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ČÁST FYSIKÁLNÍ.

An application of the method wherein the φ - and κ - methods are combined for the determination of the grating constant. Part I.

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The method, wherein the φ - and the κ -methods are combined for the precise measurement of the grating constants of crystals, is applied. Here, in this work, only the glancing angle have been directly measured and the differential glancing angle κ has been derived from the measured glancing angles. In this case the errors due to vertical displacement are neither diminished nor eliminated.

The individual fictive grating constants of zinc sulphide (ZnS) crystals, whose reflecting Bragg's planes do not agree with the cut surface, as found by Ad. Němejcová, are given. The mentioned method is used also for the measurements of the grating constants from two different orders of the same wave-length (Kunzl-Köppel method) and it has been found that the said application is very important if short wave-lengths, such as Mo K-series, are used.

The influence of the impurities on the values of the grating constant of the two used zinc sulphide crystals is found to be in the limits of 0,2 X. U.

In this article, I wish to give the results obtained by using different kinds of zinc sulphide (ZnS) crystals measuring with the method described in the preceding article.¹⁾ This method has enabled us to derive precisely the grating constants of the crystals whose surfaces are not perfect. The other known precise methods require perfect crystals for the measurement of the grating constant. Many crystals, which have a good reflecting power and a large dispersion cannot be split perfectly. As for example zinc sulphide crystal, as found by Ad. Němejcová²⁾ from her X-ray investigations and established by prof. F. Ulrich with his crystallographical measurements, is one of such a type of crystals.

As an application of the valuable advantages of the mentioned method I give here my measurements of the individual fictive grating constants of zinc sulphide crystals to prove the ex-

¹⁾ V. Dolejšek-Swami Jnanananda, Časopis 65 (1936), 97.

²⁾ Ad. Němejcová: The work in the press.

istence of the changes of the grating constant of the zinc sulphide crystals caused by the presence of impurities. The determination of the individual fictive grating constants and the said study of the changes in the grating constant of crystals naturally demand the elimination of all other possible errors due to the mentioned defects of the crystals which can cover up or falsify the results.

If we compare the results of the measurements of the grating constant of zinc sulphide crystal given with different methods by the former authors, we find that there are great divergences in the values. On looking at the results given in the Table 1, we note that this divergence varying from $d = 1907$ X. U. to 1928, 5 X. U., can be explained as a result of the difference between the reflecting plane and the cut surface of the crystal. The differences between the values of the grating constant have been attributed to the presence of different quantities of impurities in different crystals. The said presence of the different quantities of impurities in different crystals has been judged after the colours. There are, as shown in the Table 1, crystals either with

Table 1.

Grating constant a_1 XU	Author	Quality and kind of crystal
1907	Gerlach (1922)	Very transparent powdered crystal
1915,5	Goldschmidt (1927)	Chemically pure
1929,5 1928 1928 1927	Rinne (1924) Rotatory crystal method Mo $K\alpha_1, \alpha_2, \beta, \gamma$	Black Dark brown Light brown Light yellow
1922, $5 \pm 1,5$	Lehmann (1924)	
1909, 5 ± 3	Hartwig (1926)	For controlling a certain method

black, or brown, or yellow colour. In the case of the measurements of Rinne who has measured four different kinds of zinc sulphide crystals, it can be seen that there is only a difference of nearly 2 X. U. between the grating constants of the light yellow and the black zinc sulphide crystals when measured with Mo K-series with rotatory powdered crystal method. These results however differ very much from those of the other authors given in the

said Table 1 and also from the value which I obtained with Prof. V. Dolejšek for zinc sulphide crystal. Our value of the grating constant of zinc sulphide crystal is

$$d_1 = 1908,96 \text{ X. U.}$$

The crystal which we used was slightly dark brown and this value of the grating constant is obtained as the corrected one after the elimination of the possible errors due to all other defects of the crystals. I will show in this work that this value is correct. In this connection, from the former work, I wish to mention that with the said crystal we have obtained in one case in which the crystal had a great displacement ($\Delta = 0,401 \text{ mm}$) 1914,8 X. U. when calculated with the glancing angle φ_1 of Cu $K\alpha_1$ and 1923, 5 X. U. when calculated with the glancing angle φ_1 of Mo $K\alpha_1$ as the grating constant of the crystal. In the other case when the displacement is reduced, both these values have approached the value of the grating constant 1908,96 X. U. which has been derived from the corrected differential glancing angle κ and which has practically remained the same in either case.

I have measured once more the grating constant of the said crystal which was used in the measurements mentioned in the former article, only using another position of the same surface. The crystal is not uniformly coloured and hence I wanted to find whether the value of the grating constant will be the same at the two different mentioned places with two different hues of colour. The measurements and their results are given in the Tables 2 (a, b and c) and 3. The displacement of the crystal in this case has been 0,028 mm. The corrected value of the grating constant $d_1 = 1908,87 \text{ X. U.}$, which can be noted from the last mentioned Table 3, differs from the former values obtained in the preceding work by 0,09 X. U. From this slight difference only in the second decimal place of a X. U., we cannot however draw any conclusions regarding the influence of the impurities in the same crystal at different positions.

