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Some monomial curves as set-theoretic complete intersections

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Abstract. We describe associated prime ideals of some monomial curves in A^4. We use a procedure by which we prove that these curves are intersection of three hypersurfaces and we found their concrete description.

1. Introduction

It is known that k-dimensional algebraic affine variety is intersection of not less than n-k hypersurfaces in n-dimensional affine space A^n . There is the presumption that a number of these hypersurfaces is exactly n-k. In this case we can say that they are ideal-theoretic or set-theoretic complete intersections. This is also equivalent to the fact that either the associated ideal I of this variety has n-k generators (ideal-theoretic complete intersection) or the ideal I is radical of an ideal a, $a \subseteq I$, the ideal a has n-k generators (set-theoretic complete intersection). The number n-k is also height of the ideal I.

Kunz [6] showed that every monomial curve $C(n_1, n_2, n_3)$ in 3-dimensional affine space is an intersection of two hypersurfaces. This presumption is also correct for some monomial curves in 4-dimensional affine space A^4 as known by Bresinsky [2], Gastinger [5] and Solčan [7].

2. The associated prime ideal P of the monomial curve $C(n_1, n_2, n_3, n_4)$

Let n_1, n_2, n_3, n_4 be positive integers with g.c.d. equal 1 and n_1, n_2, n_3, n_4 is a minimal set of generators for the numerical semigroup $H = \langle n_1, n_2, n_3, n_4 \rangle = \{n, n = \sum a_i n_i, a_i\text{'s are nonnegative integers}\}$.

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Let K be an arbitrary field, $R = K[x_1, x_2, x_3, x_4]$ the polynomial ring in four variables over K. $C(n_1, n_2, n_3, n_4)$ a monomial curve in affine space A^4 over K having parameterization $x_i = t^{n_i}, i = 1, 2, 3, 4$. The ideal $P = P(n_1, n_2, n_3, n_4)$ of all polynomials $f(x_1, x_2, x_3, x_4) \in R$ such that $f(t^{n_1}, t^{n_2}, t^{n_3}, t^{n_4}) = 0$, t transcendental over K, is the associated prime ideal of local ring $R_{(x_1,x_2,x_3,x_4)}$ of the monomial curve $C(n_1, n_2, n_3, n_4)$. P is the corresponding ideal with $\dim(P)$=1 and height $\mathrm{ht}(P)$=3 (more information in [6]).

Let a binomial term $\prod_{i=1}^{4} x_i{}^{\gamma_i} - \prod_{i=1}^{4} x_i{}^{\vartheta_i} \in P$, where $\gamma_i \vartheta_i = 0$, $i = 1, 2, 3, 4$. It is clear that $\sum_{i=1}^{4} \gamma_i n_i = \sum_{i=1}^{4} \vartheta_i n_i$.

We have basically two types of binomial terms of P:

i) $x_i^{\gamma_i} x_j^{\gamma_j} - x_k^{\gamma_k} x_l^{\gamma_l}, \{i,j,k,l\} = \{1,2,3,4\}, \gamma_i \gamma_j \gamma_k \gamma_l \neq 0$ or

ii) $x_i^{r_i} - x_j^{\alpha_{ij}} x_k^{\alpha_{ik}} x_l^{\alpha_{il}}, \{i,j,k,l\} = \{1,2,3,4\}, r_i \neq 0.$

We denote the binomial term $x_i^{r_i} - x_j^{\alpha_{ij}} x_k^{\alpha_{ik}} x_l^{\alpha_{il}}$ by $(x_i^{r_i})$ if r_i is minimal and by $(x_i^{r_i}, x_j^{r_j})$ if $x_j^{r_j} - x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} x_l^{\alpha_{jl}} \in P$ with r_j minimal and $\alpha_{ji} = r_i, \alpha_{jk} = \alpha_{jl} = 0$. Every generating set for P contains for each $i\,(i = 1, 2, 3, 4)$ at least one polynomial $x_i^{r_i} - x_j^{\alpha_{ij}} x_k^{\alpha_{ik}} x_l^{\alpha_{il}}$ with r_i minimal. We also denote polynomial $x_i^{r_i} - x_j^{\alpha_{ij}} x_k^{\alpha_{ik}} x_l^{\alpha_{il}}$ by $(x_i^{r_i}(k,l))$ if r_i is minimal with respect to the condition either $\alpha_{ik} \neq 0$ or $\alpha_{il} \neq 0$. We define as H.Bresinsky a set B in three cases as follows:

