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ON OSCILLATIONS OF SOLUTIONS AND THEIR DERIVATIVES IN A NONLINEAR THIRD-ORDER DIFFERENTIAL EQUATION

SVATOSLAV STANĚK

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1. Introduction

Let us consider a differential equation

$$(p(t)(p(t)x')')' + 4p(t)q(t)x' + 2(p(t)q(t))'x = f(t,x,x',(p(t)x')') \tag{1}$$

whose terms are supposed to be:

$p,q \in C^0(J)$, $pq \in C^1(J)$, $f \in C^0(D)$, $p(t) > 0$ for $t \in J$, where $J = [t_0, \infty)$, $D = J \times R^3$.

By a solution of (1) we mean the right-maximal solution of (1). Let x be a solution of (1) on an interval $[t_x, T_x) \subset J$, $T_x \leqq \infty$. We say that the solution x (the derivative x' of the solution x; the second quasiderivative $(px')'$ of the solution

x) is oscillatory if the function x $(x'; (px')')$ has a zero in every left neighbourhood of the point T_x.

The aim of this paper is to find sufficient conditions for the oscillation of solutions, or for the oscillation of the derivative of solutions, or for the oscillation of the second quasiderivative of solutions of (1) on an interval $[t_x, \infty)$, respectively.

2. Lemmas

Let u, v be the solutions of the differential equation

$$(p(t)z')' + q(t)z = 0$$

(defined on J) satisfying the initial conditions $u(t_1) = 1$, $u'(t_1) = 0$, $v(t_1) = 0$, $v'(t_1) = 1$ at a point $t = t_1 \in J$. Then $y = c_1u^2 + c_2uv + c_3v^2$ is a solution of the differential equation

$$(p(t)(p(t)y')')' + 4p(t)q(t)y' + 2(p(t)q(t))'y = 0 \qquad (2)$$

and $y(t_1) = c_1$, $y'(t_1) = c_2$, $(p(t)y'(t))'_{t=t_1} = -2c_1q(t_1) + 2c_3p(t_1)$. Setting

$$a(t) = \max\{|u(t)|, |v(t)|\}, \quad b(t) = \max\{|u'(t)|, |v'(t)|\},$$
$$c(t) = |q(t)|a^2(t) + p(t)b^2(t) \text{ for } t \in J.$$

<u>Lemma 1</u> ([1]). Suppose

$$y = c_1u^2(t) + c_2u(t)v(t) + c_3v^2(t)$$

is a solution of (2) and set $\alpha = |c_1| + |c_2| + |c_3|$. Then

$$|y(t)| \leqq \alpha a^2(t),$$
$$|y'(t)| \leqq 2\alpha\, a(t)b(t),$$
$$|(p(t)y'(t))'| \leqq 2\alpha\, c(t), \qquad t \in J.$$

Lemma 2 ([1]). Suppose $T \in J$. Then equation (1) is equivalent to the integral equation

$$x(t) = y(t) + \frac{1}{2p^2(t_1)} \int_T^t (u(t)v(s)-u(s)v(t))^2 f(s,x(s),x'(s),(p(s)x'(s))')ds \qquad (3)$$

in a class of functions $\{x; x \in C^1, px' \in C^1\}$, where y is a solution of (2) satisfying the same initial conditions at the point $t = T$ as the solution x of (1) (that is $y(T) = x(T)$, $y'(T) = x'(T)$, $(p(t)y'(t))'_{t=T} = (p(t)x'(t))'_{t=T}$).

3. Oscillation of solutions and their derivatives on a halfline

Theorem 1. Let

$$f(t,x_1,x_2,x_3) \leqq F_1(t) \text{ for } (t,x_1,x_2,x_3) \in D,\ x_1 > 0, \qquad (4')$$

$$f(t,x_1,x_2,x_3) \geqq F_2(t) \text{ for } (t,x_1,x_2,x_3) \in D,\ x_1 < 0, \qquad (4'')$$

where F_1, F_2 are integrable functions on every interval, which is a part of J and

$$\limsup_{t \to \infty} (-1)^i \frac{1}{a^2(t)} \int_{t_0}^t (u(t)v(s) - u(s)v(t))^2 F_i(s)ds = \infty, \quad i = 1,2. \qquad (5)$$

Then every solution of (1) defined of a halfline $[t_x, \infty) \subset J$ is oscillatory.

