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Higher monotonicity properties of i-th derivatives of solutions of $y'' + a(x)y' + b(x)y = 0$

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HIGHER MONOTONICITY PROPERTIES OF i-th DERIVATIVES OF SOLUTIONS OF $y'' + a(x)\,y' + b(x)\,y = 0$

ELENA PAVLÍKOVÁ

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Dedicated to Prof. Miroslav Laitoch on his 60th birthday

1. Introduction and notation

In [6] J. Vosmanský derived certain higher monotonicity properties of i-th derivatives of solutions of

$$y'' + a(x)\,y' + b(x)\,y = 0, \qquad x \in (0, \infty) \tag{1}$$

in the oscillatoric case.

In this paper, using the first accompanying equation with regard to the basis α, β, where α, β are real numbers with the property $\alpha^2 + \beta^2 > 0$, we extend the above-mentioned results from [6] to the function

$$\alpha y^{(i)} + \beta\left(y^{(i+1)} + \frac{1}{2}\,a_i(x)\,y^{(i)}\right), \qquad i = 0, 1, \ldots,$$

where $y(x)$ is a solution of equation (1).

Finally, we introduce certain applications of the derived results for Bessel functions.

In [2] M. Laitoch introduced the first accompanying equation (Q) towards the differential equation

$$y'' + q(x)\,y = 0 \tag{q}$$

with regard to the basis α, β in the form

$$Y'' + Q(x)\,Y = 0, \tag{Q}$$

where

$$Q(x) = q + \frac{\alpha\beta q'}{\alpha^2 + \beta^2 q} + \frac{1}{2}\,\frac{\beta^2 q''}{\alpha^2 + \beta^2 q} - \frac{3}{4}\,\frac{\beta^4 q'^2}{(\alpha^2 + \beta^2 q)^2} \tag{2_q}$$

under the assumptions that $q(x) \in C_2$, $q(x) > 0$ for each $x \in (a, \infty)$, a is a real number, and α, β are real numbers with the property $\alpha^2 + \beta^2 > 0$.

In [2] it is proved that if $y(x)$ is a solution of (q), then the function

$$Y(x) = \frac{\alpha y + \beta y'}{\sqrt{\alpha^2 + \beta^2 q(x)}},$$

is a solution of the differential equation (Q) and conversely, if $Y(x)$ is any solution of (Q), then there exists a solution $\bar{y}(x)$ of the equation (q) such that

$$\frac{\alpha \bar{y} + \beta \bar{y}'}{\sqrt{\alpha^2 + \beta^2 q(x)}} = Y(x).$$

A function $f(x)$ is said to be n-times monotonic (or monotonic of order n) on an interval (a, ∞) if

$$(-1)^i f^{(i)}(x) \geq 0, \qquad i = 0, 1, \ldots, n, \quad x \in (a, \infty). \tag{3}$$

For such a function we write $f(x) \in M_n(a, \infty)$. If strict inequality holds throughout (3), we write $f(x) \in M_n^*(a, \infty)$. We say that $f(x)$ is completely monotonic on (a, ∞) if (3) holds for $n = \infty$.

A sequence $\{x_k\}_{k=1}^{\infty}$, denoted simply by $\{x_k\}$, is said to be n-times monotonic if

$$(-1)^i \Delta^i x_k \geq 0, \qquad i = 0, 1, \ldots, n, \quad k = 1, 2, \ldots \tag{4}$$

Here

$$\Delta^\circ x_k = x_k,\ \Delta x_k = x_{k+1} - x_k, \ldots, \Delta^n x_k = \Delta^{n-1} x_{k+1} - \Delta^{n-1} x_k.$$

For such a sequence we write $\{x_k\} \in M_n$. If strict inequality holds throughout (4), we write $\{x_k\} \in M_n^*$. The sequence $\{x_k\}$ is called completely monotonic if (4) holds for $n = \infty$.