For further investigations into the said influence of the impurities on the value of the grating constant of the crystals, I have taken another crystal and measured its grating constant. In this connection, I wish to mention that I have adjusted this crystal to the face of the stand of the crystal table as it can be seen from the sketch (Fig. 1). The fore front plane surface of the metallic crystal-stand has been adjusted to the axis of the spectrograph. The crystal has been slightly pressed and fixed to the surface of the crystal-stand. This way of adjustment is found to be more advantageous than the direct adjustment of the crystal the surface of which is not perfect as it is usual in the case of zinc

**First series with a displacement of 0,028 mm.
Crystal No. I.**

Table 2a. Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ_1 18° C
319	0,139	1' 54,1"	95° 2' 13"	23° 45' 4,73"	2,85"	23° 45' 7,58"	} 23° 45' 8,3"
320	0,147	2' 0,7"	95° 2' 11,5"	23° 45' 2,70"	3,03"	23° 45' 5,73"	
321	0,296	4' 3,0"	94° 56' 28,5"	23° 45' 7,88"	1,60"	23° 45' 9,48"	
322	0,307	4' 12,0"	94° 56' 23,5"	23° 45' 8,87"	1,64"	23° 45' 10,56"	
334	0,176	2' 24,5"	95° 2' 49,5"	23° 45' 7,20"	3,03"	23° 45' 9,28"	
335	0,188	2' 34,3"	95° 2' 52"	23° 45' 4,43"	2,65"	23° 45' 7,07"	
336	0,177	2' 25,3"	95° 2' 49,5"	23° 45' 6,05"	2,39"	23° 45' 8,44"	

Table 2b. Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ_1 18° C
330	0,243	3' 19,5"	42° 49' 15"	10° 41' 28,88"	0,96"	10° 41' 29,84"	} 10° 41' 26,1"
331	0,252	3' 26,8"	42° 49' 13"	10° 41' 26,55"	1,46"	10° 41' 28,01"	
332	0,279	3' 49,0"	42° 49' 23"	10° 41' 23,55"	0,75"	10° 41' 24,30"	
333	0,265	3' 37,5"	42° 49' 11,5"	10° 41' 21,25"	1,18"	10° 41' 22,43"	

Table 2c.

Displacement of the crystal Δ = 0,028 mm
 φ_1 Cu $K\alpha_1$ = 23° 45' 8,3"
 φ_1 Mo $K\alpha_1$ = 10° 41' 26,1"
 φ_1 Cu — φ_1 Mo = κ_1 = 13° 3' 42,2"

Table 3.

**Representation of the first series of measurements and their results
($\Delta \sim 0,028$ mm).**

$\frac{1}{2}\delta_1 = 22,0''$																
φ_1 Cu meas. 23° 45' 8,3"	$d_1 =$ 1908,62 XU	}	φ_1 Cu ~ to 1908,93 XU 23° 44' 46,3"													
mean κ_1 13° 3' 42,2"	$d_1 =$ 1908,93 XU		corr. κ_1 13° 3' 43,8"	corr. $d_1 =$ 1908,87 XU												
φ_1 Mo meas. 10° 41' 26,1"	$d_1 =$ 1907,85 XU		φ_1 Mo ~ to 1908,93 XU 10° 41' 4,1"													
$\frac{1}{2}\delta_1 = 22,0''$																
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">Δ mm</td> <td style="text-align: center;">mean Δ mm</td> <td style="text-align: center;">$\frac{1}{2}\delta$</td> <td style="text-align: center;">$\varepsilon\delta\kappa =$ $(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$ + 1,6"</td> </tr> <tr> <td style="text-align: center;">Cu 0,029</td> <td style="text-align: center;">0,028</td> <td style="text-align: center;">{21,3"</td> <td></td> </tr> <tr> <td style="text-align: center;">Mo 0,027</td> <td></td> <td style="text-align: center;">{22,8"</td> <td></td> </tr> </table>		Δ mm	mean Δ mm	$\frac{1}{2}\delta$	$\varepsilon\delta\kappa =$ $(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$ + 1,6"	Cu 0,029	0,028	{21,3"		Mo 0,027		{22,8"				
Δ mm	mean Δ mm	$\frac{1}{2}\delta$	$\varepsilon\delta\kappa =$ $(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$ + 1,6"													
Cu 0,029	0,028	{21,3"														
Mo 0,027		{22,8"														

sulphide crystals whose natural cut surfaces are not quite regular. In such cases, with the kind of adjustment mentioned above, the displacement of the crystal amounts to only some hundredths of a mm. The error due to such a displacement can be easily eliminated with our method mentioned in the former article.¹⁾ The measurements obtained with these crystals are given in the Tab-

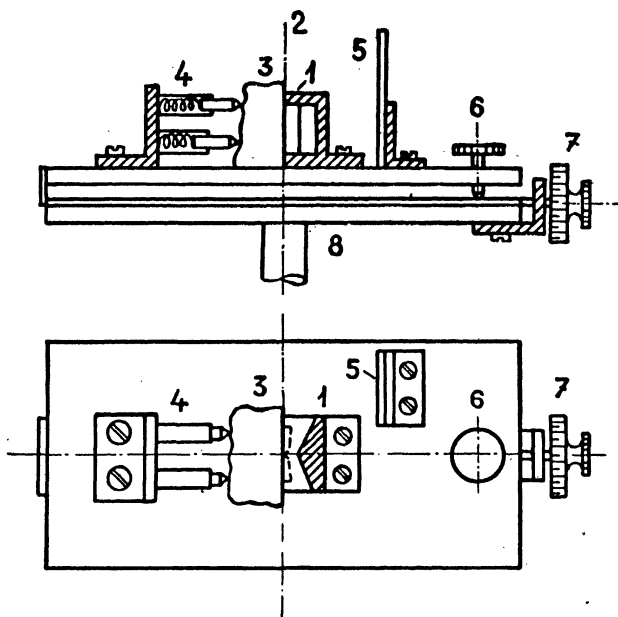


Fig. 1.

