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Asymptotic Properties of Solutions of the Third Order Quasilinear Neutral Differential Equations *

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Abstract

We estimate sufficient and necessary conditions for the existence of nonoscillatory solutions of the equation

$$(r_2(t)(r_1(t)\varphi(\{x(t)-p(t)x(h(t))\}'))')' + f(t,x(g(t))) = 0$$

with specified asymptotic behaviour as $t \to \infty$.

Key words: Quasilinear neutral differential equations, nonoscilatory solutions, Schauder–Tychonoff fixed point theorem.

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1 Introduction

We deal with quasilinear neutral differential equations of the third order in the form

$$(r_2(t)(r_1(t)\varphi(\{x(t)-p(t)x(h(t))\}'))')' + f(t,x(g(t))) = 0, \quad t \geq a \geq 0, \quad \text{(E)}$$

where

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(a) $p, r_i : [a,\infty) \to (0,\infty),\ i = 1,2$ are continuous;

(b) $h, g : [a,\infty) \to R$ are continuous, h is strictly increasing, g is nondecreasing and $h(t) < t$, $g(t) \le t$ for $t \ge a$, $\lim_{t\to\infty} h(t) = \infty$, $\lim_{t\to\infty} g(t) = \infty$;

(c) $\varphi : R \to R$ is continuous, strictly increasing and such that $u\varphi(u) > 0$ for $u \ne 0$, $\varphi(R) \equiv R$;

(d) $f : [a,\infty) \times R \to R$ is continuous and $f(t,x)$ is nondecreasing in x and such that $uf(t,u) > 0$ for $u \ne 0$ and all $t \ge a$.

Denote

$$\begin{aligned} L_o x(t) &= x(t) - p(t)x(h(t)); \\ D_1^{\varphi} x(t) &= r_1(t)\varphi(L_o' x(t)), \\ D_2^{\varphi} x(t) &= r_2(t)(D_1^{\varphi} x(t))', \quad t \ge a. \end{aligned} \tag{1.1}$$

Let $T \ge a$ be such that

$$T_o = \min\{h(T), \inf_{t \ge T} g(t)\} \ge a. \tag{1.2}$$

By a proper solution x of (E) we mean a continuous function $[T_o,\infty) \to R$ such that $L_o x(t)$, $D_1^{\varphi} x(t)$, $D_2^{\varphi} x(t)$ are continuously differentiable on $[T,\infty)$, $x(t)$ satisfies the equation (E) on $[T,\infty)$ and it is nontrivial on any neighbourhood of ∞. A proper solution $x(t)$ of (E) is nonoscillatory if there exists a $T_1 \ge T_o$ such that $x(t) \ne 0$ for all $t \ge T_1$.

The object of this paper is to give conditions for the existence of several types of nonoscillatory proper solutions of (E) with specified asymptotic behaviour as $t \to \infty$. When $p(t) \equiv 0$, $g(t) \equiv t$, then equations (E) reduces to

$$(r_2(t)(r_1(t)\varphi(x'(t)))')' + f(t,x(t)) = 0. \tag{E_1}$$

The existence of nonoscillatory solutions of the equation (E_1) has been studied in the paper [5] under the assumptions

$$\int_a^{\infty} \left|\varphi^{-1}\left(\frac{k}{r_1(t)}\right)\right| dt = \infty, \qquad \int_a^{\infty} \frac{1}{r_2(t)}\, dt = \infty.$$

A systematic study of nonocillatory properties of quasilinear neutral differential equations of second order have been done for example in the paper [3, 6–9].