Proof. Let x be a solution of (1) defined on $[t_x, \infty) \subset J$ and let x not be an oscillatory solution. Then there exists a $T \in [t_x, \infty)$ either with $x(t) > 0$ or $x(t) < 0$ for $t \geqq T$.

Let y be a solution of (2) satisfying the same initial conditions at the point T as the solution x. From Lemma 2 then follows the validity of equality (3) for $t \geqq T$.

If $x(t) > 0$ for $t \geqq T$, then

$$\frac{x(t)}{a^2(t)} \leqq \frac{y(t)}{a^2(t)} +$$

$$+ \frac{1}{2a^2(t)p^2(t_1)} \int_T^t (u(t)v(s)-u(s)v(t))^2 F_1(s)ds \text{ for } t \geqq T. \tag{6}$$

According to Lemma 1 there exists an $\alpha \in R$:

$$|y(t)| \leqq \alpha\, a^2(t), \qquad t \in J$$

and therefore

$$\liminf_{t \to \infty} \left\{ \frac{y(t)}{a^2(t)} + \frac{1}{2a^2(t)p^2(t_1)} \int_T^t (u(t)v(s)-u(s)v(t))^2 F_1(s)ds \right\} =$$

$$= -\infty,$$

which with respect to (6) contradicts the inequality

$$\liminf_{t \to \infty} \frac{x(t)}{a^2(t)} \geqq 0.$$

If $x(t) < 0$ for $t \geqq T$, then

$$\frac{x(t)}{a^2(t)} \geqq \frac{y(t)}{a^2(t)} +$$

$$+ \frac{1}{2a^2(t)p^2(t_1)} \int_T^t (u(t)v(s)-u(s)v(t))^2 F_2(s)ds \text{ for } t \geqq T \tag{7}$$

and from the relations

$$\limsup_{t \to \infty} \left\{ \frac{y(t)}{a^2(t)} + \frac{1}{2a^2(t)p^2(t_1)} \int_T^t (u(t)v(s)-u(s)v(t))^2 F_2(s)ds \right\} =$$

$$= \infty,$$

$$\limsup_{t \to \infty} \frac{x(t)}{a^2(t)} \leqq 0,$$

in contradiction to (7) above.

Remark 1. If assumptions (4′) and (5) for i=1 (or (4′′) and (5) for i=2) are fulfilled, then it follows from the proof of Theorem 1 that any solution x of (1) defined on $[t_x, \infty)$ does not take only positive (negative) values in a neighbourhood of ∞.

Remark 2. If the assumptions of Theorem 1 are satisfied and x is a solution of (1) defined on $[t_x, \infty)$, then not only the solution x but also its derivatives x', $(px')'$ are oscillatory.

It will be apparent from the following example that the fulfilment of assumptions in Theorem 1 does not guarantee the existence of any solution of (1) on the halfline J.

Example 1. Suppose

$$f(t) = \begin{cases} 0 & \text{for } t \in [0, \pi), \\ e^t \sin t & \text{for } t \in [\pi, \infty). \end{cases}$$

Equation $x''' = -6xx''^2 + f(t)$ has a solution $x = \sqrt{1-t}$ defined on the interval $[0,1)$ and p=1, q=0. Setting $t_1 = 0$ and $u(t) = 1$, $v(t) = t$, $F_1(t) = F_2(t) = f(t)$ for $t \in [0, \infty)$ yields

$$a(t) = \begin{cases} 1 & \text{for } t \in [0,1), \\ t & \text{for } t \in [1, \infty), \end{cases}$$

and for $t \in [\pi, \infty)$

$$\frac{1}{a^2(t)} \int_{\pi}^{t} (u(t)v(s) - u(s)v(t))^2 F_i(s)\,ds = \frac{1}{t^2} \int_{\pi}^{t} (t-s)^2 e^s \sin s\, ds =$$

$$= -\frac{e^t}{2t^2}(\sin t + \cos t) - \frac{e^{\pi}}{t^2}\left(\frac{t^2}{2} - \pi t + t + \frac{1}{2} - \pi + \frac{\pi^2}{2}\right),$$

where i=1,2. Since $\limsup_{t \to \infty} \frac{e^t}{2t^2}(\sin t + \cos t) = \infty$,