les 4 (a, b and c) and 6 (a, b and c). The measurements in the Tables 6 (a, b and c) are made with a smaller displacement than in the case of the measurements given in the Tables 4 (a, b and c). From the measurements in the Tables 4 (a, b and c). I have calculated the displacement which is $-0,047$ mm, and I have readjusted the crystal to remove the displacement. With this new adjustment I have made the measurements mentioned in the Tables 6 (a, b and c). In the Table 7 we can see that the displacement has diminished to an amount of $0,0145$ mm. As it can be seen from the results mentioned in the Tables 5 and 7 the value of the grating constant, after the displacement errors are eliminated, are $1908,72$ and $1908,66$ X. U. respectively. In the second series of measurements given in the Tables 6 (a, b and c) there is only a very minute amount of displacement.

**Second series with a displacement of 0,047 mm.
Crystal No. II.**

Table 4a.

Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for $\Delta t^\circ \text{C}$	φ 18° C	mean value of φ 18° C
303	0,235	3' 12,9"	94° 58' 51,75"	23° 45' 31,15"	1,12"	23° 45' 32,27"	} 23° 45' 29,2"
304	0,153	2' 5,6"	95° 3' 52"	23° 45' 26,60"	1,33"	23° 45' 27,93"	
305	0,156	2' 8,0"	95° 3' 51"	23° 45' 25,75"	1,05"	23° 45' 26,80"	
306	0,172	2' 21,2"	94° 59' 40,5"	23° 45' 30,43"	1,89"	23° 45' 32,32"	
312	0,158	2' 9,7"	95° 3' 30,75"	23° 45' 20,26"	3,29"	23° 45' 23,55"	
313	0,192	2' 37,6"	94° 59' 20,75"	23° 45' 29,58"	2,89"	23° 45' 32,47"	

Table 4b.

Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for $\Delta t^\circ \text{C}$	φ 18° C	mean value of φ 18° C
310	0,215	2' 56,5"	42° 49' 58,5"	10° 41' 45,50"	0,76"	10° 41' 46,26"	} 10° 41' 44,1"
311	0,212	2' 54,0"	42° 49' 56,5"	10° 41' 45,63"	0,82"	10° 41' 46,45"	
314	0,200	2' 44,2"	42° 44' 7,5"	10° 41' 42,93"	1,17"	10° 41' 44,10"	
315	0,177	2' 25,3"	42° 44' 5,5"	10° 41' 37,70"	1,29"	10° 41' 38,99"	
316	0,215	2' 56,5"	42° 49' 40,5"	10° 41' 41,00"	1,19"	10° 41' 42,19"	
317	0,190	2' 36,0"	42° 49' 39,5"	10° 41' 45,88"	0,62"	10° 41' 46,50"	

Table 4c.

Displacement of the crystal Δ = 0,047 mm
 $\varphi_1\text{Cu } K\alpha_1$ = 23° 45' 29,2"
 $\varphi_1\text{Mo } K\alpha_1$ = 10° 41' 44,1"
 $\varphi_1\text{Cu} - \varphi_1\text{Mo} = \kappa_1$ = 13° 3' 45,1"

Table 5.

Representation of the second series of measurements and their results ($\Delta = 0,047$ mm).

$$\begin{array}{l}
 \left. \begin{array}{l} \varphi_1\text{Cu meas.} \\ 23^\circ 45' 29,2'' \end{array} \right\} \begin{array}{l} d_1 = \\ 1908,08 \text{ XU} \end{array} \\
 \left. \begin{array}{l} \text{mean } \kappa_1 \\ 13^\circ 3' 45,1'' \end{array} \right\} \begin{array}{l} d_1 = \\ 1908,82 \text{ XU} \end{array} \\
 \left. \begin{array}{l} \varphi_1\text{Mo meas.} \\ 10^\circ 41' 44,1'' \end{array} \right\} \begin{array}{l} d_1 = \\ 1906,96 \text{ XU} \end{array} \\
 \left. \begin{array}{l} \varphi_1\text{Cu} \sim \text{to } 1908,82 \text{ XU} \\ 23^\circ 44' 51,5'' \end{array} \right\} \begin{array}{l} \text{corr. } \kappa_1 \\ 13^\circ 3' 47,8'' \end{array} \\
 \left. \begin{array}{l} \varphi_1\text{Mo} \sim \text{to } 1908,82 \text{ XU} \\ 10^\circ 41' 6,3'' \end{array} \right\} \begin{array}{l} \text{corr. } d_1 = \\ 1908,72 \text{ XU} \end{array}
 \end{array}$$

$$\left[\begin{array}{l} \Delta \text{ mean } \Delta \\ \text{mm mm} \end{array} \right. \left. \begin{array}{l} \frac{1}{2} \delta \\ \left. \begin{array}{l} \text{Cu } 0,050 \\ \text{Mo } 0,047 \end{array} \right\} \begin{array}{l} \{36,4'' \\ \{39,1'' \end{array} \end{array} \right. \left. \begin{array}{l} \varepsilon \delta \kappa = \\ - (\frac{1}{2} \delta \text{Cu} - \frac{1}{2} \delta \text{Mo}) \\ + 2,7'' \end{array} \right]$$