Next we will assume that either

$$\int_a^{\infty} \left|\varphi^{-1}\Big(\frac{k}{r_1(t)}\Big)\right| dt = \infty, \qquad \int_a^{\infty} \frac{1}{r_2(t)} dt < \infty \tag{1.3}$$

or

$$\int_a^{\infty} \left|\varphi^{-1}\Big(\frac{k}{r_1(t)}\Big)\right| dt < \infty, \qquad \int_a^{\infty} \left|\varphi^{-1}\Big(\frac{k}{r_1(t)} \int_0^t \frac{ds}{r_2(s)}\Big)\right| dt = \infty \tag{1.4}$$

for every $k \neq 0$, where φ^{-1} is the inverse function to φ. In the paper [2], where $\int^{\infty} dt/r_2(t) = \infty$ there are studied asymptotic properties of solutions of the equation (E).

We denote

$$\phi_{k,T}(r_1, r_2 : t) = \int_T^t \varphi^{-1}\left(\frac{1}{r_1(s)} \int_T^s \frac{k}{r_2(\tau)} d\tau\right) ds, \tag{1.5}$$

$$\phi_k(r_1, r_2 : t) = \phi_{k,a}(r_1, r_2 : t) \qquad t \geq T \geq a, \ k \neq 0.$$

From (1.5) in view of (1.3) or (1.4) we get that

$$\phi_{k,T}(r_1, r_2 : \infty) = \infty.$$

Let $x(t)$ be a nonoscillatory solutions of (E) defined on $[t_o, \infty)$, $t_0 \geq a$. From the equation (E) and assumptions (a)–(d) it follows that the function $L_o x(t)$ has to be eventually of a constant sign, so that

$$x(t)L_o x(t) > 0 \quad \text{or} \quad x(t)L_o x(t) < 0$$

for all sufficiently large t. We denote by N the set of all proper nonoscillatory solutions of (E) and define

$$N^+ = \{x \in N : x(t)L_o x(t) > 0 \quad \text{for all large } t\},$$

$$N^- = \{x \in N : x(t)L_o x(t) > 0 \quad \text{for all large } t\}.$$

We introduce the notation:

$$\gamma(t) = \sup(s \geq a : g(s) \leq t) \tag{1.6}$$

$$\gamma_h(t) = \sup(s \geq a : h(s) < t).$$

$$h^{[0]}(t) \equiv t, \ h^{[k]}(t) = h^{[k-1]}(h(t)), \quad k = 1, 2, \ldots \tag{1.7}$$

$$P_o(t) \equiv 1, \ P_k(t) = \prod_{i=0}^{k-1} p(h^{[i]}(t)), \quad k = 1, 2, \ldots \tag{1.8}$$

Let $x(t) \in N^+$ for $t \geq t_1 \geq \gamma(t_0)$. Then from (1.1) with regard to the last relations we obtain

$$x(t) = \sum_{k=0}^{n(t)-1} P_k(t) L_o x(h^{[k]}(t)) + P_{n(t)}(t) x(h^{[n(t)]}(t)), \quad t \geq t_{n(t)} = \gamma_h(t_{n(t)-1}), \tag{1.9}$$

where $n(t)$ denotes the least positive integer such that $t_o \leq h^{[n(t)]}(t) \leq t_1$.

2 Existence of nonoscillatory solutions

Let T, T_0 be defined by (1.2) Let $C[T_o,\infty)$ be a Frechet space of all continuous functions defined in $[T_o,\infty)$ with the topology of the uniform convergence on any compact subintervals of $[T_o,\infty)$.

1) Let $0 \le p(t) \le \lambda_1 < 1$. Define the operator $\phi_{\lambda_1} : C[T_o,\infty) \to C[T_o,\infty)$ as follows

$$\tilde{x}(t) = \phi_{\lambda_1} y(t) = \begin{cases} \sum\limits_{k=0}^{n(t)-1} P_k(t) y[h^{[k]}(t)] + P_{n(t)}(t)\frac{y(T)}{1-p(T)}, & t \ge T, \\ \frac{y(T)}{1-p(T)}, \quad t \in [T_o, T] \end{cases} \tag{2.1}$$

where $n(t)$ denotes the least positive integer such that $T_o \le h^{[n(t)]}(t) \le T$.

