.

All methods known from the theory of linear discrete control systems (see for example [5]) can be used in this system. In the present paper a simple, clear and sufficient stability criterion will be derived for the above mentioned special case. Taking this criterion as a starting point we shall derive a sufficient stability criterion for the non linear multi-parameter digital control system.

The i-th component of the actuating signals vector can be expressed in this case as follows

$$\psi_i(n+1) = -[a_{ii}x_i(n+1) + \sum_{\substack{j=1 \\ j \neq i}}^{l} a_{ij}x_j(n+1)] + b_i \,. \tag{5.1}$$

If we take the expressions for $x_i(n+1)$ and $x_j(n+1)$ from (4.3) and substitute them into (5.1) then after a small modification we shall obtain the following expression

$$\psi_i(n+1) = (1 - a_{ii}c_{ii})\,\psi_i(n) - \sum_{\substack{j=1 \\ j \neq i}}^{l} a_{ij}c_{jj}\,\psi_j(n) \tag{5.2}$$

and we can write

$$|\psi_i(n+1)| \leqq |1 - a_{ii}c_{ii}|\,|\psi_i(n)| + \sum_{\substack{j=1 \\ j \neq i}}^{l} |a_{ij}|\,|c_{jj}|\,|\psi_j(n)| \,. \tag{5.3}$$

When summing up inequalities (5.3) for $i = 1, 2, \ldots, l$, we shall obtain the following inequality

$$\sum_{i=1}^{l} |\psi_i(n+1)| \leqq \sum_{i=1}^{l} h_i\,|\psi_i(n)| \,, \tag{5.4}$$

where

$$h_i = |1 - a_{ii}c_{ii}| + \sum_{\substack{j=1 \\ j \neq i}}^{l} |a_{ji}|\,|c_{ii}| > 0 \,. \tag{5.5}$$

When designating

$$h = \max h_i \,, \tag{5.6}$$

then $0 < h_i \leqq h$ and the expression (5.4) can be modified to

$$\sum_{i=1}^{l} |\psi_i(n+1)| \leqq h \sum_{i=1}^{l} |\psi_i(n)| \,, \tag{5.7}$$

and finally to

$$\sum_{i=1}^{l} |\psi_i(n)| \leqq h^n \sum_{i=1}^{l} |\psi_i(0)| \,. \tag{5.8}$$

It follows from the expression (5.8) that the control process will be stable under any initial conditions, provided the values of gain factor of partial controllers fulfill the inequality

$$0 < h < 1 . \tag{5.9}$$

Then

$$\sum_{i=1}^{l} |\psi_i(n + 1)| < \sum_{i=1}^{l} |\psi_i(n)| , \tag{5.10}$$

$$\lim_{n\to\infty} \sum_{i=1}^{l} |\psi_i(n)| = 0 , \tag{5.11}$$

$$\lim_{n\to\infty} |\psi_i(n)| = 0 . \tag{5.12}$$

The condition (5.9) will be fulfilled when h_i defined by the expression (5.5) satisfies the inequality

$$h_i < 1 . \tag{5.13}$$

When taking expression (5.5) for h_i and substituing it in (5.13), then we can easily deduce that condition (5.13) will be satisfied for $c_{ii} > 0$ and $a_{ii}c_{ii} \leqq 1$ if

$$0 < c_{ii} \leqq 1/a_{ii} \tag{5.14}$$

and

$$a_{ii} > \sum_{\substack{j=1 \\ j \neq i}}^{l} |a_{ji}| . \tag{5.15}$$

In a similar way we can deduce that for $c_{ii} > 0$ and $1 < a_{ii}c_{ii}$ the condition (5.13) will hold if

$$1/a_{ii} < c_{ii} < 2/(a_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{l} |a_{ji}|) . \tag{5.16}$$

It follows from inequality (5.16) that this inequality holds if inequality (5.15) is fulfilled.