Such an adjustment with only the said minute displacement is possible to obtain only with perfect crystals adjusted with the usual optical methods. Here it can be noted that there are no considerable differences between the value of the grating constant obtained directly from the glancing angles φ of Cu $K\alpha_1$ and Mo $K\alpha_1$, and the value obtained through our method. If we survey all the values of the grating constants of the zinc sulphide crystals directly derived from the measured glancing angle φ , we will not be in a position to say that there exists any difference between the values of the grating constants of the used first and second crystals. If we however compare our corrected values of d_1 obtained from the measurements of either one of the crystals, we see that all the corrected values obtained with the second crystal which has been lighter in colour than the first crystal, are smaller than those corrected values of the first one. As the mean values of the grating constants of the first and the second crystals we obtain the following ones:

- I. crystal (ZnS) $d_1 = 1908,93$ X. U.
 II. „ „ „ $d_1 = 1908,66$ X. U.

It has been our view only to see whether there exists the said influence of impurities on the grating constants of the crystals which was said to be amounting to the value of some X. U. The results of our investigation with this method testify that the divergences in the values of the grating constants of zinc sulphide crystals given by the former authors (Table 1) are due to the fact that Bragg's reflecting surface does not coincide with the cut surfaces of the crystal which are not regularly plane faced. It is quite evident that even with such imperfect crystals, we can with our method determine the precise value of the constant of crystal grating.

We are however not in a position to say that the difference between the values of the grating constant of the two crystals amounting to 0,2—0,3 X. U. is due to the influence of the impurities. It will be shown in the latter part of this work that the value of the constant obtained for the first crystal, corresponds to the fictive value d_2 of the second crystal, when looked from the view of the index of refraction.

Here I should however remark that the crystals which I have at my disposal and which have given sharp spectral lines are not of uniform colour. The differences in the amounts of impurities at different places of the same crystal might have caused differences in the values of the grating constant of one and the same crystal if the reflection takes place at different places of the reflecting surface. The differences in the results of

**Third series with a displacement of 0,0145 mm.
Crystal No. II.**

Table 6a. Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_i	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
341	0,188	2' 34,3"	95° 2' 53,5"	23° 45' 4,82"	1,28"	23° 45' 6,10"	} 23° 45' 9,9"
342	0,205	2' 48,3"	95° 3' 5"	23° 45' 4,18"	1,69"	23° 45' 5,87"	
343	0,191	2' 36,8"	95° 2' 54"	23° 45' 4,03"	2,50"	23° 45' 6,80"	
345	0,266	3' 38,3"	95° 4' 3"	23° 45' 6,18"	1,43"	23° 45' 7,61"	
346	0,254	3' 28,5"	95° 4' 2,5"	23° 45' 8,48"	0,76"	23° 45' 9,24"	
348	0,230	3' 8,8"	94° 57' 43,5"	23° 45' 13,08"	2,62"	23° 45' 15,70"	
349	0,217	2' 58,1"	94° 57' 46,5"	23° 45' 11,15"	2,71"	23° 45' 13,86"	
350	0,214	2' 55,7"	94° 57' 52,5"	23° 45' 12,05"	2,18"	23° 45' 14,23"	

Table 6b. Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_i	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
352	0,236	3' 13,7"	42° 42' 12,0"	10° 41' 21,42"	0,85"	10° 41' 22,27"	} 10° 41' 20,6"
353	0,204	2' 47,4"	42° 42' 3"	10° 41' 12,60"	1,50"	10° 41' 13,10"	
354	0,231	3' 9,6"	42° 42' 2,5"	10° 41' 18,03"	1,17"	10° 41' 19,20"	
355	0,216	2' 57,3"	42° 42' 9,5"	10° 41' 16,70"	1,37"	10° 41' 18,07"	
356	0,271	3' 42,4"	42° 48' 53"	10° 41' 17,93"	1,20"	10° 41' 19,13"	
357	0,246	3' 21,9"	42° 48' 53"	10° 41' 22,72"	1,42"	10° 41' 24,14"	
358	0,238	3' 15,4"	42° 48' 56"	10° 41' 25,15"	1,11"	10° 41' 26,26"	
359	0,241	3' 17,8"	42° 48' 44"	10° 41' 21,55"	1,56"	10° 41' 23,11"	

Table 6c.

Displacement of the crystal Δ = 0,0145 mm
 φ_1 Cu $K\alpha_1$ = 23° 45' 9,9"
 φ_1 Mo $K\alpha_1$ = 10° 41' 20,6"
 φ_1 Cu — φ_1 Mo = α_1 = 13° 3' 49,2"

Table 7.

**Representation of the third series of measurements and their results
($\Delta \sim 0,0145$ mm).**

	$\frac{1}{2}\delta_1 = 11,3''$		
φ_1 Cu meas.	$d_1 =$	}	φ_1 Cu \sim to 1908,67 XU 23° 44' 58,9" corr. α_1 = 13° 3' 49,7" corr. $d_1 =$ 1908,65 XU φ_1 Mo \sim to 1908,67 XU 10° 41' 11,9"
23° 45' 9,9"	1908,63 XU		
mean α_1	$d_1 =$		
13° 3' 49,2"	1908,67 XU		
φ_1 Mo meas.	$d_1 =$		
10° 41' 20,6"	1908,12 XU		
	$\frac{1}{2}\delta_1 = 11,3''$		
Δ mm	mean Δ mm	$\frac{1}{2}\delta$	$\varepsilon\delta\alpha =$
Cu 0,015	0,0145	{ 10,9"	- ($\frac{1}{2}\delta$ Cu — $\frac{1}{2}\delta$ Mo)
Mo 0,014		{ 11,7"	+ 0,8"

the individual groups of measurements, as it can be seen from their respective tables, are within the said limits of 0,2—0,3 X. U.