2) Let $p(t) \ge \lambda_2 > 1$. Let $C_\lambda[T_o,\infty)$ stand for a subject of $C[T_o,\infty)$ consiting of all functions $y(t)$ such that the series $\sum_{k=1}^{\infty} \lambda_2^{-k} |y(h^{[k]}(t))|$ are unformly convergent on every compact subinterval of $[T,\infty)$.

Define the operator $\phi_{\lambda_2} : C_{\lambda_2}[T,\infty) \to C[T,\infty)$ as follows

$$\tilde{x}(t) = \phi_{\lambda_2} y(t) = \sum_{k=1}^{\infty} \frac{y[h^{-[k]}(t)]}{P_k[h^{-[k]}(t)]}, \quad t \ge T_o, \tag{2.2}$$

where $h^{-[k]}$ is the inverse function to $h^{[k]}$.

Lemma 2.1 *If $y \in C[T,\infty)$, then*

a) $\tilde{x} = \phi_{\lambda_1} y$ satisfies the functional equation

$$\tilde{x}(t) - p(t)\tilde{x}(h(t)) = y(t), \quad t \ge T. \tag{2.3}$$

b) $\tilde{x} = \phi_{\lambda_2} y$ satisfies the functional equation

$$\tilde{x}(t) - p(t)\tilde{x}(h(t)) = -y(t), \quad t \ge T. \tag{2.4}$$

Proof of Lemma follows immediately from (2.1) and (2.2) respectively.

Theorem 2.1 *Let the assumptions (a)-(d) and either (1.3) or (1.4) hold. In addition let either*

$$0 \le p(t) \le \lambda_1 < 1 \quad or \quad 1 < \lambda_2 \le p(t) \le p_o < \infty. \tag{2.5}$$

Suppose that

$$\int_a^\infty \left| \varphi^{-1}\left(\frac{1}{r_1(t)} \int_t^\infty \frac{1}{r_2(s)} \int_s^\infty f(r,d) dr ds \right) \right| dt < \infty \tag{2.6}$$

for some constant $d \ne 0$.

Then there exists a nonoscillatory solution x of (E) such that

$$\lim_{t\to\infty} |L_o x(t)| = c > 0, \quad \lim_{t\to\infty} |D_1^\varphi x(t)| = 0, \quad \lim_{t\to\infty} |D_2^\varphi x(t)| = 0. \tag{2.7}$$

Proof Suppose that (2.6) holds for some constant $d > 0$. Let $c > 0$ be a constant such that either $2c \leq d(1-\lambda_1)$, $0 < \lambda_1 < 1$ or $2c \leq d(\lambda_2 - 1)$ for $\lambda_2 > 1$.

We choose $T \geq a$ so large that (1.2) and

$$\int_T^\infty \varphi^{-1}\left(\frac{1}{r_1(t)}\int_t^\infty \frac{1}{r_2(s)}\int_s^\infty f(r,d)drds\right)dt \leq \frac{c}{2}. \tag{2.8}$$

1) Let $0 \leq p(t) \leq \lambda_1 < 1$ on $[T_o, \infty)$.

With each $y \in C[T_o,\infty)$ we define the mapping $\tilde{x} : [T_o,\infty) \to R$ by (2.1). In view of Lemma 2.1 $\tilde{x}(t) = \phi_{\lambda_1} y(t)$ satisfies the relation (2.3).

Define a convex subset Y of $C[T_o,\infty)$ as follows:

$$Y = \{y \in C[T_o,\infty) : c \leq y(t) \leq 2c \text{ on } [T,\infty) \text{ and } y(t) = y(T) \text{ on } [T_o, T]\}. \tag{2.9}$$

If $y \in Y$ then using (2.1) and (2.3) we obtain

$$c \leq \tilde{x}(t) \leq \frac{2c}{1-\lambda_1} \leq d, \quad t \geq T. \tag{2.10}$$

2) Let $1 < \lambda_2 \leq p(t) \leq p_o < \infty$. For each $y \in C[T,\infty)$ we define the mapping: $\tilde{x} : [T,\infty) \to R$ by (2.2). With regard to Lemma 2.1, $\tilde{x} = \phi_{\lambda_2} y$ satisfies the relation (2.4).