$\liminf_{t \to \infty} \frac{e^t}{2t^2}(\sin t + \cos t) = -\infty$, the assumptions of Theorem 1 are fulfilled, whereby the solution $x = \sqrt{1-t}$ of the equation considered, is not defined on the halfline $[0, \infty)$.

Theorem 2. Suppose $q(t) \leqq 0$ for $t \in J$ and

$$f(t,x_1,x_2,x_3) \leqq F_1(t) \text{ for } (t,x_1,x_2,x_3) \in D,\ x_2 > 0, \qquad (8')$$

$$f(t,x_1,x_2,x_3) \geqq F_2(t) \text{ for } (t,x_1,x_2,x_3) \in D,\ x_2 < 0, \qquad (8'')$$

where F_1, F_2 are integrable functions on each interval, which is a part of J and

$$\limsup_{t \to \infty} (-1)^i \frac{1}{p(t)a(t)b(t)} \int_{t_o}^{t} \int_{t_o}^{s} \Big[p(s)(u'(s)v(\tau) - u(\tau)v'(s))^2 - q(s)(u(s)v(\tau) - u(\tau)v(s))^2\Big] F_i(\tau)\, d\tau\, ds = \infty, \quad i = 1,2. \qquad (9)$$

Then the derivative x' of each solution x of (1) defined on a halfline $[t_x, \infty) \subset J$, is oscillatory.

Proof. Suppose x is a solution of (1) defined on $[t_x, \infty) \subset J$ and its derivative x' is not oscillatory. Then there exists a $T \geqq t_x$ being either $x'(t) > 0$ or $x'(t) < 0$ for $t \geqq T$. According to Lemma 2, there exists a solution y of (2), satisfying the same initial conditions at the point t = T as the solution x, so that equality (3) holds for $t \geqq T$. A slight modification of this equality yields

$$(p(t)x'(t))' = (p(t)y'(t))' + \frac{1}{p^2(t_1)} \int_T^t \Big[p(t)(u'(t)v(s) - u(s)v'(t))^2 - q(t)(u(t)v(s) - u(t)v(s))^2\Big] \cdot f(s,x(s),x'(s),(p(s)x'(s))')ds \qquad (10)$$

whence, on integrating (10) from T to t ($\geq$ T),

$$x'(t) = y'(t)+ \frac{1}{p(t)p^2(t_1)} \int_T^t \int_T^s \Big[p(s)(u'(s)v(\tau)-u(\tau)v'(s))^2-$$

$$-q(s)(u(s)v(\tau)-u(\tau)v(s))^2\Big] f(\tau, x(\tau), x'(\tau), (p(\tau)x'(\tau))')\,d\tau\, ds. \tag{11}$$

If $x'(t) > 0$ for $t \geq T$, then we get from (11) and (8')

$$\frac{x'(t)}{a(t)b(t)} \leq \frac{y'(t)}{a(t)b(t)} +$$

$$+ \frac{1}{p(t)a(t)b(t)p^2(t_1)} \int_T^t \int_T^s \Big[p(s)(u'(s)v(\tau)-u(\tau)v'(s))^2 -$$

$$- q(s)(u(s)v(\tau) - u(\tau)v(s))^2\Big] . \; F_1(\tau)\; d\tau ds. \tag{12}$$

According to Lemma 1 there exists an $\alpha \in R$:

$$|y'(t)| \leq \alpha\, a(t)b(t), \qquad t \in J$$

and consequently from (9)