The measurements and their results given in the Tables 8 (a, b and c) and 9 and also those given in the Tables 10 (a, b and c) and 11 have been taken with one and the same kind of crystal adjustment. In these cases the vertical adjustment of the crystal is made more precise than those of the preceding series of measurements with the said second crystal. In this investigation a precise vertical crystal adjustment is important. I have measured the glancing angles with the precise method of Siegbahn. From the measured glancing angles φ of Cu $K\alpha_1$ and φ of Mo $K\alpha_1$ I have deduced the differential glancing angles κ . I have shown in the preceding work that there is practically no difference between the directly measured glancing angle φ and the differential glancing angle κ derived from the measured glancing angles. Therefore in this work I have measured only the glancing angles φ from which I have deduced the angle κ . In this case through the combination of the φ - and κ -methods the error due to the displacement in the horizontal position has been eliminated just in the way shown in our preceding work.¹⁾ But the error due to the vertical adjustment in this case is not diminished as in the case of the direct measurement of the differential glancing angle κ . A simple consideration makes it clear, that the errors due to the displacements in the vertical and horizontal positions are diminished only when the angle κ is directly measured. This fact has been shown and experimentally verified in the works of Kunzl and Köppel³⁾ and those of Bouchal and Dolejšek.⁴⁾ It is natural that when the angle κ is not measured but is only derived from the measured glancing angles φ , only the error due to horizontal displacement is diminished, but the error due to vertical displacement that enters into the value of the differential glancing angle κ remains the same as that which enters into the value of the glancing angle. So in this direction, in so far as there is a displacement in the vertical adjustment of the crystal, the directly measured angles κ are not equivalent to the angles κ derived from the measured glancing angles. In other words when we directly measure the angles κ , the errors due to vertical and horizontal displacements are both diminished in the values of the grating constants. If we derive the angle κ from the directly measured glancing angle and calculate the grating constant with the said angle κ , the errors due to horizontal displacement is dimi-

³⁾ V. Kunzl-J. Köppel, C. R. **196** (1933), 787; **196** (1933), 940; Časopis **63** (1934), 109; Journ. de Phys. **5** (1934), 145.

⁴⁾ F. Bouchal-V. Dolejšek, Časopis **65** (1935), 33; C. R. **199** (1934), 1054.

**Fourth series with a displacement of 0,023 mm.
Crystal No. II.**

Table 8a. Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for $\Delta t^\circ \text{C}$	φ 18° C	mean value of φ 18° C
403	0,281	3' 50,6"	95° 2' 40"	23° 44' 42,35"	2,10"	23° 44' 44,45"	} 23° 44' 46,2"
404	0,280	3' 49,8"	95° 2' 38"	23° 44' 42,05"	2,45"	23° 44' 44,50"	
407	0,172	2' 21,2"	94° 56' 41,5"	23° 44' 45,68"	2,97"	23° 44' 48,65"	
408	0,157	2' 8,9"	94° 56' 39"	23° 44' 41,98"	3,20"	23° 44' 45,18"	
409	0,208	2' 50,7"	94° 56' 29"	23° 44' 49,92"	2,27"	23° 44' 52,19"	
410	0,212	2' 54,0"	94° 56' 3"	23° 44' 44,25"	2,45"	23° 44' 46,70"	
411	0,290	3' 58,0"	95° 2' 47"	23° 44' 42,25"	2,56"	23° 44' 44,81"	
412	0,303	4' 8,7"	95° 2' 49"	23° 44' 40,08"	3,15"	23° 44' 43,23"	

Table 8b. Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for $\Delta t^\circ \text{C}$	φ 18° C	mean value of φ 18° C
372	0,181	2' 28,6"	42° 45' 47"	10° 40' 49,60"	1,12"	10° 40' 50,72"	} 10° 40' 53,9"
376	0,175	2' 23,6"	42° 45' 57"	10° 40' 53,35"	1,17"	10° 40' 54,52"	
377	0,175	2' 23,6"	42° 45' 57"	10° 40' 53,35"	1,12"	10° 40' 54,47"	
419	0,164	2' 14,6"	42° 41' 23,5"	10° 40' 54,50"	1,23"	10° 40' 55,73"	

Table 8c.

Displacement of the crystal $\Delta \dots = 0,023$ mm
 $\varphi_1\text{Cu } K\alpha_1 \dots \dots \dots = 23^\circ 44' 46,2''$
 $\varphi_1\text{Mo } K\alpha_1 \dots \dots \dots = 10^\circ 40' 53,9''$
 $\varphi_1\text{Cu} - \varphi_1\text{Mo} = \kappa_1 \dots \dots \dots = 13^\circ 3' 52,3''$

Table 9.

Representation of the fourth series of measurements and their results ($\Delta \sim 0,023$ mm).