- Let $(x_s^{r_s}), s = i, j, k, l, \{i,j,k,l\} = \{1,2,3,4\}$, with at least two exponents α_{sh} not zero, $h \in \{i,j,k,l\} - \{s\}$. $B = \{(x_i^{r_i}), (x_j^{r_j}), (x_k^{r_k}), (x_l^{r_l})\}$
- $(x_i^{r_i}, x_j^{r_j}) \in P$, but $(x_k^{r_k}, x_l^{r_l}) \notin P$. Either $B = \{(x_i^{r_i}, x_j^{r_j}), (x_k^{r_k}), (x_l^{r_l})\}$ or $B = \{(x_i^{r_i}, x_j^{r_j}), (x_k^{r_k}), (x_l^{r_l}), (x_j^{r_j'}(k,l))\}$,
- $B = \{(x_i^{r_i}, x_j^{r_j}), (x_k^{r_k}, x_l^{r_l})\} \cup C$, $C \subseteq \{(x_j^{r_j'}(k,l)), (x_l^{r_l'}(i,j))\}$.

We write $x_i^{\gamma_{i1}} x_j^{\gamma_{j1}} - x_k^{\gamma_{k1}} x_l^{\gamma_{l1}} \not< x_i^{\gamma_{i2}} x_j^{\gamma_{j2}} - x_k^{\gamma_{k2}} x_l^{\gamma_{l2}}$ if between the first (second) monomials of this polynomials it is holding either $\gamma_{i1} > \gamma_{i2}\,(\gamma_{k1} > \gamma_{k2})$ and $\gamma_{j1} < \gamma_{j2}\,(\gamma_{l1} < \gamma_{l2})$ or the inequalities are reversed.

We next define a set $D_{ij}, i \neq j, \{i,j\} \subset \{1,2,3,4\}$, $D_{ij} = \{f = x_i^{\gamma_i} x_j^{\gamma_j} - x_k^{\gamma_k} x_l^{\gamma_l}$, $\{k,l\} \subset \{1,2,3,4\} - \{i,j\}, \gamma_h < r_h$ for the polynomials $(x_h^{r_h}(k,l))$ if $h \in \{i,j\}$, for the polynomials $(x_h^{r_h}(i,j))$ if $h \in \{k,l\}$ and for each binomial term $f' = x_i^{\gamma_{i1}} x_j^{\gamma_{j1}} - x_k^{\gamma_{k1}} x_l^{\gamma_{l1}} \in P, f' \neq f$ is $f' \not< f\}$.

In [1], H.Bresinsky gives the following theorem.

Theorem 2.1. $M = B \cup D_{ij} \cup D_{ik} \cup D_{il}, \{i,j,k,l\} = \{1,2,3,4\}$ is a minimal generating set for the associated prime ideal $P = P(n_1, n_2, n_3, n_4)$ of the monomial curve in A^4.

We can find in [5], Lemma 7.1, the next property of minimal generating set for a prime ideal P.

Lemma 2.1. Let $g_j = \prod_{i=1}^{4} x_i{}^{\gamma_i}, j = 1, \ldots, t$ be a monomial term in R. Let $M = \{x_1^{r_1} - g_1, x_2^{r_2} - g_2, g_3 - g_4, \ldots, g_{t-1} - g_t\}$ be a minimal generating set for the associated prime ideal P of the monomial curve in A^4. If $x_2 \mid g_1$ and $x_1 \mid g_2$, then there is $k, 3 \leq k \leq t$ with $g_k = x_1^{\delta_1} x_2^{\delta_2}$.