We may say that a sufficient condition of stability for the given system, and for any initial condition $x_i(0)$, is defined by the inequality (5.15) and by the inequality

$$0 < c_{ii} < 2/(a_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{l} |a_{ji}|) . \tag{5.17}$$

When defining the Jacobian matrix of absolute values

$$J = (|a_{ij}|) , \tag{5.18}$$

 where

$$a_{ij} = \partial\psi_i/\partial x_j\,, \quad j = 1, 2, \ldots, l\,, \tag{5.19}$$

then the condition (5.15) means that values of diagonal elements of matrix (5.18) must in each column be greater than the sum of values non-diagonal elements of the respective column.

Sufficient conditions for MS stability can be derived by similar reasoning as in the case of one-parameter non-linear digital control system described in paper [8].

Suppose we have a multi-parameter non-linear digital t-invariant control system with non-linearities $N_{ii}(x_i)$, $N_{ij}(x_j)$ and with reference inputs b_i.

It is obvious from the graphical analysis of the control process that under any starting conditions $x_i(0)$ we can consider only corresponding abscisae in each step instead of original non-linearities. These abscisae are marked by thick lines in Fig. 2 and their tangents $a_{ii}(n)$ and $a_{ji}(n)$ vary in dependence on n.

It is clear directly from the graphical analysis of control process of the non-linear multi-parameter system being considered that for any $x_i(n)$ and for corresponding values of $\psi_i(n)$, the values of actuating signals $\psi_i(n + 1)$ in the next step may in each case be determined as values $\psi_i(1)$ of the linear multi-parameter system in which $\psi_i(0) = \psi_i(n)$. Supposing that sufficient conditions (5.15) and (5.17) have been satisfied in each step and corresponding values of $a_{ii}(n)$ and $a_{ji}(n)$ have been substituted for a_{ii} and a_{ji}, and taking into consideration the expression (5.10) we can easily find out that the following expression holds good also for the non-linear multiparameter system under consideration

$$\sum_{i=1}^{l} |\psi_i(n + 1)| < \sum_{i=1}^{l} |\psi_i(n)|\,,$$

i.e. the control process is stable.

If non-linearities $N_{ii}(x_i)$ and $N_{ij}(x_j)$ changed into other non-linearities $N_{ii}(x_i, t_1)$ and $N_{ij}(x_j, t_1)$ over a certain time t_1, then it would be clear that according to previous reasoning the control process must be stable, provided conditions (5.15) and (5.17) are fulfilled in each step. Again the values of a_{ii} and a_{ji} are replaced in each step by corresponding values of $a_{ii}(n)$ and $a_{ji}(n)$ in these expressions. Because no reference input b_i occur in expressions (5.15) and (5.17), the system will be stable also for reference inputs $b_i(t)$, these being functions of time.

It follows from what has been stated, that the general sufficient stability criterion can be expressed as fulfillment of conditions (5.15) and (5.17) in each step, where the values of a_{ii} and a_{ij} are replaced by corresponding values of $a_{ii}(n)$ and $a_{ji}(n)$ in each step. Various practical stability criteria for MS can be derived from this general sufficient stability criterion. One of them is described in the following text.

If we use denotation $\langle x_{id}, x_{ih}\rangle$ for intervals comprising steady-state values of output signals x_i^* of partial controllers $R_{ii}(z)$, then MS will be stable for any $x_i(0) \in$

$\in \langle x_{id}, x_{ih} \rangle$ (i.e. conditions (5.15) and (5.17) will be satisfied for corresponding values $a_{ii}(n)$ and $a_{ji}(n)$ in each step), when:

I. Condition (5.15) is satisfied if we substitute

$$a_{ii} = \min |\partial\psi_i(\mathbf{x}, t)/\partial x_i| , \tag{5.20}$$

$$|a_{ji}| = \max |\partial\psi_j(\mathbf{x}, t)/\partial x_i| , \tag{5.21}$$

where values of a_{ii} and $|a_{ji}|$ are also specified for $x_i \in \langle x_{id}, x_{ih} \rangle$ and $t \in \langle 0, \infty \rangle$.