	$\frac{1}{2}\delta = 17,9''$			
$\varphi_1\text{Cu meas.}$	$d_1 =$	}	$\varphi_1\text{Cu} \sim \text{to } 1908,55 \text{ XU}$	$23^\circ 45' 4,1''$
23° 44' 46,2"	1908,93 XU			
mean κ_1	$d_1 =$		corr. κ_1	corr. $d_1 =$
13° 3' 52,3"	1908,55 XU		13° 3' 50,7"	1908,61 XU
$\varphi_1\text{Mo meas.}$	$d_1 =$	}	$\varphi_1\text{Mo} \sim \text{to } 1908,55 \text{ XU}$	$10^\circ 41' 11,8''$
10° 40' 53,9"	1909,43 XU			
	$\frac{1}{2}\delta = 17,9''$			

$$\left[\begin{array}{c} \Delta \quad \text{mean } \Delta \\ \text{mm} \quad \text{mm} \end{array} \right. \left. \begin{array}{c} \frac{1}{2}\delta \\ \left\{ \begin{array}{l} 17,7'' \\ 19,3'' \end{array} \right\} \end{array} \right] + \left(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo} \right) = \begin{array}{c} \varepsilon\delta\kappa = \\ - 1,6'' \end{array}$$

nished but the error due to the vertical displacement remains the same. In this work I have directly measured the glancing angles φ and derived from them the angle \varkappa . Therefore it is essential to regard the precision of the vertical adjustment of the crystal. In the mentioned series of measurements given in the Tables 4 and 6, the vertical adjustment has been made and corrected by measuring the distances between the spectral lines at different vertical positions and by calculating from them the amount of the vertical displacement of the crystal. In the two series of measurements given in the Tables 8 (a, b and c) and 10 (a, b and c) a more precise vertical adjustment of the crystal has been obtained. I have been able to get the said precision in the vertical adjustment with a reflecting mirror, vertically fixed on the crystal table, and with the help of a telescope and scale by precisely readjusting the crystal to the calculated inclined angle of the crystal in its vertical position. In this way the crystal has been vertically adjusted quickly and satisfactorily. The precision of the measurements and their results given in the Tables 8 and 10 can therefore be considered to be better than those of the former series of measurements and their results. As the corrected value of the grating constant derived from the measurements given in the Table 8 (a, b and c), we obtain $d_1 = 1908,61$ X. U. and the mean value of the grating constant d_1 of all the series of measurements with the second crystal is 1908,69 X. U. From the minute differences between the three obtained values of the grating constant d_1 , we cannot however draw any further conclusions than the afore said one.

From the series of measurements shown in the Tables 8 and 9, I will show a further possibility of the application of the method shown in the preceding article for deriving the individual fictive grating constants. In this case the vertical position of the reflecting surface of the crystal has been very precisely parallel to the axis of the spectrograph. The horizontal displacement from the axis of the spectrograph has been 0,024 mm. Before I begin to discuss the individual fictive constants, I will show once more how far the horizontal displacement of the crystal plays a rôle in the grating constant derived from the glancing angle φ . As it can be seen from the Table 9, the grating constant derived from the glancing angle φ of Cu $K\alpha_1$ is 1908,93 X. U., while that derived from φ of Mo $K\alpha_1$ is 1909,43 X. U. Both these values differ from the value of the grating constant derived from the differential glancing angle \varkappa . As it can be seen from the Table 7, where the horizontal displacement has been approximately half the displacement value in the case of the measurements shown in the Tables 8 and 9, the values of the grating constant derived from the glancing angles have approached the

Fifth series with a displacement of 0,007 mm.
Crystal No. II.

Table 10a.

Cu, φ_2

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
385	0,525	7' 10,9"	214° 39' 14,5"	53° 38' 0,90"	8,09"	53° 38' 8,99"	53° 38' 19,2"
387	0,460	6' 17,6"	214° 39' 15,5"	53° 38' 14,48"	8,09"	53° 38' 22,57"	
388	0,504	6' 53,7"	214° 39' 19"	53° 38' 6,43"	6,11"	53° 38' 12,54"	
389	0,502	6' 52,0"	214° 39' 17"	53° 38' 6,25"	8,09"	53° 38' 14,34"	
390	0,548	7' 29,8"	214° 39' 17"	53° 37' 56,80"	9,17"	53° 38' 5,97"	
395	0,331	4' 31,7"	214° 37' 13"	53° 38' 10,32"	8,45"	53° 38' 18,77"	
396	0,344	4' 42,4"	214° 37' 13,5"	53° 38' 7,78"	5,67"	53° 38' 13,45"	
399	0,563	7' 42,1"	214° 26' 9"	53° 38' 27,78"	5,13"	53° 38' 32,91"	
400	0,558	7' 38,0"	214° 26' 7"	53° 38' 26,25"	8,99"	53° 38' 35,24"	
414	0,399	5' 27,5"	214° 28' 2"	53° 38' 22,38"	8,00"	53° 38' 30,38"	
418	0,335	4' 35,0"	214° 28' 1"	53° 38' 9,00"	7,37"	53° 38' 16,37"	

Table 10b.

Mo, φ_2

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
380	0,209	2' 51,5"	87° 5' 46"	21° 45' 43,63"	2,06"	21° 45' 45,69"	21° 45' 47,9"
381	0,153	2' 5,6"	87° 1' 3"	21° 45' 47,15"	2,14"	21° 45' 49,29"	
382	0,155	2' 7,2"	87° 1' 2"	21° 45' 47,30"	2,33"	21° 45' 49,63"	
379	0,192	2' 37,6"	87° 5' 46"	21° 45' 47,10"	2,51"	21° 45' 47,10"	

Table 10c.