2. New basic results

1. In this section we consider a second order linear differential equation (1), where $a(x) \in C_3(0, \infty)$, $b(x) \in C_2(0, \infty)$

The transformation

$$u(x) = y(x) \exp\left[\frac{1}{2} \int a(x)\, dx\right]$$

transforms (1) into the differential equation

$$u'' + f(x)\, u = 0, \tag{5}$$

where

$$f(x) = b(x) - \frac{1}{2} a'(x) - \frac{1}{4} a^2(x). \tag{6}$$

Let $f(x) \in C_2$, $f(x) > 0$ on $(0, \infty)$. The first accompanying equation towards differential equation (5) with regard to the basis α, β has the form

$$U'' + F(x)\, U = 0, \tag{7}$$

where $F(x)$ is given by formula (2_f).

Thus, some of the results of [1] can be applied to equation (5) to give information on solutions of differential equation (1).

Lemma 1. *Let α, β be real numbers such that $\alpha^2 + \beta^2 > 0$, $\alpha\beta \leq 0$ and let $n \geq 2$ be an integer. For the function $f(x)$ defined by* (6) *suppose that*

$$f(x) > 0, \quad f'(x) > 0, \quad f'(x) \in M_n(0, \infty), \quad x \in (0, \infty). \tag{8}$$

Then for the carrier $F(x)$ of the first accompanying equation (7) *towards differential equation* (5) *with regard to the basis α, β we have*

$$F'(x) > 0, \qquad F'(x) \in M_{n-2}(0, \infty), \qquad x \in (0, \infty)$$

and

$$0 < F(\infty) = f(\infty) \leq \infty.$$

Proof. (see paper [4], Lemma 2).

Let us denote, for fixed $\lambda > -1$,

$$R_k = \int_{x_k}^{x_{k+1}} W(x) \exp\left[\frac{\lambda}{2} \int a(x)\, \mathrm{d}x\right] \left| \frac{\alpha y + \beta\left(y' + \frac{1}{2} a(x)\, y\right)}{\sqrt{\alpha^2 + \beta^2 f(x)}} \right|^{\lambda} \mathrm{d}x, \qquad k = 1, 2, \ldots, \tag{9}$$

where $y(x)$ is an arbitrary solution of (1) and $\{x_k\}$ is a sequence of consecutive zeros of the function $\alpha z(x) + \beta\left(z'(x) + \frac{1}{2} a(x)\, z(x)\right)$, where $z(x)$ is any solution of (1) which may or may not be linearly independent of $y(x)$. The function $W(x)$ is any sufficiently monotonic function.

Theorem 1. *Let α, β be real numbers such that $\alpha^2 + \beta^2 > 0$, $\alpha\beta \leqq 0$, and $n \geqq 2$ be an integer. For the function $f(x)$ defined by* (6) *suppose that*

$$f(x) > 0, \quad f'(x) > 0, \quad f'(x) \in M_n(0, \infty), \quad x \in (0, \infty).$$

Let

$$W(x) > 0, \quad W(x) \in M_{n-2}(0, \infty), \quad x \in (0, \infty). \tag{10}$$

Then for R_k defined by (9) *there holds*

$$\{R_k\} \in M^*_{n-2}. \tag{11}$$

Proof. Let $y(x)$, $z(x)$ be solutions of the differential equation (1). Then the functions

$$u(x) = y(x) \exp\left[\frac{1}{2} \int a(x)\, \mathrm{d}x\right],$$

$$v(x) = z(x) \exp\left[\frac{1}{2} \int a(x)\, dx\right],$$

are solutions of the differential equation (5).

It follows from [2] that the functions

$$Y(x) = \frac{\alpha u + \beta u'}{\sqrt{\alpha^2 + \beta^2 f(x)}},$$

$$Z(x) = \frac{\alpha v + \beta v'}{\sqrt{\alpha^2 + \beta^2 f(x)}},$$

are solutions of the differential equation (7).