If $y \in Y$, then using (2.2) and (2.4) we get

$$0 < \frac{c}{p_o} \leq \tilde{x}(t) \leq \frac{2c}{\lambda_2 - 1} \leq d, \quad t \geq T. \tag{2.11}$$

We now define an operator $F : Y \to C[T_o,\infty)$ by

$$(Fy)(t) = \begin{cases} c + \int_t^\infty \varphi^{-1}\left(\frac{1}{r_1(\tau)}\int_\tau^\infty \frac{1}{r_2(s)}\int_s^\infty f(r,\tilde{x}(g(r)))drds\right)d\tau, & t \geq T, \\ (Fy)(T), \quad T_o \leq t \leq T. \end{cases} \tag{2.12}$$

We will show that the Schauder–Tychonoff fixed point theorem ensures the existence of a fixed element $y_o = Fy_o \in Y$ and this

$$y_o(t) = \tilde{x}_o(t) - p(t)\tilde{x}_o(h(t)) = L_o\tilde{x}(t)$$

satisfies the desired asymptotic properties (2.7). The Schauder–Tychonoff fixed point theorem can be applied to the operator F if:

i) F maps Y into Y;

ii) F is continuous on Y;

iii) $F(Y)$ is a relatively compact.

i) Let $y \in Y$, then from (2.12) in view of (2.10), (2.11) the assumption (b) and (2.9) we get

$$c \le (Fy)(t) \le c + \int_T^\infty \varphi^{-1}\Big(\frac{1}{r_1(\tau)}\int_\tau^\infty \frac{1}{r_2(s)}\int_s^\infty f\Big(r, \frac{2c}{|\lambda - 1|}\Big)drds\Big)d\tau$$
$$\le \frac{3}{2}c < 2c, \quad t \ge T_o,$$

where $\lambda = \lambda_1$ or $\lambda = \lambda_2$.

ii) F is continuous on Y. Let $y_n, y \in Y$ $(n = 1, 2, \dots)$ and $y_n \to y$ as $n \to \infty$ in the space $C[T_o, \infty)$. This means that $y_n(t) \to y(t)$ as $n \to \infty$. Using the Lebesque dominanted theorem we can show that $(Fy_n)(t) \to (Fy)(t)$ as $n \to \infty$ uniformly on every compact subinterval of $[T_o, \infty)$.

iii) $F(Y)$ is a relatively compact. By the Arzela–Ascoli theorem, it is sufficiently to prove that $F(Y)$ is uniformly bounded and equicontinuous at every point $t \in [T_o, \infty)$. The uniformly bounded of F(Y) is clear since $c \le (Fy)(t) \le 2c$, $t \ge T_o$ for any $y \in Y$.

The equicontinuity of $F(Y)$ follows from the relation

$$0 \le (Fy)'(t) \le \varphi^{-1}\Big(\frac{1}{r_1(t)}\int_t^\infty \frac{1}{r_2(s)}\int_s^\infty f(r, \tilde{x}(g(r)))drds\Big)$$
$$\le \varphi^{-1}\Big(\frac{1}{r_1(t)}\int_T^\infty \frac{1}{r_2(s)}\int_s^\infty f(r, d)drds\Big), \quad t \ge T$$

holds for any $y \in Y$ and the right-hand side of the above given inequality is independent on $y \in Y$.

Then we can apply the Schauder–Tychonoff fixed point theorem to the operator $F : Y \to Y$. Then, from (2.13) we get

$$y(t) = c + \int_t^\infty \varphi^{-1}\Big(\frac{1}{r_1(\tau)}\int_\tau^\infty \frac{1}{r_2(s)}\int_s^\infty f(r, \tilde{x}(g(r)))drds\Big)d\tau, \quad t \ge T, \tag{2.13}$$

where $y(t) = \tilde{x}(t) - p(t)\tilde{x}(h(t))$.