$$\limsup_{t \to \infty} \left\{ \frac{y'(t)}{a(t)b(t)} + \frac{1}{p(t)a(t)b(t)} \int_T^t \int_T^s \Big[p(s)(u'(s)v(\tau) - \right.$$

$$- u(s)v'(\tau))^2 - q(s)(u(s)v(\tau) - u(\tau)v(s))^2\Big] F_1(\tau)\; d\tau ds =$$

$$= - \infty ,$$

which, with respect to (12), contradicts the fact that

$$\limsup_{t \to \infty} \frac{x'(t)}{a(t)b(t)} \geq 0.$$

In applying (8''), it may likevise be proved that the assumption $x'(t) < 0$ for $t \geq T$ leads to a contradiction.

Remark 3. If the assumptions of Theorem 2 are fulfilled and x is a solution of (11) defined on a halfline $[t_x, \infty)$, then not only x' but also $(px')'$ is oscillatory.

Remark 4. If $q(t) \leqq 0$ for $t \in J$ and besides the assumptions $(8')$ and (9) for i=1 (or $(8'')$ and (9) for i=2) are fulfilled, then it follows from the proof of Theorem 2 that the derivative of any solution of (1) defined on a halfline, does not take only positive (negative) values in a neighbourhood of ∞.

The following example will show that the fulfilment of assumptions in Theorem 2 does not guarantee the existence of any solution of (1) on the halfline J.

Example 2. Consider a differential equation

$$x''' = -\frac{1}{3}\left(\frac{4}{3}\right)^3 x' x''^4 + f(t) \tag{13}$$

on the interval $J = [0, \infty)$, where

$$f(t) = \begin{cases} 0 & \text{for } t \in [0, \pi], \\ t^2 \sin t & \text{for } t \in (\pi, \infty). \end{cases}$$

Let $t_1 = 0$. Then $p = 1$, $q = 0$, $u = 1$, $v = t$ and further

$$a(t) = \begin{cases} 1 & \text{for } t \in [0,1], \\ t & \text{for } t \in (1, \infty), \end{cases}$$

$b = 1$ and assumptions $(8')$ and $(8'')$ are fulfilled for the functions $F_1 = F_2 = f$. The fulfilment of assumption (9) follows from the relations

$$\limsup_{t \to \infty} \frac{1}{a(t)} \int_0^t \int_0^s f(\tau)\,d\tau\,ds = \limsup_{t \to \infty} \frac{1}{t} \int_0^t \int_0^s \tau^2 \sin\tau\,d\tau\,ds =$$

$$= \limsup_{t \to \infty} \frac{1}{t} (6\sin t - 4t\cos t - t^2 \sin t - 2t) = \infty$$

and

$$\liminf_{t\to\infty} \frac{1}{a(t)} \int_0^t \int_0^s f(\tau)d\tau ds = -\infty ,$$

whereby the function $x(t) = (1-t)^{3/2}$ is a solution of (13) just on the interval $[0,1)$.

Theorem 3. Suppose

$$f(t,x_1,x_2,x_3) \leq F_1(t) \text{ for } (t,x_1,x_2,x_3)\in D,\ x_3>0, \quad (14')$$

$$f(t,x_1,x_2,x_3) \geq F_2(t) \text{ for } (t,x_1,x_2,x_3)\in D,\ x_3<0, \quad (14'')$$

where F_1, F_2 are integrable functions on any interval being a part of J. Then every solution of (1) is defined on J.

If moreover $q(t) \leq 0$ for $t\in J$ and

$$\limsup_{t\to\infty} (-1)^i \frac{1}{c(t)} \int_0^t \Big[p(s)(u'(t)v(s) - u(s)v'(t))^2 - \quad (15)$$

$$- q(s)(u(t)v(s) - u(s)v(t))^2\Big] F_i(s)ds = \infty ,$$

$$i = 1,2,$$

then for every solution x of (1), its second quasiderivative $(px')'$ is oscillatory.