The preceding lemma and theorem give necessary conditions of a minimal set of generators for an associated prime ideal P of a monomial curve. Put $N = \{x_i^{r_i} - x_j^{\alpha_{ij}} x_k^{\alpha_{ik}} x_l^{\alpha_{il}}, x_j^{r_j} - x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} x_l^{\alpha_{jl}}, x_k^{r_k} - x_i^{\alpha_{ki}} x_l^{\alpha_{kl}}, x_l^{r_l} - x_i^{\alpha_{li}} x_j^{\alpha_{lj}}\}$, $\{i,j,k,l\} = \{1,2,3,4\}$, where only α_{il} or α_{ji} can be zero. By using the Lemma 2.1 we obtain, the set N can't be minimal generating set for the ideal P, but the set $N \cup \{x_i^{\gamma_i} x_k^{\gamma_k} - x_j^{\gamma_j} x_l^{\gamma_l}\}$ can be. When we use every conditions of the Theorem 2.1 we get the following relations:

$$
\begin{aligned}
r_i &= \alpha_{ji} + \alpha_{ki} + \alpha_{li} \\
r_j &= \alpha_{ij} + \alpha_{lj} \\
r_k &= \alpha_{ik} + \alpha_{jk} \\
r_l &= \alpha_{il} + \alpha_{jl} + \alpha_{kl} \\
\gamma_i &= \alpha_{ji} + \alpha_{li}, \gamma_j = \alpha_{ij}, \gamma_k = \alpha_{jk}, \gamma_l = \alpha_{il} + \alpha_{kl}
\end{aligned} \tag{1}
$$

3. Set-theoretic complete intersection

In this section we suppose that the ideal P has the minimal generating set $N \cup \{x_i^{\gamma_i} x_k^{\gamma_k} - x_j^{\gamma_j} x_l^{\gamma_l}\}$ with respected the relations (1).

Theorem 3.1. Let P be the associated prime ideal of the monomial curve $C(n_1, n_2, n_3, n_4)$ in the notation as above. Then this monomial curve $C(n_1, n_2, n_3, n_4)$ is a set-theoretic complete intersection.

Proof. We have to prove that $P = \mathrm{Rad}(f_1, f_2, f_3)$, $f_s \in P, s = 1,2,3$. Let $f_2 = x_j^{r_j} - x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} x_l^{\alpha_{jl}}$, $f_3 = x_k^{r_k} - x_i^{\alpha_{ki}} x_l^{\alpha_{kl}}$, $\{i,j,k,l\} = \{1,2,3,4\}$. When we add a linear combination of polynomials f_2, f_3 to a polynomial $f \in R$ we denote it as $\longrightarrow$. Let $f_4 = x_l^{r_l} - x_i^{\alpha_{li}} x_j^{\alpha_{lj}}$, hence

$$f_4^{r_j r_k} = (-1)^{r_j r_k} \sum_{h=0}^{r_j r_k} \binom{r_j r_k}{h} (-1)^h (x_i^{\alpha_{li}} {x_j}^{\alpha_{lj}})^{r_j r_k - h} x_l^{r_l h}.$$

We denote $b_h = \binom{r_j r_k}{h} (-1)^{r_j r_k + h}$, $h = 0, \ldots, r_j r_k$. $f_4^{r_j r_k} \longrightarrow$

$$
\begin{aligned}
\longrightarrow \quad & \sum_{h=0}^{\alpha_{lj} r_k - 1} b_h x_i^{\alpha_{li}(r_j r_k - h) + \alpha_{ji}(\alpha_{lj} r_k - h)} {x_j}^{\alpha_{ij} h} {x_k}^{\alpha_{jk}(\alpha_{lj} r_k - h)} x_l^{(\alpha_{il} + \alpha_{kl})h + \alpha_{jl} \alpha_{lj} r_k} \\
+ \quad & \sum_{h=\alpha_{lj} r_k}^{r_j r_k} b_h (x_i^{\alpha_{li}} {x_j}^{\alpha_{lj}})^{r_j r_k - h} x_l^{r_l h}
\end{aligned}
$$

$[{x_j}^{\alpha_{lj}(r_j r_k - h)} \longrightarrow {x_j}^{\alpha_{ij} h} (x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} x_l^{\alpha_{jl}})^{(\alpha_{lj} r_k - h)}$, if $h < \alpha_{lj} r_k]$

$$\longrightarrow \quad \sum_{h=0}^{\alpha_{jk} \alpha_{lj} - 1} b_h x_i^{\alpha_{li} r_j r_k + \alpha_{lj}(\alpha_{ji} r_k + \alpha_{ki} \alpha_{jk}) - r_i h} (x_j^{\alpha_{ij}} {x_k}^{\alpha_{ik}})^h {x_l}^{\alpha_{il} h + \alpha_{lj}(\alpha_{jl} r_k + \alpha_{jk} \alpha_{kl})}$$