Lemma 1 implies that $F'(x) > 0$ on $(0, \infty)$, $F'(x) \in M_{n-2}(0, \infty)$ and $0 < F(\infty) \leq \leq \infty$. So, the conditions of ([3], Theorem 3.1) are fulfilled. Using this theorem we have

$$\{N_k\} \in M^*_{n-2},$$

where N_k is defined by

$$N_k = \int_{s_k}^{s_{k+1}} W(x)\, |Y(x)|^\lambda\, dx, \qquad \lambda > -1, \quad k = 1, 2, \ldots,$$

where $Y(x)$ is the solution of equation (7), $\{s_k\}$ denotes the sequence of consecutive zeros of the solution $Z(x)$ of (7).

Since $Z(x)\sqrt{\alpha^2 + \beta^2 f(x)} = \alpha v(x) + \beta v'(x)$ we have $\{s_k\} = \{t_k\}$, where $\{t_k\}$ denotes the sequence of consecutive zeros of the function $\alpha v(x) + \beta v'(x)$.

But, $\alpha v(x) + \beta v'(x) = \exp\left[\frac{1}{2} \int a(x)\, dx\right]\left(\alpha z(x) + \beta z'(x) + \frac{1}{2} a(x)\, z(x)\right)$, so that $\{t_k\} = \{x_k\}$, where $\{x_k\}$ denotes the sequence of consecutive zeros of the functions $\alpha z(x) + \beta\left(z'(x) + \frac{1}{2} a(x)\, z(x)\right)$.

Hence it follows that

$$N_k = \int_{t_k}^{t_{k+1}} W(x) \left| \frac{\alpha u + \beta u'}{\sqrt{\alpha^2 + \beta^2 f(x)}} \right| dx = R_k,$$

so that (11) holds, and the theorem is proved.

Corollary 1. *Under the hypotheses of Theorem* 1 *we have*

$$\left\{ \int_{x_k}^{x_{k+1}} W(x) \exp\left[\frac{\lambda}{2} \int a(x)\, dx\right] \left| \alpha y + \beta\left(y' + \frac{1}{2} a(x)\, y\right) \right|^\lambda dx \right\} \in M^*_{n-2},$$

for $\lambda \in (-1, 0\rangle$, $k = 1, 2, \ldots$.

Proof of this corollary follows directly from Theorem 1, because (11) remains valid when $W(x)$ is replaced by

$$W(x)\,(\alpha^2 + \beta^2 f(x))^{\lambda/2}, \qquad \lambda \in (-1, 0\rangle,$$

since the last function belongs to $M_{n-2}(0, \infty)$.

Corollary 2, *Let the conditions of Theorem* 1 *be satisfied. Let* $a(x) > 0$, $a(x) \in M_{n-1}(0, \infty)$, $x \in (0, \infty)$. *Then for* $\bar{R}_k$ *defined by*

$$\bar{R}_k = \int_{x_k}^{x_{k+1}} \left| \frac{\alpha y + \beta\left(y' + \frac{1}{2} a(x)\, y\right)}{\sqrt{\alpha^2 + \beta^2 f(x)}} \right|^{\lambda} \mathrm{d}x, \qquad \lambda \geq 0, \quad k = 1, 2, \ldots,$$

where $\{x_k\}$ *and* $y(x)$ *have the same meaning as in* (9), *there holds*

$$\{\bar{R}_k\} \in M^*_{n-2}.$$

Proof. Let us choose the function $W(x)$ in the form $W(x) = \exp\left[-\frac{\lambda}{2} \int a(x)\, \mathrm{d}x\right]$. It is easy to see that under the assumptions of Corollary 2 $W(x)$ satisfies (10) for $\lambda \geq 0$. Hence from Theorem 1 we obtain $\{\bar{R}_k\} \in M^*_{n-2}$, and the corollary is proved.

Remark 1. If in the above considerations we choose $\alpha = 1$, $\beta = 0$, then we get the results from [6] concerning the monotonicity of the sequence of consecutive zeros of any arbitrary solution $y(x)$ of equation (1).