Displacement of the crystal Δ = — 0,007 mm
 φ_1 Cu $K\alpha_1$ = 53° 38' 19,2"
 φ_1 Mo $K\alpha_2$ = 21° 45' 47,9"
 φ_1 Cu — φ_2 Mo = κ_2 = 31° 53' 31,3"

value of the grating constant derived from the angle κ . In all these cases only a minute change appears in the values derived from the corrected values of the angle κ . In the case of the determination of the individual fictive constants with the measurements of the glancing angles φ , a slight displacement of the crystal can cause such errors in the values of the glancing angles that the values of the individual fictive constants become erroneous. These errors are greater in the case of the usage of shorter wave-lengths, than in the case of using longer wave-lengths. The fictive constant d_1 derived from φ of Mo $K\alpha_1$ is even larger than the fictive constant of the second order d_2 , which can therefore by no means be correct.

In agreement with the fact, that for the greater glancing angles of longer wave-lengths the errors due to the displacement

of the crystal in the grating constant appear smaller than those that enter into the value of the grating constant derived from the glancing angles of shorter wave-lengths, the errors due to the displacement have smaller influence on the grating constant in the second order d_2 as can be noted from the Table 10. From the results (Table 10) it can be noted that the values of the grating constant derived from the glancing angles φ , from the differential glancing angles \varkappa and that derived from the corrected angle \varkappa agree with one other in the limits of the possible errors of observation. These measurements have been made with the same crystal-adjustment as in the case of the measurements given in the Tables 8 and 9. Now from what I have said it is evident that in the case of my measurement of the fictive grating constant in the second order, there is no practical value to correct the value of the angle \varkappa , if no greater precision is available than what I have obtained with my apparatus. The measurements of the first and the second orders (Tables 8 and 10) have been made with the same position of the crystal, and the amount of displacement in either case therefore ought to be the same. But from our results the amount of displacement in the second order has practically disappeared. This said disappearance of the displacement could be due to errors which are within the limits of observation. I will once more point out that in the second order, in this case, the value obtained from Bouchal-Dolejšek's equation⁵⁾ does not require the correction of the angle \varkappa as it has been done in the first order.

Table 11.

Representation of the fifth series of measurements and their results ($\Delta \sim -0,007\text{mm}$).

		$\frac{1}{2}\delta = 5,8''$		
$\varphi_2\text{Cu meas.}$	$d_2 =$	$\varphi_2\text{Cu} \sim \text{to } 1909,15 \text{ XU}$	$53^\circ 38' 13,4''$	
$53^\circ 38' 19,2''$	$1909,11 \text{ XU}$			
mean \varkappa_2	$d_2 =$	$\varphi_2\text{Cu} \sim \text{to } 1909,15 \text{ XU}$	$53^\circ 38' 13,4''$	
$31^\circ 53' 31,3''$	$1909,15 \text{ XU}$			corr. \varkappa_2
$\varphi_2\text{Mo meas.}$	$d_2 =$	$\varphi_2\text{Mo} \sim \text{to } 1909,15 \text{ XU}$	$21^\circ 45' 44,4''$	
$21^\circ 45' 47,9''$	$1909,07 \text{ XU}$			
		$\frac{1}{2}\delta = 3,5''$		

Δ	mean Δ	$\frac{1}{2}\delta$	$\varepsilon\delta\varkappa =$
mm	mm		$(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$
Cu 0,0072	0,0074	{4,5''}	+ 1,1''
Mo 0,0076		{5,6''}	

⁵⁾ F. Bouchal-V. Dolejšek, Časopis 65 (1935), 33; C. R. 199 (1934), 1054.

From the two above mentioned facts, it can be judged that the application of the method mentioned in the preceding article will be valuable for the precise determination of the fictive grating constants $d_{m,n}$ derived from the equation of Kunzl and Köppel⁶⁾ with the differential glancing angle $\kappa_{m,n}$ of the same wave-length in two different orders, m and n . If we calculate these values of $d_{1,2}$ with the angles $\kappa_{1,2}$ derived from our measurements given in the Tables 9 and 11, using the said Kunzl-Köppel's equation,

$$d_{m,n} = \frac{m\lambda}{2 \sin \kappa_{m,n}} \sqrt{\left(\frac{n}{m}\right)^2 - 2 \frac{n}{m} \cos \kappa_{m,n} + 1},$$

where m and n are the orders ($n > m$) and $\kappa_{m,n} = \varphi_n - \varphi_m$, we obtain from Cu $K\alpha_1$ the value $d_{1,2} = 1909,19$ X. U. and with the corrected angle $\kappa_{1,2}$ of Cu $K\alpha_1$ the corrected value $d_{1,2} = 1909,42$ X. U. Again we obtain from $\kappa_{1,2}$ of Mo $K\alpha_1$ the value of $d_{1,2}$ equal to 1908,75 X. U., and with the corrected $\kappa_{1,2}$ of the same line, the value of $d_{1,2}$ is 1909,84 X. U. If we consider the value of $d_{m,n}$ obtained from the angle $\kappa_{1,2}$ of Mo $K\alpha_1$, it can be seen, that though the displacement error, which enters into d derived from the angle $\kappa_{1,2}$ is diminished, in comparison to the d obtained from the glancing angle φ , is not entirely eliminated. The difference between the value of $d_{1,2}$ obtained from the value of $\kappa_{1,2}$ of Mo $K\alpha_1$ and the value from the same angle $\kappa_{1,2}$ corrected with our mentioned method, is considerable. Hence in the case of measuring the grating constant $d_{m,n}$ from the angle $\kappa_{1,2}$, the application of the method of correction will be of great value.