Proof. Suppose x is a solution of (1) on a interval $[t_0,T_x)$ and $T_x<\infty$. Without any loss of generality we may assume $F_1(t) \geq 0$, $F_2(t) \leq 0$ for $t\in [t_0,T_x]$. Let us put

$$F(t) =\{\max F_1(t),\ -F_2(t)\},\quad Q(t) = \max_{t_0\leq s\leq t} p(s)|q(s)| ,$$

$$\alpha(t) = (p(t)q(t))' \quad \text{for } t\in J$$

and

$$\beta(t) = p(t)(p(t)x'(t))' \quad \text{for } t\in [t_0,T_x).$$

Let $t_1 \in [t_o, T_x)$ be such a number that

$$\int_{t_1}^{T_x} \frac{1}{p(t)} \int_{t_1}^{t} \frac{1}{p(s)} \left[4Q(s) + \int_{t_1}^{s} |\alpha(\tau)|\, d\tau\, ds dt < \frac{1}{2}\right. \tag{16}$$

Let further $J_1 = [t_1, T_x)$ and

$$X(t) = \max_{t_1 \leqq s \leqq t} |x(s)| \quad \text{for } t \in J_1 .$$

If $\beta(t) > 0$ for $t \in (a,b) \subset J_1$, then the equality

$$\beta'(t) = -4(p(t)q(t)x(t))' + 2\alpha(t)x(t) +$$
$$+ f(t, x(t), x'(t), (p(t)x'(t))'), \quad t \in [t_o, T_x)$$

upon integration from a to t ($\geqq$ a) yields

$$\beta(t) = \beta(a) - 4(p(t)q(t)x(t) - p(a)q(a)x(a)) +$$
$$+ 2\int_a^t \alpha(s)x(s)ds + \int_a^t f(s, x(s), x'(s), (p(s)x'(s))')ds, \tag{17}$$
$$t \in [t_o, T_x)$$

and further on making use of (14´) and of the evident estimates gives

$$0 < \beta(t) \leqq \beta(a) - 4(p(t)q(t)x(t) - p(a)q(a)x(a)) +$$
$$+ 2\int_a^t \alpha(s)x(s)ds + \int_a^t F_1(s)ds \leqq \beta(a) + \tag{18}$$
$$+ 2X(t)\left[4Q(t) + \int_{t_1}^{t} |\alpha(s)|\, ds\right] + \int_{t_1}^{t} F(s)ds,$$
$$t \in (a,b).$$

If $\beta(t) < 0$ for $t \in (a,b) \subset J_1$, then it follows from (17) and (14´´) that

$$0 > \beta(t) \geqq \beta(a) - 4(p(t)q(t)x(t) - p(a)q(a)x(a)) +$$

$$+ 2\int_a^t \alpha(s)x(s)ds + \int_a^t F_2(s)ds \geqq \beta(a) - 2X(t)\Big[4Q(t) + \qquad (19)$$

$$+ \int_{t_1}^t |\alpha(s)|\,ds\Big] - \int_{t_1}^t F(s)ds, \quad t \in (a,b).$$

From (18) and (19) it follows that the estimate

$$|\beta(t)| \leqq |\beta(a)| + 2X(t)\Big[4Q(t) + \int_{t_1}^t |\alpha(s)|\,ds\Big] + \int_{t_1}^t F(s)ds \qquad (20)$$

holds on every interval $(a,b) \subset J_1$, where $\beta(t) \neq 0$, which yields

$$|\beta(t)| \leqq |\beta(t_1)| + 2X(t)\Big[4Q(t) + \int_{t_1}^t |\alpha(s)|\,ds\Big] + \int_{t_1}^t F(s)ds, \quad t \in J_1. \qquad (21)$$