If we choose $\alpha = 0$, $\beta = 1$, then we obtain the results from [6] for the monotonicity of the sequence of consecutive zeros of the function $y'(x) + \frac{1}{2} a(x)\, y(x)$.

2. Consider the differential equation (1). Let $a_0(x) = a(x)$, $b_0(x) \equiv b(x) \neq 0$ be continuous and sufficiently differentiable functions on $(0, \infty)$. Let $a_i(x)$, $b_i(x)$ be defined recurrently for $i = 1, 2, \ldots$ by formulas

$$a_i(x) = a_{i-1} \frac{b'_{i-1}}{b_{i-1}},$$

$$b_i(x) = b_{i-1} + a'_{i-1} - a_{i-1} \frac{b'_{i-1}}{b_{i-1}}. \qquad (12_i)$$

Suppose that $b_i(x) \neq 0$ for $x \in (0, \infty)$ and all needed i.

In ([6], Lemma 2.1) it is proved that if $y(x)$, $z(x)$ are non-trivial linearly independent solutions of

$$y'' + a_0(x)\, y' + b_0(x)\, y = 0, \qquad (13_0)$$

then $y^{(i)}(x)$, $z^{(i)}(x)$ are non-trivial linearly independent solutions of

$$y'' + a_i(x)\, y' + b_i(x)\, y = 0. \qquad (14_i)$$

Let $a_i(x)$, $b_i(x)$ be defined by (12_i). The transformation

$$u(x) = y(x) \exp\left[\frac{1}{2} \int a_i(x)\,dx\right], \tag{15_i}$$

transforms (14_i) into the differential equation

$$u'' + f_i(x)\,u = 0, \tag{16_i}$$

where $f_i(x)$ is defined by

$$f_i(x) = b_i(x) - \frac{1}{2} a_i'(x) - \frac{1}{4} a_i^2 x, \qquad i = 0, 1, \dots [6]. \tag{17_i}$$

Let $f_i(x) \in C_2$, $f_i(x) > 0$ for $x > 0$ and an arbitrary but fixed integer. The first accompanying equation towards the differential equation (16_i) with regard to the basis α, β has the form

$$U'' + F_i(x)\,U = 0,$$

where $F_i(x)$ is given by formula (2_{f_i}).

In this section we shall study sequences $\{R_k^{(i)}\}$, where $R_k^{(i)}$ is defined for fixed $\lambda > -1$ by

$$R_k^{(i)} = \int_{x_k^{(i)}}^{x_{k+1}^{(i)}} W(x) \exp\left[\frac{\lambda}{2} \int a_i(x)\,dx\right] \left| \frac{\alpha y^{(i)} + \beta\left(y^{(i+1)} + \frac{1}{2} a_i(x)\,y^{(i)}\right)}{\sqrt{\alpha^2 + \beta^2 f_i(x)}} \right|^{\lambda} dx, \tag{18}$$

where $y(x)$ is an arbitrary solution of (1) and $\{x_k^{(i)}\}$ is a sequence of consecutive zeros of the function $\alpha z^{(i)}(x) + \beta\left(z^{(i+1)}(x) + \frac{1}{2} a_i(x)\,z^{(i)}(x)\right)$, where $z(x)$ is any solution of (1) which may or may not be linearly independent of $y(x)$. The function $a_i(x)$ is defined recurrently by (12_i). The function $W(x)$ is any sufficiently monotonic function.

Theorem 2. *Let $n \geqq 2$, $i \geq 1$ be arbitrary but fixed integers and let α, β be real numbers such that $\alpha^2 + \beta^2 > 0$, $\alpha\beta \leq 0$. Let the coefficients $a(x) \equiv a_0(x)$, $b(x) \equiv$ $\equiv b_0(x)$ of (1) $\equiv$ (13_0) be such that $a_j(x)$ $(j = 0, 1, \dots, i)$, $b_j(x) \neq 0$ $(j = 0, 1, \dots,$ $\dots, i - 1)$ defined by (12_j) are differentiable. For the function $f_i(x)$ defined by (17_i) suppose that*

$$f_i(x) > 0,\ f_i'(x) > 0,\ f_i'(x) \in M_n(0, \infty), \qquad x \in (0, \infty).$$