II. Condition (5.17) is satisfied when substituting

$$a_{ii} = \max |\partial\psi_i(\mathbf{x}, t)/\partial x_i| , \tag{5.22}$$

$$|a_{ji}| = \max |\partial\psi_j(\mathbf{x}, t)/\partial x_i| , \tag{5.23}$$

where values of a_{ii} and $|a_{ji}|$ are also specified for $x_i \in \langle x_{id}, x_{ih} \rangle$ and $t \in \langle 0, \infty \rangle$.

III. The following relationship holds good in each step for values of output signals from partial controllers

$$x_i(n) \in \langle x_{id}, x_{ih} \rangle .$$

Comments

1. There is no need to consider the sufficient stability condition III for a linear multi-parameter digital control system which is a special variant of MS.

2. The sharper the inequality (5.15) which is a part of condition I, i.e. the larger the value of the left hand side of inequality (5.15) than its right-hand side for each $i = 1, 2, \ldots, l$, the larger the probability that condition III is also fulfilled when sufficient stability conditions I and II are satisfied.

3. It follows from expression (5.17), the elements of which are specified by expressions (5.22) and (5.23) that the sharper the inequality (5.15) containing elements specified by expressions (5.22) and (5.23) the nearer the values of gain factor c_{ii} to boundary values $2/a_{ii}$, where values of a_{ii} are determined by expression (5.22) (compare with boundary gain factor of one-parameter non-linear digital control system described in the paper [8]).

4. Upper bounds of intervals $\langle x_{id}, x_{ih} \rangle$ can be estimated for example by systematic trials in such a way that it may hold for all values of x_{ih} simultaneously $\psi_i < 0$.

Lower bounds can be estimated in a similar way with the result that it holds good for all x_{id} simultaneously $\psi_i > 0$.

The smaller the difference between values of output signals from partial controllers and the sharper the inequalities (5.15) which are part of the sufficient stability condition I, the simpler the described procedure.

Some methods of random searching with and without learning ability [4], or possibly other methods can also be utilized for the determination of the intervals $\langle x_{id}, x_{ih} \rangle$.

5. When there is a need to satisfy the sufficient stability condition I, can of methods from two groups be employed.

Group 1. The original MS can be easily modified in such a way that an appropriate aditional multi-parameter system characterized by a matrix of operators $\mathscr{F}_p^*$ (a diagonal matrix with linear partial operators which are time independent) is connected in paralel to the multi-parameter controlled system being described by a matrix of operators $\mathscr{F}^*$. However this can be done only when we do not insist on steady-state values of output signals from partial controllers in the modified MS being exactly the same as in the original MS (for example when specifying approximate values of x_i^*, further when specifying appropriate initial conditions $x_i(0)$, etc.).

Group 2. The original MS described by the matrix equation $-\boldsymbol{N} + \boldsymbol{b} = \boldsymbol{0}$ can in each case be transformed by various methods into a system which is described by an equivalent matrix equation $-\boldsymbol{N}_N + \boldsymbol{b}_N = \boldsymbol{0}$ and a corresponding required matrix (5.18), if it is required that steady-state values of output signals from partial controllers in the modified MS be the same as in the original MS. Here are some examples:

a) The elements on the left-hand side of matrix equation (1.7) are arranged properly.

b) These elements are multiplied by proper constants, or appropriate substitutions are introduced.

c) We can prove easily that the equivalent matrix equation

$$\boldsymbol{M}\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}) = \boldsymbol{0} \tag{5.26}$$

contains the matrix (5.18) in the form

$$\boldsymbol{J}_{\mathrm{N}} = |D_{\boldsymbol{A}}| \, (|{}^{\mathrm{N}}a_{ij}|) \,, \tag{5.27}$$

where $\boldsymbol{M} = ({}^{\mathrm{N}}a_{ij})$ is a square matrix of properly selected constants, $\boldsymbol{A}^{\mathrm{T}}$ is a transponed matrix of subdeterminants of matrix $\boldsymbol{A}$, and $D_{\boldsymbol{A}}$ is a determinant of matrix $\boldsymbol{A}$, if this is a special case of MS with a linear t-invariant matrix of operators $\mathscr{F}^* = (a_{ij})$, where $a_{ij} > 0$ and with a corresponding matrix equation (1.7) in the form

$$\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b} = \boldsymbol{0} \,, \tag{5.25}$$

where $\boldsymbol{A} = (a_{ij})$.