Here I should like to point out that the aforesaid fact will be important in the second part of the work, in which I propose to make the use of the angle κ instead of the angle φ for the measurement of the wave-lengths of X-radiations. For this proposed method of measuring the wave-length it is necessary to measure the angle $\kappa_{1,2}$ of the wave-length, which is to be measured, in two different orders and to have the knowledge of the values of the individual fictive constants of the grating crystal which is used as a diffraction grating. The results regarding the individual fictive grating constants will be discussed in the second part where the directly measured true value of the grating constant will be given. From the results obtained in this work, it can be seen, that the fictive grating constant d_2 has greater value than d_1 as it is to be expected. The difference $d_2 - d_1 = 0,51$ X. U. obtained with the corrected value of differential glancing angle κ is smaller than the difference $d_2 - d_1 = 0,60$ X. U. derived from the

⁶⁾ V. Kunzl-J. Köppel, C. R. 196 (1933), 787; 196 (1933), 940; Časopis 68 (1934), 109; Journ. de Phys. 5 (1934), 145.

value κ which is not corrected. The differences are taken only from the last two series of measurements in which the vertical adjustment has been improved. From the preceding series of measurements the value of d_1 is somewhat higher than the mentioned last one. If we compare the above mentioned difference $d_2 - d_1$ with that which is expected from the point of view of the theoretical value of the index of refraction, which, when calculated from the value of the density of zinc sulphide given in the tables of Landolt-Bernstein, is

$$\frac{\delta}{\lambda^2} = 5,2 - 5,7 \cdot 10^{-12},$$

we see, that this value of δ/λ^2 agrees better with the difference $d_2 - d_1$, if we take for d_1 the value of the first measured crystal, than that measured from the second. That is why we have not concluded that the difference between the values of the grating constant of the first and the second crystals can be said to be true. The decision of this problem will be given after testing the results by the direct measurement of the true grating constant as mentioned already.

Here I should particularly like to point out, that the applied correction to the differential glancing angle κ as is done in this work is of special importance for smaller glancing angles (shorter wave-lengths). In the usual cases for longer wave-lengths or for higher orders, when the glancing angles are large enough, the use of the angle κ without correction is practically sufficient for the precise determination. In the cases of the usage of that angle κ , where large and small glancing angles φ have been simultaneously utilised as in the case of the angle $\kappa_{m,n}$ of the same wave-length, the application of the method of correction of the said angle $\kappa_{m,n}$ is important. As it can be noted from our results, the importance of these corrections is increasing with the increasing wave-lengths. Therefore in the measurement of the wave-lengths in the proposed method, wherein the angle $\kappa_{1,2}$ is utilised, the mentioned correction of this will be of decisive advantage for the precise measurements of the wave-lengths.

This work has been done in the Spectroscopic Institute of Prof. Dr. V. Dolejšek (Charles University, Prague), to whom I offer my sincerest thanks for kindly putting all the necessary requisites at my disposal.

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Aplikace metody vzniklé kombinací metod φ a κ na určení mřížkové konstanty.

(Obsah předešlého článku.)

Autor užívá metody k přesnému měření mřížkových konstant krystalů popsané s prof. Dolejškem¹⁾, při čemž postupuje nyní tím způsobem, že měří pouze úhly sklonu φ metodou Siegbahnovou, a to pro dvě vlnové délky a dva řády při téže pozici krystalu. Úhel κ nyní neměří zvláště přímo, jako v předchozí práci, nýbrž určuje jej z měřených úhlů sklonu φ . V tomto případě ovšem jsou do úhlu κ přeneseny chyby, které vcházejí do měřeného úhlu φ vlivem sklonu reflexní roviny od osy krystalu (vertikální justace), takže krystal nutno nyní justovati velmi přesně, což provádí autor přímým měřením na deskách a metodou pomocného zrcátka a škály.

Jako aplikaci metody určuje autor fiktivní mřížkové konstanty sfaleritu (ZnS), jenž náleží do skupiny krystalů, jichž štěpné plochy nesouhlasí s reflekcními rovinami Braggovskými, jak zjistila v dosud nepublikované práci Ad. Němejcová souhlasně s měřeními krystalografickými prof. Ulricha. V takovýchto případech je přesné měření mřížkových konstant normálními metodami nemožné.

Uvedenou metodou autor určuje jednotlivé fiktivní hodnoty a ukazuje, v kterých případech má význam použití kombinace úhlů φ a κ , a v kterých případech (dlouhé vlny a vyšší řády) již pouhým přímým použitím úhlu κ k určení mřížkové konstanty lze docílit v mezích docílené přesnosti správných výsledků.

Dále ukazuje autor, že vliv přímísenin v krystalu ZnS na změnu jeho mřížkové konstanty je při dvou jím použitých krystalech menší než 0,2 X-jedn.

Autor určil hodnoty fiktivních mřížkových konstant, aby jich mohl užít pro přesné měření vlnových délek pomocí κ -metody; to bude obsahem druhé části této práce.

¹⁾ V. Dolejšek-Swami Jnanananda, Časopis 65 (97), 1936.