Let

$$W(x) > 0,\ W(x) \in M_{n-2}(0, \infty), \qquad x \in (0, \infty).$$

Then for $R_k^{(i)}$ defined by (18) *there holds*

$$\{R_k^{(i)}\} \in M_{n-2}^*. \tag{19}$$

Proof. Let $y(x)$, $z(x)$ be solutions of the differential equation (1). It follows from [6] that the functions $y^{(i)}(x) = y_i(x)$, $z^{(i)}(x) = z_i(x)$ are solutions of the differential equation (14_i). This implies that if $\{x_k^{(i)}\}$ denotes the sequence of consecutive zeros of the function $\alpha z^{(i)}(x) + \beta\left(z^{(i+1)}(x) + \frac{1}{2}a_i(x)\, z^{(i)}(x)\right)$, then this sequence represents the sequence of cosecutive zeros of the function $\alpha z_i(x) + \beta\left(z_i'(x) + \frac{1}{2}a_i(x)\, z_i(x)\right)$.

Theorem 2 follows now from Theorem 1 if we replace equation (1) by (14_i).

Corollary 3. *Under the hypotheses of Theorem 2 we have*

$$\left\{\int_{x_k^{(i)}}^{x_{k+1}^{(i)}} W(x) \exp\left[\frac{\lambda}{2}\int a_i(x)\,\mathrm{d}x\right] \left|\alpha y^{(i)} + \beta\left(y^{(i+1)} + \frac{1}{2}a_i(x)\, y^{(i)}\right)\right|^{\lambda} \mathrm{d}x\right\} \in M_{n-2}^{*}$$

for $\lambda \in (-1, 0\rangle$.

Proof of this corollary follows directly from Theorem 2. Assertion (19) remains valid when $W(x)$ is replaced by

$$W(x)\,(\alpha^2 + \beta^2 f_i(x))^{\lambda/2}, \qquad \lambda \in (-1, 0\rangle.$$

Corollary 4. *Let the conditions of Theorem 2 be satisfied. Let* $a_i(x) > 0$, $a_i(x) \in M_{n-1}(0, \infty)$. *Then for* $\bar{R}_k^{(i)}$ *defined by*

$$\bar{R}_k^{(i)} = \int_{x_k^{(i)}}^{x_{k+1}^{(i)}} \left|\frac{\alpha y^{(i)} + \beta\left(y^{(i+1)} + \frac{1}{2}a_i(x)\, y^{(i)}\right)}{\sqrt{\alpha^2 + \beta^2 f_i(x)}}\right|^{\lambda} \mathrm{d}x, \quad \lambda > 0, \qquad k = 1, 2, \ldots$$

where $\{x_k^{(i)}\}$ *and* $y^{(i)}(x)$ *have the same meaning as in* (18), *there holds*

$$\{\bar{R}_k^{(i)}\} \in M_{n-2}^{*}.$$

Proof. In Theorem 2, we set $W(x) = \exp\left[-\frac{\lambda}{2}\int a_i(x)\,\mathrm{d}x\right]$, $\lambda > 0$.

3. Applications to Bessel functions

Throughout this section we suppose that α, β are real numbers such that $\alpha^2 + \beta^2 > 0$, $\alpha\beta \le 0$.

Let $C_\nu(x)$ denote any Bessel (cylinder) function of order ν, i.e. any nontrivial solution of the Bessel equation

$$y'' + \frac{1}{x}y' + \left(1 - \frac{\nu^2}{x^2}\right)y = 0, \qquad x \in (0, \infty). \qquad (20_\nu)$$

Let $x > \nu$ and let $\{a'_{\nu k}\}_{k=1}^{\infty}$ denote the sequence of consecutive positive zeros of the function

$$\alpha C_\nu'(x) + \beta\left(C_\nu''(x) + \frac{1}{2}a_{\nu 1}(x)\, C_\nu'(x)\right)$$

and let $\{b'_{\nu k}\}_{k=1}$ denote the analogous sequence of the function

$$\alpha \bar{C}'_\nu(x) + \beta\left(\bar{C}''_\nu(x) + \frac{1}{2} a_{\nu 1}(x)\, \bar{C}'_\nu(x)\right),$$

where $a_{\nu 1}(x)$ is defined by (12_1) and $\bar{C}_\nu(x)$ denotes any Bessel function of order ν, possibly $C_\nu(x)$ again.