It follows from the described situation that a equivalent matrix equation (5.26) with a prescribed matrix $\boldsymbol{J}_{\mathrm{N}}$, i.e. with the matrix (5.18), can be constructed in each case. It is recomended to choose the elements of matrix $\boldsymbol{J}_{\mathrm{N}}$ in such a way that all elements a_{ii} are equal and ratio

(5.28) $$p = \sum_{\substack{j=1 \\ j \neq i}}^{l} |a_{ji}|/a_{ii} \ll 1$$

is also equal (for example $p = 0{,}1$).

Inequalities (5.15) are then sufficiently sharp.

When elements of matrix (1.7) are non-linear functions, then elements a_{ij} which are needed for the construction of matrix $\mathbf{A}^{\mathrm{T}}$ can be determined for certain values of $x_i \in \langle x_{id}, x_{ih} \rangle$ by a proper linearization.

d) If – instead of values of a_{ii} – we consider such positive numbers v_{ii} in the sufficient stability condition I that the condition is satisfied, then the values $c_{ii} = 1/v_{ii}$ can be utilized as approximate stable values of gain factors.

6. The sufficient stability condition III can be satisfied, if need be, by selecting different initial conditions, by decreasing gain factor c_{ii}, by expanding intervals $\langle x_{id}, x_{ih} \rangle$ and by calculating new values of gain factors c_{ii}, by properly defining new non-linearities for $x_{ih} < x_i < x_{id}$, by utilizing some of the methods mentioned in comment 5, or by combinations of the above mentioned possibilities.

7. If nonlinearities $N_{ii}(x_i, t)$ turn into negative values and then have an ascending character, or if they have descending character, or if they alternately descend and ascend, then we can use a similar procedure as in the case of one-parameter non-linear digital control systems [7; 8].

8. When some or all nonlinearities $N_{ii}(x_i, t)$ are descending, then these systems may be solved as the MS which was defined in the introductory part of this paper by introducing proper substitutions.

6. QUALITY OF THE CONTROL PROCESS

The quality of the control process can be judged according to the size of the absolute control area. The latter will be defined by the following expression

(6.1) $$P_{MA} = T \sum_{n=0}^{\infty} \sum_{i=1}^{l} |\psi_i(n)| .$$

Optimum gain factors of the transfer matrix of controllers (3.7) will be such gain factor c_{ii}, for which the absolute control area P_{MA} is minimal, or at least nearly minimal.

The values for selection of optimum gain factor c_{ii} in MS will be derived next.

Suppose we have a multi-parameter non-linear digital control system with non-linearities $N_{ii}(x_i)$, $N_{ij}(x_j)$ and with reference inputs b_i. Let us further suppose that sufficient conditions of stability defined in chapter 5 are satisfied for this system.

Expression (5.10) holds good for such a system. The expression (6.1) can be written in the form

$$P_{MA} = T\left[\sum_{i=1}^{l} |\psi_i(0)| + \sum_{i=1}^{l} |\psi_i(1)| + \sum_{n=2}^{l} \sum_{i=1}^{l} |\psi_i(n)|\right]. \tag{6.2}$$

As expression (5.10) holds good, it follows from expression (6.2) that:

a) The absolute control area P_{MA} will depend on the selection of initial conditions $x_i(0)$ and it will be the smaller, the smaller the sum $\sum_{i=1}^{l} |\psi_i(0)|$ for the chosen initial conditions. If we knew the steady-state values of partial controllers output signals x_i^* and if we chose $x_i(0) = x_i^*$, than it would hold good that $\psi_i(0) = 0$, $\sum_{i=1}^{l} |\psi_i(0)| = 0$ and $P_{MA} = 0$. Actuating signals would equal zero already at the beginning of the first sampling period. If there are some non-zero actuating signals resulting from changes of non-linearities or control variables, these can be eliminated for any stable values of gain factors c_{ii}. This can be proved by performing the calculation according to expression (4.3).