From (2.13) in view of (2.6) we get

$$\lim_{t\to\infty} y(t) = c, \quad \lim_{t\to\infty} r_1(t)\varphi(y'(t)) = 0, \quad \lim_{t\to\infty} r_2(t)(r_1(t)\varphi(y'(t)))' = 0.$$

Theorem 2.2 *Let the assumptions (a)–(d), either (1.3) or (1.4) hold. Let*

$$0 \le p(t) \le \lambda_1 < 1, \tag{2.14}$$

and

$$\int_{\gamma(a)}^\infty |f(t, c\phi_k(r_1, r_2 : g(t)))|\, dt < \infty \tag{2.15}$$

for some constants $c \ne 0$, $k \ne 0$, $kc > 0$.

If

$$\lim_{l\to 0, kl>0} \frac{\phi_{l,T}(r_1, r_2 : t)}{\phi_{k,T}(r_1, r_2 : t)} = 0 \tag{2.16}$$

uniformly on any subinterval $[T_1, \infty) \subset [T, \infty)$ *and*

$$\int_a^\infty \left| \varphi^{-1}(\frac{1}{r_1(t)} \int_0^t \frac{1}{r_2(s)} \int_s^\infty f(r, d) dr ds) \right| dt = \infty \tag{2.17}$$

for any $d \neq 0$, *then the equation (E) has a nonoscillatory solution of the type*

$$\lim_{t\to\infty} |L_o x(t)| = \infty, \quad \lim_{t\to\infty} |D_1^\varphi x(t)| = b_1 > 0, \quad \lim_{t\to\infty} D_2^\varphi x(t) = 0. \tag{2.18}$$

Proof We consider the case $k > 0$, $c > 0$ and $d > 0$. Let c_o be such that $0 < c_o < c$. In view of (2.15), (2.16) there exist positive constants $l : l < k$ and $T \geq a$ such that (1.2),

$$c_o + \phi_l(r_1, r_2 : t) \leq c\phi_k(r_1, r_2 : t), \quad t \geq T \tag{2.19}$$

and

$$\int_T^\infty f(t, (c_o + \phi_l(r_1, r_2 : g(t))/(1 - \lambda_1))dt < l. \tag{2.20}$$

Define the set $Y_o \subset C[T_o, \infty)$ where $C[T_o, \infty)$ is the space defined in the proof of Theorem 1 and the mapping $F : Y \to C[T_o, \infty)$ as follows.

$$\begin{aligned} Y = \{y \in C[T_o, \infty) : c_o \leq y(t) \leq c_o + \phi_l(r_1, r_2 : t), \\ t \in [T, \infty);\ y(t) = y(T),\ t \in [T_o, T]\}. \end{aligned} \tag{2.21}$$

$$(Fy)(t) = \begin{cases} c_o + \int_T^t \varphi^{-1}\left(\frac{1}{r_1(\tau)} \int_T^\tau \frac{1}{r_2(s)} \int_s^\infty f(r, \tilde{x}(g(r))) dr ds\right) d\tau, & t \geq T \\ c_o, \quad t \in [T_o, T], \end{cases} \tag{2.22}$$

where $\tilde{x}(t)$ is the function defined via (2.1) and satisfies (2.3). Then in view of (2.1) and (2.21) we have

$$c_o \leq y(t) \leq \tilde{x}(t) \leq \frac{1}{1 - \lambda_1}(c_o + \phi_l(r_1, r_2; t)), \quad t \geq T. \tag{2.23}$$