Lemma 2. *Let $f_{\nu 1}(x)$ be defined by (17_1) for $x > \nu$. Then there exists one and only one number $a \in (\nu, \infty)$ such that $f_{\nu 1}(a) = 0$.*

Proof. Using (17_1) we have $f_{\nu 1}(x) = 1 - \dfrac{\nu^2 - \frac{1}{4}}{x^2} - \dfrac{1}{x^2 - \nu^2} - \dfrac{3\nu^2}{(x^2 - \nu^2)^2}$ for $x > \nu$. It is obvious that $\lim\limits_{x \to \nu-} f_{\nu 1}(x) = -\infty$.

Since $\lim\limits_{x \to \infty} f_{\nu 1}(x) = 1$ and $f'_{\nu 1}(x) \in M^*_n(\nu, \infty)$ ([5], Theorem 3.1) there exists one and only one number $a \in (\nu, \infty)$ such that $f_{\nu 1}(a) = 0$.

Theorem 3. *Let $n \geq 2$ be an integer and $\nu \geq 0$ an arbitrary number. Let $a_{\nu 1}(x)$ be defined by (12_1), $f_{\nu 1}(x)$ be defined by (17_1) for $x > \nu$, and $f_{\nu 1}(a) = 0$, $a > \nu$. Let*

$$W(x) > 0, \quad W(x) \in M_{n-2}(a, \infty), \qquad x \in (a, \infty)$$

and let $R'_{\nu k}$ be defined for $x \in (a, \infty)$ and $\lambda > -1$ by

$$R'_{\nu k} = \int_{b'_{\nu k}}^{b'_{\nu, k+1}} W(x) \exp\left[\frac{\lambda}{2} \int a_{\nu 1}(x)\, \mathrm{d}x\right] \left| \frac{\alpha C'_\nu + \beta\left(C''_\nu + \frac{1}{2} \alpha_{\nu 1}(x)\, C'\right)}{\sqrt{\alpha^2 + \beta^2 f_{\nu 1}(x)}} \right|^\lambda \mathrm{d}x. \tag{21}$$

Let m be the smallest integer satisfying $a \leq b'_{\nu m}$. Then

$$\{R'_{\nu k}\}_{k=m}^{\infty} \in M^*_{n-2}. \tag{22}$$

Proof. Theorem 3 is a direct corollary of Theorem 2.

Since $f_{\nu 1}(a) = 0$ we obtain from $f'_{\nu 1}(x) \in M^*_n(\nu, \infty)$ ([5], Theorem 3.1) that $f_{\nu 1}(x) > 0$ on (a, ∞).

So, the conditions of the modified form of Theorem 2 are satisfied for any $n \geq 2$ if the interval $(0, \infty)$ is replaced by (a, ∞).

The expression $R_k^{(i)}$ defined in (18) is of the form (21) so that (22) holds and the theorem is proved.

Corollary 5. *Let the assumptions of Theorem 3 hold. Let $W(x)$ be a positive, completely monotonic function on (a, ∞). Let $R_{\nu k}$ be defined by (21). Then*

$$\{R'_{\nu k}\}_{k=m}^{\infty} \in M^*_\infty.$$

The corollary is the case $n = \infty$ in Theorem 3.