From the equality

$$x(t) = x(t_1) + p(t_1)x'(t_1)\int_{t_1}^t \frac{ds}{p(s)} + \int_{t_1}^t \frac{1}{p(s)}\int_{t_1}^s \frac{\beta(\tau)}{p(\tau)}\,d\tau\,ds,$$

we find the estimate to be

$$|x(t)| \leqq |x(t_1)| + p(t_1)|x'(t_1)|\int_{t_1}^t \frac{ds}{p(s)} +$$

$$+ \int_{t_1}^t \frac{1}{p(s)}\int_{t_1}^s \frac{|\beta(\tau)|}{p(\tau)}\,d\tau\,ds$$

whence and from (21) we obtain

$$|x(t)| \leqq |x(t_1)| + p(t_1)|x'(t_1)| \int_{t_1}^{t} \frac{ds}{p(s)} +$$

$$+ \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tilde{\tau})} \Big[|\beta(t_1)| + 2X(\tilde{\tau}) \Big[4Q(\tilde{\tau}) + \int_{t_1}^{\tilde{\tau}} |\alpha(\nu)| d\nu \Big] +$$

$$+ \int_{t_1}^{\tilde{\tau}} F(\nu) d\nu \Big] d\tilde{\tau}\, ds \leqq |x(t_1)| +$$

$$+ p(t_1)|x'(t_1)| \int_{t_1}^{t} \frac{ds}{p(s)} + |\beta(t_1)| \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tilde{\tau})} d\tilde{\tau} ds +$$

$$+ 2X(t) \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tilde{\tau})} \Big[4Q(\tilde{\tau}) + \int_{t_1}^{\tilde{\tau}} |\alpha(\nu)| d\nu \Big] d\tilde{\tau}\, ds +$$

$$+ \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tilde{\tau})} \int_{t_1}^{\tilde{\tau}} F(\nu) d\nu\, d\tilde{\tau}\, ds$$

therefore

$$X(t) \leqq 2X(t) \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tilde{\tau})} \Big[4Q(\tilde{\tau}) + \int_{t_1}^{\tilde{\tau}} |\alpha(\nu)| d\nu \Big] d\tilde{\tau}\, ds +$$

$$+ |x(t_1)| + p(t_1)|x'(t_1)| \int_{t_1}^{t} \frac{ds}{p(s)} +$$

$$+ |\beta(t_1)| \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tilde{\tau})} d\tilde{\tau}\, ds +$$

$$+ \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{t} \frac{1}{p(\tilde{\tau})} \int_{t_1}^{\tilde{\tau}} F(\nu)\, d\nu\, d\tilde{\tau}\, ds$$

and

$$X(t) \leq \Big(1-2\int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tau)} \big(4Q(\tau) + \int_{t_1}^{\tau}|\alpha(\nu)|\,d\nu\big)d\tau\,ds\Big)^{-1} .$$

$$\cdot \Big(|x(t_1)| + p(t_1)|x'(t_1)| \int_{t_1}^{t} \frac{ds}{p(s)} +$$

$$+ |\beta(t_1)| \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tau)}\, d\tau ds +$$

$$+ \int_{t_1}^{t} \frac{1}{p(s)} \int_{t_1}^{s} \frac{1}{p(\tau)} \int_{t_1}^{\tau} F(\nu)d\nu\, d\tau\, ds\Big), \quad t \in J_1 .$$

From the last inequality and from (16) follows the boundedness of X on J_1. Thus the solution x of (1) is bounded on $[t_0, T_x)$, and with respect to (21) the function β is bounded there, as well. Then, naturally, there exist (finite) $\lim_{t\to T_x^-} x'(t)$ and $\lim_{t\to T_x^-} x(t)$. From the continuity of the function f on the set D follows the boundedness of the function $f(t,x(t),x'(t),(p(t)x'(t))')$ on $[t_0,T_x)$, and thus the boundedness of the function β' there, as well. Then, naturally, there exists a $\lim_{t\to T_x^-} (p(t)x'(t))'$ which enables us to extend the solution x of (1) to the right of the point $t = T_x$, in contradiction to $T_x < \infty$. This proves the assertion that assumptions (14') and (14'') guarantee the existence of each solution of (1) on the halfline J.

Let x be a solution of (1) and its second quasiderivative $(px')'$ not be oscillatory on J. Hence, there exists a T $(\geq t_0)$ with either $(p(t)x'(t))' > 0$ or $(p(t)x'(t))' < 0$ for $t \geq T$. According to Lemma 2 there exists a solution y of (2) satisfying the same initial conditions at the point $t = T$ as the solution x, such that equality (10) holds on $[T, \infty)$.

