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SOLUTION OF A GIVEN PROBLEM IN THE BACK TIME

KAREL BENEŠ

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On electronic analog computers can be calculated problems in which the independent variable is an increasing quantity. If we have to perform the solution in the interval $t \in \langle t_a; t_b \rangle$ with initial conditions being reserved for the argument t_1 and $t_a < t_1 \leqq t_b$, the problem will be solved in two parts:

1. The given problem will be solved in the usual way, i.e. in the interval $t_1 \leqq t \leqq t_b$ with initial conditions being reserved at the point $t = t_1$.

2. The problem will be solved in the back time, i.e. in the interval $t_1 \geqq t \geqq t_a$ with initial conditions at the point $t = t_1$.

There are then two possible ways for carrying out the solution in the back time.

a) With the aid of the transformation of the independent variable and the coefficient $M_t = -1$ we find the equation (equations) and the programme diagramm which will solve the given problem (the differential equation) in the coordinate system with an inversely oriented axis of the independent variable.

b) The fact that the integrators operate with a negative coefficient of integration enables us to perform the time transformation with a negative coefficient directly on a computer. The equation will be programmed in the usual way, the integrators being assumed with a positive coefficient of integration i.e. not changing the sign. The negative time scale will be formed by using the integrators with a negative coefficient of integration. The correctness of this proceeding will be shown in this article.

The solution of a problem in the back time in the way of a) i.e. via the transformation of the independent variable and the coefficient $M_t = -1$, is evident. Suppose, for example, we have a differential equation of the n-th order

$$y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1y' + a_0y = k \qquad (1)$$

with initial conditions $y(t_1^{(j)})$, $j = 0, 1, 2, \ldots, n - 1$. For simplicity, we shall consider the equation of (1) with constant coefficients and having its righthand side equal to a constant. By applying the coefficient $M_t = -1$ and using $y^{(j)} = M_t^j Y^{(j)}$ the equation of (1) will be carried over to a machinery equation

$$(-1)^n Y^{(n)} + (-1)^{n-1} a_{n-1} Y^{(n-1)} + \dots - a_1 Y' + a_0 Y = k \tag{2}$$

with initial conditions $Y^{(j)}(t_1) = \dfrac{y^{(j)}(t_1)}{(-1)^j}$, $j = 0, 1, 2, \dots, n-1$. This equation has the form

$$Y^{(n)} - a_{n-1} Y^{(n-1)} + \dots - a_1 Y' + a_0 Y = k \tag{3}$$

for n even and the form

$$-Y^{(n)} + a_{n-1} Y^{(n-1)} - \dots - a_1 Y' + a_0 Y = k \tag{4}$$

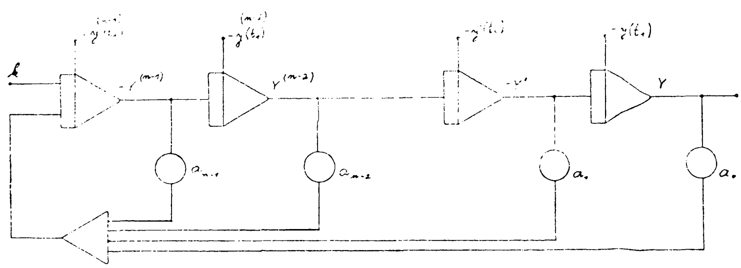

Fig. 1

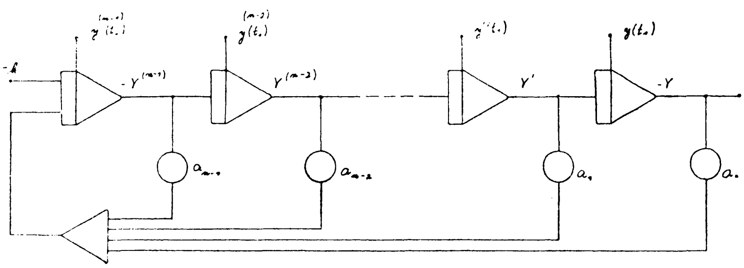

Fig. 2

for n odd. Let us now programme equation (3) in the form

$$Y^{(n)} = k + a_{n-1} Y^{(n-1)} - \dots + a_1 Y' - a_0 Y \tag{3a}$$

by the programme diagram shown in Figure 1 and equation (4) in the form

$$Y^{(n)} = -k + a_{n-1} Y^{(n-1)} + \dots - a_1 Y' + a_0 Y = k \tag{4a}$$

by the programme diagram shown in Figure 2.