We can prove that F maps Y_o into Y_o. For any $y \in Y_o$, in view of (2.19), (2.20), (2.23) and the assumption (b) we have

$$\begin{aligned} & c_o \leq (Fy)(t) \\ & \leq c_o + \int_T^t \varphi^{-1}\left(\frac{1}{r_1(\tau)} \int_T^\tau \frac{1}{r_2(s)} \int_T^\infty f(r, \frac{1}{1 - \lambda_1}(c_o + \phi_l(r_1, r_2 : g(r))) dr ds\right) dt \\ & \leq c_o + \int_T^t \varphi^{-1}\left(\frac{1}{r_1(\tau)} \int_T^\tau \frac{1}{r_2(s)} ds\right) d\tau = c_o + \phi_l(r_1, r_2 : t), \quad t \geq T. \end{aligned}$$

We can similarly as in the proof of Theorem 1 to verify that F is the continuous operator and FY_o is a compact in $C[T_o, \infty)$. Then by the Schauder–Tychonoff fixed point theorem there exists a fixed element $y_o = Fy_o \in Y_o$, which satisfies the equation

$$y_o(t) = \begin{cases} c_o + \int_T^t \varphi^{-1}\left(\frac{1}{r_1(\tau)}\int_T^\tau \frac{1}{r_2(s)}\int_s^\infty f(r,\tilde{x}_o(g(r)))drds\right)dt, \ t \geq T, \\ c_o, \quad t \in [T_o, T], \end{cases} \tag{2.24}$$

where $y_o(t) = \tilde{x}_o(t) - p(t)\tilde{x}_o(g(t))$, $t \geq T$ and $\tilde{x}_o(t)$ is a solution of (E). From (2.20) in view of the monotonicity of the function f, (2.17) and the fact that $\tilde{x}(g(t)) \geq c_o > 0$ for $t \geq \gamma(T)$ we obtain that

$$\lim_{t\to\infty} y_o(t) = \lim_{t\to\infty} L_o\tilde{x}(t) = \infty.$$

Differentiating (2.22) and then adaptation it, we get

$$D_1^\varphi \tilde{x}(t) = r_1(t)\varphi(L_o'\tilde{x}(t))' = \int_T^t \frac{1}{r_2(s)}\int_s^\infty f(r,\tilde{x}(g(r))drds,$$

$$D_2^\varphi \tilde{x}(t) = r_2(t)(D_1^\varphi \tilde{x}(t))' = \int_t^\infty f(r;\tilde{x}(g(r)))\,ds. \tag{2.25}$$

In view of the monotonicity of $D_1^\varphi \tilde{x}$, (2.23), (2.15) we obtain that there exists a positive limit of $D_1^\varphi \tilde{x}(t)$. From (2.25), in view of (2.16) we get that

$$\lim_{t\to\infty} D_2^\varphi x(t) = 0.$$

We proved that $\tilde{x}(t)$ is a nonoscillatory solution of the type (2.18).

Theorem 2.3 *Suppose that (a)–(d), (1.3), (2.14) and (2.16) hold. Then equation (E) has a nonoscillatory solution of the type*

$$\lim_{t\to\infty} |L_o x(t)| = \infty, \quad \lim_{t\to\infty} |D_1^\varphi x(t)| = b_1 > 0, \quad \lim_{t\to\infty} |D_2^\varphi x(t)| = c_1 > 0$$

if and only if (2.15) holds for some constants k, c such that $kc > 0$.

Proof of this theorem is the same as the proof of the Theorem 1 (the "only if" part) and the proof of Theorem 2 (the "if" part) in the paper [2]. Therefore we omit it.

Theorem 2.4 *Let the assumptions (a)–(d), (1.4), (2.14) and (2.16) hold. Then the equation (E) has a nonoscillatory solution of the type*

$$\lim_{t\to\infty} |L_o x(t)| = \infty, \quad \lim_{t\to\infty} |D_1^\varphi x(t)| = \infty, \quad \lim_{t\to\infty} |D_2^\varphi x(t)| = a_1 > 0$$

if and only if (2.15) holds for some constants k, c such that $kc > 0$.

The proof of Theorem 2.4 is the same as the proof of Theorem 2.3.

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