b) If we select $x_i(0) \neq x_i^*$, then the smaller the sum $\sum_{i=1}^{l} |\psi_i(1)|$ for the same values of $x_i(0)$, the smaller the absolute control area P_{MA}. It is obvious that the sum $\sum_{i=1}^{l} |\psi_i(1)|$ may be influenced by the selection of gain factor c_{ii}.The smaller each value of $|\psi_i(1)|$, or possibly the smaller the differences $|x_i(1) - x_i^*|$ (see graphical analysis of the control process in Fig. 2), the smaller the sum $\sum_{i=1}^{l} |\psi_i(1)|$.

It goes from expression (4.3), that in this case $|\psi_i(1)|$ can be equal to zero after one step, if we choose

$$c_{ii} = (x_i^* - x_i(0))/\psi_i(0)\,, \tag{6.3}$$

i.e. if the steady-state values of partial controllers output signals x_i^* are known. However the gain factors c_{ii} calculated by means of expression (6.3) must be situated in regions of stability, because in the reverse case a non-stable control process could occur even for small changes of non-linearities or reference inputs.

When steady-state values of output signals from partial controllers are not known (this is the case when calculating the extremum of the multi-variable object function, when designing an adaptive system for automatic seeking of extremum, etc.), then the optimum gain factor c_{ii} can be determined only approximately. For example, the approximate value of $\tilde{x}_i^*$ can be substituted for x_i^* in expression (6.3) and this approximate value can be calculated numerically as follows: Values of gain factors c_{ii} are selected in such a way that an oscillating control process occurs, i.e. for example the following inequality holds good

$$x_i(n) < x_i(n+2) < x_i(n+1)\,, \tag{6.4}$$

where n is a serial number of a certain step, then the value of x_i^* can be taken as the first estimate

$$\tilde{x}_i^* = 1/2\,.\,[x_i(n) + x_i(n+2)/2 + x_i(n+1)]\,. \tag{6.5}$$

c) A simple criterion for the selection of optimum gain factors c_{ii} can be derived by the following reasoning:

Let us first suppose that we have a linear t-invariant multi-parameter digital control system. The sharper the inequalities (5.15) the more justified the replacement of this system by l mutually independent one-parameter linear digital control system (see paper [8]), as far as the control process is concerned. The optimum gain factors of these systems can be specified by using expression

$$c_{ii} = 1/a_{ii}\,. \tag{6.6}$$

When we have a t-invariant non-linear multi-parameter digital control system or possibly also a t-variant non-linear multi-parameter system with small changes of non-linearities and reference inputs, and when inequalities (5.15) with elements specified by the expressions (5.20) and (5.21) are sufficiently sharp and intervals $\langle x_{id}, x_{ih} \rangle$ are not very large, then parts of non-linearities can be substituted by lines and, taking into consideration the previous reasoning, the recommended optimum values of gain factors c_{ii} can be calculated in this case according to the expression (6.6), in which the values specified by the expression (5.22) are utilized.

The sharper the inequalities (5.15) with elements specified by expressions (5.20) and (5.21), the greater the changes of non-linearities and reference inputs for which the described recommended optimum values of gain factors c_{ii} are valid.

Comments

1. If we have a t-variant non-linear digital control system, we can define a certain state of non linearities and reference inputs, for which the optimum values of c_{ii} can be derived by some of the described procedures.

2. If inequalities (5.15) with elements specified by expressions (5.20) and (5.21) are not sharp enough, large oscillations can occur in the given system for values of c_{ii} specified by expression (6.6) and the quality of the control process is low. In this case we can use some of the methods for securing the sufficient condition I of MS to increase the sharpness of inequalities (5.15).