Remark 2. As a direct conclusion of Theorem 3 we obtain

$$\{(a'_{\nu,k+1})^{\gamma} - (a'_{\nu k})^{\gamma}\}_{k=m}^{\infty} \in M_{\infty}^{2}, \qquad 0 < \gamma \leqq 1, \tag{23}$$

$$\left\{\lg \frac{a'_{\nu,k+1}}{a'_{\nu k}}\right\}_{k=m}^{\infty} \in M_{\infty}^{*}. \tag{24}$$

Assertion (23) is an immediate consequence of Theorem 3 with $\lambda = 0$, $\bar{C}_\nu(x) \equiv C_\nu(x)$ and $W(x) = \gamma x^{\gamma - 1}$.

Assertion (24) follows from Theorem 3 with $\lambda = 0$, $\bar{C}_\nu(x) = C_\nu(x)$ and $W(x) = x^{-1}$.

Remark 3. Let the assumptions of Theorem 3 hold and let $\gamma > 0$. Then

$$\{(a'_{\nu k})^{-\gamma}\}_{k=m}^{\infty} \in M_{\infty}^{*}, \tag{25}$$

$$\{(\lg a'_{\nu k})^{-\gamma}\}_{k=m}^{\infty} \in M_{\infty}^{*}, \quad a'_{\nu m} > 1, \tag{26}$$

$$\{\exp(-\gamma a'_{\nu k})\}_{k=m}^{\infty} \in M_{\infty}^{*}. \tag{27}$$

Assertions (25), (26) and (27) follow from Theorem 3 with $\bar{C}_\nu(x) = C_\nu(x)$, $\lambda = 0$ and

$$W(x) = -[x^{-\gamma}]',$$
$$W(x) = -[(\lg x)^{-\gamma}]'$$

and

$$W(x) = -[e^{-\gamma x}]',$$

respectively.

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Souhrn

POZNÁMKA O VLASTNOSTIACH VYŠŠEJ MONOTÓNNOSTI i-tej DERIVÁCIE RIEŠENÍ ROVNICE $y'' + a(x)\, y' + b(x)\, y = 0$

ELENA PAVLÍKOVÁ

V práci [6] J. Vosmanský odvodil vlastnosti vyššej monotónnosti i-tej derivácie riešení diferenciálnej rovnice

$$y'' + a(x)\, y' + b(x)\, y = 0, \qquad x \in (0, \infty) \tag{1}$$

v oscilatorickom prípade.

V tejto práci, na základe prvej sprievodnej rovnice vzhľadom na bázu α, β, kde α, β sú reálne čísla s vlastnosťou $\alpha^2 + \beta^2 > 0$, sú rozšírené výsledky z [6] na funkciu

$$\alpha y^{(i)} + \beta\left(y^{(i+1)} + \frac{1}{2} a_i(x)\, y^{(i)}\right), \qquad i = 0, 1, \ldots,$$

kde $y(x)$ je riešením rovnice (1).

V závere sú uvedené aplikácie dosiahnutých výsledkov na Besselove funkcie.

Резюме

ЗАМЕТКА О СВОЙСТВАХ ВЫСШЕЙ МОНОТОННОСТИ i-той ПРОИЗВОДНОЙ РЕШЕНИЙ УРАВНЕНИЯ $y'' + a(x)y' + b(x)y = 0$

ЕЛЕНА ПАВЛИКОВА

В работе [6] Я. Восмански исследовал свойства высшей монотонности i-той производной решений дифференциального уравнения

$$y'' + a(x)y' + b(x)y = 0, \qquad x \in (0, \infty) \tag{1}$$

в колебательном случае.

В этой работе, с помощью первого сопроводительного уравнения при базисе α, β где α, β произвольные вещественные постоянные с свойством $\alpha^2 + \beta^2 > 0$, обобщены результаты из [6] на функции

$$\alpha y^{(i)} + \beta\left(y^{(i+1)} + \frac{1}{2} a_i(x)\, y^{(i)}\right), \qquad i = 0, 1, \ldots,$$

где $y(x)$ решение дифференциального уравнения (1).

В заключении приведены приложения полученых результатов к теории бесселевых фвнкций.