$$
\begin{aligned}
&+ \sum_{h=\alpha_{jk}\alpha_{lj}}^{\alpha_{lj}r_k-1} b_h x_i^{\alpha_{li}(r_jr_k-h)+\alpha_{ji}(\alpha_{lj}r_k-h)} {x_j}^{\alpha_{ij}h} {x_k}^{\alpha_{jk}(\alpha_{lj}r_k-h)} x_l^{(\alpha_{il}+\alpha_{kl})h+\alpha_{jl}\alpha_{lj}r_k} \\
&+ \sum_{h=\alpha_{lj}r_k}^{r_jr_k} b_h (x_i^{\alpha_{li}} {x_j}^{\alpha_{lj}})^{r_jr_k-h} x_l^{r_lh}
\end{aligned}
$$

$[{x_k}^{\alpha_{jk}(r_k\alpha_{lj}-h)} \longrightarrow {x_k}^{\alpha_{ik}h}(x_i^{\alpha_{ki}} x_l^{\alpha_{kl}})^{(\alpha_{jk}\alpha_{lj}-h)}$, if $h < \alpha_{jk}\alpha_{lj}]$

$$
\begin{aligned}
= \quad & x_l^{\alpha_{lj}(\alpha_{jl}r_k+\alpha_{jk}\alpha_{kl})} \left(\sum_{h=0}^{\alpha_{jk}\alpha_{lj}-1} b_h x_i^{\alpha_{li}r_jr_k+\alpha_{lj}(\alpha_{ji}r_k+\alpha_{ki}\alpha_{jk})-r_ih} (x_j^{\alpha_{ij}} {x_k}^{\alpha_{ik}})^h {x_l}^{\alpha_{il}h} \right. \\
+ \quad & \sum_{h=\alpha_{jk}\alpha_{lj}}^{\alpha_{lj}r_k-1} b_h x_i^{\alpha_{li}(r_jr_k-h)+\alpha_{ji}(\alpha_{lj}r_k-h)} {x_j}^{\alpha_{ij}h} {x_k}^{\alpha_{jk}(\alpha_{lj}r_k-h)} x_l^{\alpha_{il}h+\alpha_{kl}(h-\alpha_{jk}\alpha_{lj})} \\
+ \quad & \left. \sum_{h=\alpha_{lj}r_k}^{r_jr_k} b_h (x_i^{\alpha_{li}} {x_j}^{\alpha_{lj}})^{r_jr_k-h} x_l^{\alpha_{il}h+\alpha_{jl}(h-\alpha_{lj}r_k)+\alpha_{kl}(h-\alpha_{jk}\alpha_{lj})} \right) \\
= \quad & x_l^{\alpha_{lj}(\alpha_{jl}r_k+\alpha_{jk}\alpha_{kl})} f_1
\end{aligned}
$$

We denote the generator $x_i^{\alpha_{ji}+\alpha_{li}} x_k^{\alpha_{jk}} - x_j^{\alpha_{ij}} x_l^{\alpha_{il}+\alpha_{kl}}$ of the ideal P as f_5. Take an equation $x_j^{\alpha_{lj}} f_5 = -x_l^{\alpha_{il}+\alpha_{kl}} f_2 - x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} f_4$. It is clear that

$$x_j^{\alpha_{lj}} f_5 \equiv -x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} f_4 \mod(f_2, f_3),$$

also

$$x_j^{\alpha_{lj}r_jr_k} f_5^{r_jr_k} \equiv (-1)^{r_jr_k} x_i^{\alpha_{ji}r_jr_k} x_k^{\alpha_{jk}r_jr_k} f_4^{r_jr_k} \quad \mod(f_2, f_3)$$

and

$$f_4^{r_jr_k} \equiv x_l^{\alpha_{lj}(\alpha_{jl}r_k+\alpha_{jk}\alpha_{kl})} f_1 \mod(f_2, f_3). \tag{2}$$