3. It may happen in some cases during the control process calculation, that because of the selected calculation precision the solution "will come to a stop" even if elements of the actuating signals vector are not yet equal to zero. In this case we can continue in such a way that we increase the gain factors c_{ii} in a stability region, select other initial conditions $x_i(0)$ or increase the precision of calculation.

4. The expression for the estimation of absolute control area P_{MA} can be derived by similar reasoning as in paper [8], but the resulting expression is more complex.

5. From what has been stated in this paper we can formulate the principles for selecting proper gain factors, which vary in each step or after a certain number of steps.

Note that the approach used in this paper can be utilized for other, more complex types of nonlinearities too.

Let us solve one simple example to illustrate at least one of many possible applications of methods described in the paper.

EXAMPLE OF APPLICATION

Find the extremum of the object function

$$E(x_1, x_2) = 4e^{x_1} + 5e^{x_2} + x_1x_2 + x_1^2 + x_2^2 - 18{,}676x_1 - 24{,}375x_2 ,$$

by selecting proper constants c_{ii} and by transforming this problem into the MS solution.

Solution. Firts the components of the object function gradient will be specified and modified into forms

$$[\psi_1 = -(4e^{x_1} + 2x_1 + x_2) + 18{,}676 , \quad \psi_2 = -(5e^{x_2} + 2x_2 + x_1) + 24{,}375] .$$

In order to solve the given problem we have to find the needed gain factor in a certain fictitious MS, which is specified by a transfer matrix of controllers (3·7) with non-linearities and reference inputs not varying in time. Steady-state values of output signals from partial controllers are coordinates of the extremum being sought.

Estimate the intervals $\langle x_{id}, x_{ih} \rangle$. For example we can select $x_{1d} = x_{2h} = 0{\cdot}00$ in the given case because $\psi_1(x_{1d}, x_{2d}) > 0$, $\psi_2(x_{1d}, x_{2d}) > 0$, and $x_{1h} = x_{2h} = 1{\cdot}60$, because $\psi_1(x_{1h}, x_{2h}) < 0$, $\psi_2(x_{1h}, x_{2h}) < 0$.

Construct the matrix (5.18) with elements specified by expressions (5.20 and (5.21):

$$J = \begin{pmatrix} 4e^{0,00} + 2 & 1 \\ 1 & 5e^{0,00} + 2 \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ 1 & 7 \end{pmatrix},$$

where it holds for columns $6 > 1$ and $1 < 7$.

Table 1.

n	$x_1(n)$	$\psi_1(n)$	$c_{11}\psi_1(n)$	$x_2(n)$	$\psi_2(n)$	$c_{22}\psi_2(n)$
0	1·600	−5·936	−0·273	1·600	−5·190	−0·192
1	1·327	−0·466	−0·021	1·408	−0·208	−0·008
2	1·306	−0·100	−0·005	1·400	−0·006	−0·000
3	1·301	−0·018	−0·001	1·400	−0·001	−0·000
4	**1·300**	**0·000**	0·000	**1·400**	**0·000**	0·000

The elements of matrix J satisfy inequalities (5.15). Because these inequalities are sharp enough, we can calculate approximate optimum values of gain factors simply by using expression (6.6), in which the values of a_{ii} are calculated by means of expression (5.22): $c_{11} = 1/(4e^{1,6} + 2) \doteq \doteq 0.046$, $c_{22} = 1/(5e^{1,6} + 2) \doteq 0.037$.

The steady-state values of output signals from partial controllers can be calculated one after the other by means of expression (4.3). From Table 1 we can see the course of the control process for the calculated values of c_{ii} and for the initial conditions $x_1(0) = x_2(0) = 1.600$. We can see that if three decimal places are valid, then the control process will be finished after the fourth step.

The following coordinates of the object function extremum can be read from the last row of the Tab. 1: $x_1^* = 1.300$, $x_2^* = 1.400$. Min $E(x_1, x_2) = E(x_1^*, x_2^*) = -17.983$.

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