Next by Lemma 1

$$\left|(p(t)y'(t))'\right| \leqq k.c(t), \quad t \in J, \tag{22}$$

where $k > 0$ is a convenient number.

Let $(p(t)x'(t))' > 0$ for $t \geqq T$. Then from (11), (14'') and from $q(t) \leqq 0$ for $t \in J$ we obtain

$$\frac{(p(t)x'(t))'}{c(t)} \leqq \frac{(p(t)y'(t))'}{c(t)} + \frac{1}{c(t)p^2(t_1)} \int_T^t \Big[p(t)(u'(t)v(s)-u(s)v'(t))^2 - q(t)(u(t)v(s)-u(s)v(t))^2\Big] F_1(s)ds, \quad t \geqq T. \tag{23}$$

Because of (15) and (22) we have

$$\limsup_{t \to \infty} \Big[\frac{(p(t)y'(t))'}{c(t)} + \frac{1}{c(t)p^2(t_1)} \int_T^t \Big[p(t)(u'(t)v(s)-u(s)v'(t))^2 - q(t)(u(t)v(s)-u(s)v(t))^2\Big] F_1(s)ds = -\infty,$$

which, however, contradicts the fact of (23) with respect to $\limsup_{t \to \infty} \frac{(p(t)x'(t))'}{c(t)} \geqq 0$

We can similarly prove that the assumption $(p(t)x'(t))' < 0$ for $t \geqq T$ also yields a contradiction.

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OSCILACE ŘEŠENÍ A JEJICH DERIVACÍ NELINEÁRNÍ DIFERENCIÁLNÍ ROVNICE 3.ŘÁDU

Souhrn

Je vyšetřována nelineární diferenciální rovnice 3.řádu

$$(p(t)(p(t)x')')' + 4p(t)q(t)x' + 2(p(t)q(t))'x = f(t,x,x', (p(t)x')'), \qquad (1)$$

kde $p,\ q \in C^0(J)$, $pq \in C^1(J)$, $f \in C^0(D)$, $p(t) > 0$ pro $t \in J$ a $J = [t_0, \infty)$, $D = J \times R^3$.

Nechť x je (napravo) maximální řešení rovnice (1) definované na intervalu $[t_x, T_x)$ ($T_x \leqq \infty$). Řekneme, že řešení x (derivace x' řešení x; druhá kvaziderivace $(px')'$ řešení x) je oscilatorické, jestliže v každém levém okolí bodu T_x existuje nulový bod funkce x (x'; $(px')'$).

V práci jsou uvedeny podmínky, které jsou postačující k tomu, aby každé řešení x rovnice (1) (derivace x' řešení x; druhá kvaziderivace $(px')'$ řešení x), které je definované na polopřímce $[t_x, \infty)$, bylo oscilatorické.

КОЛЕБАНИЕ РЕШЕНИЙ И ИХ ПРОИЗВОДНЫХ НЕЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ 3-ГО ПОРЯДКА

Резюме

Изучается нелинейное дифференциальное уравнение 3-го порядка

$$(p(t)(p(t)x')')' + 4p(t)\dot{q}(t)x' + 2(p(t)q(t))'x = \\ = f(t,x,x',(p(t)x')'), \qquad (1)$$

где $p,\ q \in C^0(J),\ pq \in C^1(J),\ f \in C^0(D),\ p(t) > 0$ для $t \in R$ и $J = [t_0, \infty),\ D = J \times R^3$.

Пусть x (направо) полное решение уравнения (1) определенное на интервале $[t_x, T_x)$ ($T_x \leq \infty$). Решение x (производная x' решения x; вторая квазипроизводная (px')' решения x) называется колеблющееся, если в каждой левой окрестности точки T_x существует нулевая точка функции x (x'; (px')') .

В работе приводятся достаточные условия для того, чтобы все решения x (производные x' решений x; вторые квазипроизводные(px') ' решений x) уравнения (1), которые определены на полупрямой $[t_x, \infty)$ были колеблющиеся.

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