We know that $R/(f_2, f_3)$ is a module over $K[x_i, x_l]$ and $\{f_2, f_3\}$ is a Gröbner basis for (f_2, f_3) with respect to the lexicographic order, taking $x_j > x_k > x_i > x_l$. By [4], Chapter 1, §3, Exercise 4 each element $\overline{f} \in R/(f_2, f_3)$ is uniquely expressed $\overline{f} = a_1^1 \cdot 1 + \ldots + a_1^{r_k} \cdot x_k^{r_k-1} + a_2^1 \cdot x_j + \cdots + a_2^{r_k} \cdot x_j x_k^{r_k-1} + \ldots + a_{r_j}^1 \cdot x_j^{r_j-1} + \ldots + a_{r_j}^{r_k} \cdot x_j^{r_j-1} x_k^{r_k-1} + (f_2, f_3)$, $a_d^n \in K[x_i, x_l]$, $n = 1, 2, \ldots, r_k$, $d = 1, 2, \ldots, r_j$. Clearly, the module $R/(f_2, f_3)$ has a basis $\{\overline{1}, \ldots, \overline{x_j^{r_j-1} x_k^{r_k-1}}\}$ (that is, a generating set that is linearly independent over $K[x_i, x_l]$), thus $R/(f_2, f_3)$ is free module over $K[x_i, x_l]$ and its rank is r_jr_k. Therefore

$$f_5^{r_jr_k} \equiv (-1)^{r_jr_k} x_i^{\alpha_{ij}(\alpha_{jk}\alpha_{ki}+\alpha_{ji}r_k)} x_l^{\alpha_{jk}\alpha_{kl}r_j} f_1 \mod(f_2, f_3). \tag{3}$$

Let $g = x_i^{r_i} - x_j^{\alpha_{ij}} {x_k}^{\alpha_{ik}} {x_l}^{\alpha_{il}}$ be another generator of the ideal P. Moreover,

$$x_j^{\alpha_{lj}} g \equiv -x_i^{\alpha_{ji}+\alpha_{ki}} f_4 \mod(f_2, f_3).$$

We use the same properties as above and we have a consequence,

$$g^{r_jr_k} \equiv (-1)^{r_jr_k} x_i^{\alpha_{ij}\alpha_{ji}r_k+\alpha_{ki}r_jr_k-\alpha_{jk}\alpha_{ki}\alpha_{lj}} f_1 \mod(f_2, f_3). \tag{4}$$

If P is the associated prime ideal of the monomial curve, then $P = \mathrm{Rad}(P)$ and $f_1 \in P$. That is way $(f_1, f_2, f_3) \subseteq P$ and this inclusion induces $\mathrm{Rad}(f_1, f_2, f_3) \subseteq P = \mathrm{Rad}(P)$. If we know (2),(3),(4) then we easily get $P \subseteq \mathrm{Rad}(f_1, f_2, f_3)$. There is $P = \mathrm{Rad}(f_1, f_2, f_3)$ and our proof is completed.

Let

$$P = (x_i^{r_i} - x_j^{\alpha_{ij}} x_k^{\alpha_{ik}} x_l^{\alpha_{il}}, x_j^{r_j} - x_i^{\alpha_{ji}} x_k^{\alpha_{jk}} x_l^{\alpha_{jl}}, x_k^{r_k} - x_i^{\alpha_{ki}} x_l^{\alpha_{kl}},$$
$$x_l^{r_l} - x_i^{\alpha_{li}} x_j^{\alpha_{lj}}, x_i^{\alpha_{ji}+\alpha_{li}} x_k^{\alpha_{jk}} - x_j^{\alpha_{ij}} x_l^{\alpha_{il}+\alpha_{kl}})$$

be the ideal from the Theorem 3.1.

We have four cases:

a) $\alpha_{il} = 0 \wedge \alpha_{ji} = 0$

b) $\alpha_{il} \neq 0 \wedge \alpha_{ji} \neq 0$

c) $\alpha_{il} \neq 0 \wedge \alpha_{ji} = 0$

d) $\alpha_{il} = 0 \wedge \alpha_{ji} \neq 0$

It is known that associated prime ideal P of monomial curve $C(n_1, n_2, n_3, n_4)$ is from case a), when numerical semigroup $H = \langle n_1, n_2, n_3, n_4 \rangle$ is symmetric and the curve is not ideal-theoretic intersection (see [2],[3]).