In solving equation (1) let us perform the transformation of the independent variable in the way outlined in b) i.e. directly on a computer utilizing the fact that the integrators change the sign simultaneously. Let us programme equation (1) first in the form

$$y^{(n)} = k - a_{n-1}y^{(n-1)} - \dots - a_1 y' - a_0 y \tag{5}$$

by the programme diagram in Figure 3 and then in the form

$$-y^{(n)} = -k + a_{n-1}y^{(n-1)} + \dots + a_1 y' + a_0 y \tag{6}$$

by the programme diagram in Figure 4. Since the original variable y and the machinery variable Y are associated with the relation $y^{(j)} = (-1)^j\ Y^{(j)}$ the programme

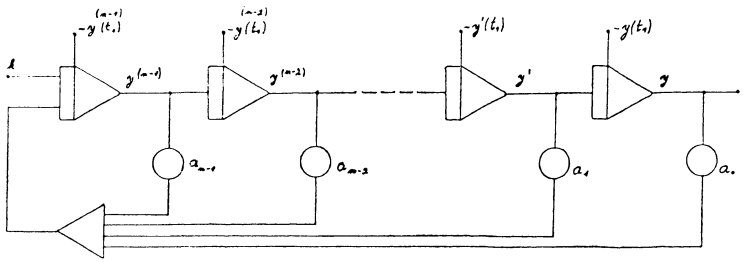

Fig. 3

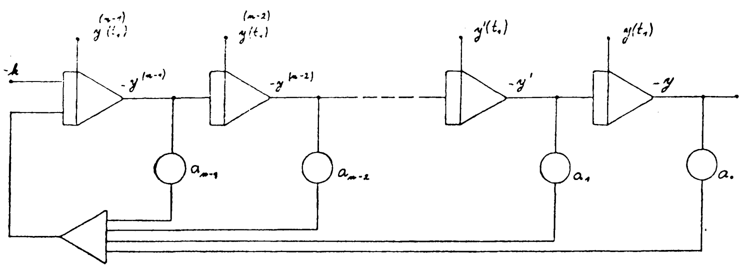

Fig. 4

diagram in Figure 3 is equivalent to that in Figure 1 for the solution of equation (1) in the back time for n even; the programme diagram in Figure 4 is equivalent to that in Figure 2 in solving equation (1) for n odd.

The way suggested under b), showing the transformation of the independent variable directly on a computer is simpler and thus more favourable, too.

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Shrnutí

ŘEŠENÍ DANÉHO PROBLÉMU VE ZPĚTNÉM ČASE

KAREL BENEŠ

V článku je popsán způsob řešení úloh (diferenciálních rovnic) ve zpětném čase bez převodu dané rovnice na strojovou rovnici se záporným měřítkovým koeficientem nezávisle proměnné. Při programování úlohy se využívá záporného koeficientu integrace elektronických integrátorů a časová transformace se záporným měřítkovým koeficientem je prováděna přímo počítačem.

Резюме

РЕШЕНИЕ ДАННОЙ ЗАДАЧИ В ОБРАТНОМ ВРЕМЕНИ

КАРЕЛ БЕНЕШ

В статье описан метод решения задач (дифференциальных уравнений) в обратном времени без перевода уравнения на машинное уравнени с отрицательным масштабным коэффициентом независимо переменной. При программированию задачи использовается отрицательный коэффициент интегрирования электронических интеграторов и трансформация времени производится прямо вычислительной машиной.