In [8] we can find two groups of monomial curves

$$C(4s-1, 4s, 4s+1, 6s) \qquad \text{and}$$
$$C(4s+1, 4s+2, 4s+3, 6s+3),\ s \in N,\ s \geq 2.$$

Minimal generating sets of their associated prime ideals are

$$\{x_i^{s+1} - x_j x_k {x_l}^{s-1}, x_j^2 - x_i x_k x_l, x_k^2 - x_i x_l, x_l^{s+l} - x_i^{s-1} x_j, x_i^s x_k - x_j {x_l}^s\}$$

and

$$\{x_i^{s+2} - x_j x_k {x_l}^{s-1}, x_j^2 - x_i x_k x_l, x_k^2 - x_i x_l, x_l^{s+l} - x_i^s x_j, x_i^{s+1} x_k - x_j {x_l}^s\}$$

for $(i,j,k,l) = (3,4,2,1)$ and $(i,j,k,l) = (1,4,2,3)$. By a direct computation, we know that the relations (1) hold for exponents of these polynomials, so we can say that these ideals are from the case b).

We obtain ideal from the case d) [case c)] by indexes permutation of generators for an ideal from the case c) [case d)]. Now, we show two examples of monomial curves which associated prime ideals are from new case c) or d). The prime ideals for this curves $C(n_1, n_2, n_3, n_4)$ are given by Computer algebra system Macaulay created by D.Bayer and M.Stillman.

Example 3.1. Let $C(8, 9, 15, 19)$ be a monomial curve. $Q = (x_1^3 - x_2 x_3, {x_2}^3 - x_1 x_4, x_3^3 - x_1 x_2^2 x_4, x_4^2 - x_1 x_3^2, x_1^2 x_2^2 - x_3 x_4)$ is the associated prime ideal of this curve and it has description for $(i,j,k,l) = (3,2,4,1)$ as the ideal in the Theorem 3.1 from case c) or for $(i,j,k,l) = (1,3,2,4)$ from case d). Hence, the curve $C(8, 9, 15, 19)$ is set-theoretic intersection and

$$Q = \mathrm{Rad}(f_1, x_2^3 - x_1 x_4, x_4^2 - x_1 x_3^2), \text{ where}$$

$$f_1 = x_3^8 - 6x_1x_2^2x_3^5x_4 + \sum_{h=2}^{6}(-1)^{6+h}\binom{6}{h}(x_2x_3)^{6-h}x_1^{3h-3} \quad \text{[case c)] or}$$

$$Q = \operatorname{Rad}(f_1', x_2^3 - x_1x_4, x_3^3 - x_1x_2^2x_4), \text{ where}$$

$$f_1' = \sum_{h=0}^{3}(-1)^{9+h}\binom{9}{h}x_1^{19-3h}x_2^hx_3^h + \sum_{h=4}^{5}(-1)^{9+h}\binom{9}{h}x_1^{15-h}x_2^hx_3^{2(6-h)}x_4^{h-4} +$$

$$\sum_{h=6}^{9}(-1)^{9+h}\binom{9}{h}(x_1x_3^2)^{9-h}x_4^{2h-10} \quad \text{[case d)].}$$

Example 3.2. Let $C(12, 13, 20, 23)$ be a monomial curve. Its associated prime ideal is $J = (x_1^3 - x_2x_4, {x_2}^4 - x_1x_3^2, x_3^3 - x_1^2x_2x_4, x_4^2 - x_2^2x_3, x_1^2x_2^3 - x_3^2x_4)$. The ideal J has description as the ideal in the Theorem 3.1 from case d) for $(i, j, k, l) = (2, 3, 1, 4)$ or from case c) for $(i, j, k, l) = (3, 1, 4, 2)$. Therefore, this curve is set-theoretic intersection and

$$J = \operatorname{Rad}(f_1, x_1^3 - x_2x_4, x_3^3 - x_1^2x_2x_4), \text{ where}$$

$$f_1 = -x_2^{23} + 9x_1x_2^{19}x_3^2 - 36x_1^4x_2^16x_3x_4 + \sum_{h=3}^{9}(-1)^{9+h}\binom{9}{h}(x_2^2x_3)^{9-h}x_4^{2h-5}$$

$$\text{[case d)] or}$$

$$J = \operatorname{Rad}(f_1', x_1^3 - x_2x_4, x_4^2 - x_2^2x_3), \text{ where}$$

$$f_1' = x_3^{13} - 6x_1^2x_2x_3^{10}x_4 + \sum_{h=2}^{6}(-1)^{6+h}\binom{6}{h}(x_1x_3^2)^{6-h}x_2^{4h-4} \quad \text{[case c)].}$$

Question. Did we give a description of all associated ideals P of monomial curves in A^4 generated by 5 elements?

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