Libuše Marková

A specialization of an anholonomial subvarietes system of an n-dimensional variety in unimodular $n+2$-dimensional space

Sborník prací Přírodovědecké fakulty University Palackého v Olomouci. Matematika, Vol. 13 (1973), No. 1, 47--54

Persistent URL: http://dml.cz/dmlcz/120024

1973 — ACTA UNIVERSITATIS PALACKIANAE OLOMUCENSIS
FACULTAS RERUM NATURALIUM — TOM 41

Katedra algebry a geometrie přírodovědecké fakulty University Palackého v Olomouci
Vedoucí katedry: Doc. RNDr. Josef Šimek

A SPECIALIZATION OF AN ANHOLONOMIAL SUBVARIETIES SYSTEM OF AN N-DIMENSIONAL VARIETY IN UNIMODULAR N+2-DIMENSIONAL SPACE

LIBUŠE MARKOVÁ

(Received June 20, 1972)

1. Introduction

In article [1] has been given a construction of a semicanonical moving frame of variety for a case $n = 3$. A special interest of the present article is to follow this procedure and to show a possibility of its generalization for a case $n > 2$. The case $n = 1$ is trivial. This way, however, cannot be used for the case $n = 2$ as described further in text.

Since the fundamental concepts and assumptions which occur here have been expressed in [1] we shall proceed without repeating them to the own construction using the notation introduced in [1].

2. Construction of a semicanonical moving frame

We begin with a given differentiable variety Φ_n in $\mathbf{A}^{n+2}$ with an anholonomial subvarieties system $\mathbf{S}$. The application of the system $\mathbf{S}$ necessitates the application of n independent subvarieties ψ_1, which can be thought of as being coordinate. The system of differential equations determinating them is then

$$\omega^{\alpha_{i_1}} = \omega^{\alpha_{i_2}} = \omega^{\alpha_{i_3}} = \ldots\ldots = \omega^{\alpha_{i_{n-1}}} = 0, \tag{1}$$

where $\alpha_{i_1}, \ldots, \alpha_{i_{n-1}}, \alpha_{i_n}$ are all permutations of indexes 1, 2, ..., n. These equations are supposed not to be completely integrable.

Convention: From now on let the indexes i, j, k, ... be running through values 1, ..., n, n + 1, n + 2, and the indexes $\alpha, \beta, \gamma, \ldots$ running through values 1, ..., n and μ, ν, λ running through values n + 1, n + 2.

Then the derivate formulas of the moving frame are:

$$\mathrm{dm} = \omega^i \mathbf{e}_i, \qquad \mathrm{d}\mathbf{e}_i = \omega_i^k \mathbf{e}_k, \tag{2}$$
$$\mathrm{d}\omega^i = \omega^j \wedge \omega_j^i, \qquad \mathrm{d}\omega_i^k = \omega_i^j \wedge \omega_j^k, \qquad \omega_i^i = 0,$$

and the forms $\omega_\beta^\alpha(\alpha \neq \beta)$ are the prominent forms of a moving frame. Identifying a top of a moving frame **M** with a point of variety Φ_n then the forms ω^α, ω^μ become the principal ones and so far at this point the vectors of a moving frame $e_1, \ldots, e_n$ belong to the tangent n-plane of variety Φ_n at this point and then

$$\omega^\mu = 0. \tag{3}$$

By exterior differentiation of (3) and using the Cartan's lemma we come to

$$\omega_\alpha^\mu = R_{\alpha\beta}^\mu \omega^\beta, \qquad R_{\alpha\beta}^\mu = R_{\beta\alpha}^\mu, \tag{4}$$

yielding for variation relative to the secondary parameters

$$\delta R_{\alpha\beta}^\mu = R_{\gamma\beta}^\mu \pi_\alpha^\gamma + R_{\alpha\gamma}^\mu \pi_\beta^\gamma - R_{\alpha\beta}^\nu \pi_\nu^\mu. \tag{5}$$

Let us look for a focal hyperplane Γ of a tangent n-plane of variety Φ_n. Let

$$\mathbf{Y} = \mathbf{M} + X^\alpha \mathbf{e}_\alpha$$

be an arbitrary point of a tangent n-plane $(\mathbf{M}, \mathbf{e}_1, \ldots, \mathbf{e}_n)$. Thus from the condition in a form $\mathbf{Y} \in \Gamma$. $\mathrm{d}Y \in \Gamma$ we obtain

$$\mathrm{d}\mathbf{Y} = X^\alpha \mathbf{e}_\alpha + X^{n+1} \mathbf{e}_\varrho, \quad \text{where} \quad \mathbf{e}_\varrho = \mathbf{e}_{n+1} - t\,\mathbf{e}_{n+2}.$$

We can rewrite this condition in a form

$$x^\beta \omega_\beta^{n+2} + t x^\beta \omega_\beta^{n+1} = 0. \tag{6}$$

Substituting (4) into (6) we get

$$x^\beta R_{\beta\alpha}^{n+2} \omega^\alpha + t x^\beta R_{\beta\alpha}^{n+1} \omega^\alpha = 0.$$

This equation must be satisfied for an arbitrary point of the tangent n-plane. Hence we get for ω^α a system of n homogeneous equations of a type

$$\omega^\alpha (R_{\alpha\beta}^{n+2} + t R_{\alpha\beta}^{n+1}) = 0.$$

For a nontrivial solution of this system it is necessary and sufficient that

$$\det \| R_{\alpha\beta}^{n+2} + t R_{\alpha\beta}^{n+1} \| = 0,$$

which leads to an equation

$$\binom{n}{0} R^{n+1,\ldots,n+1} t^n + \binom{n}{1} R^{n+1,\ldots,n+1,n+2} t^{n-1} + \ldots$$
$$\ldots + \binom{n}{n-1} R^{n+2,\ldots,n+2,n+1} t + \binom{n}{n} R^{n+2,\ldots,n+2} = 0,$$

where

$$R^{\mu_1\mu_2\ldots\mu_n} = \frac{\omega_1^{(\mu_1} \wedge \omega_2^{\mu_2} \wedge \ldots \wedge \omega_n^{\mu_n)}}{\omega^1 \wedge \omega^2 \wedge \ldots \wedge \omega^n} = \det \| R_{1\beta}^{(\mu^1} R_{2\beta}^{\mu_2} \ldots R_{n\beta}^{\mu_n)} \| . \tag{7}$$

The round brackets at superscripts express the symmetrization according to these indexes.

From (5) we obtain for $\delta R^{\mu_1\ldots\mu_n}$

$$-\delta R^{\mu_1\ldots\mu_n} = -2R^{\mu_1\ldots\mu_n}\pi_\alpha^\alpha + R^{\nu\mu_2\ldots\mu_n}\pi_\nu^{\mu_1} + \ldots + R^{\mu_1\ldots\mu_{n-1}\nu}\pi_\nu^{\mu_n}.$$

On the assumption that

$$\mathbf{R} = (n-1)^2 R^{n+1,\ldots,n+1,n+2,n+2} \, . \, R^{n+2,\ldots,n+2,n+1,n+1} - \\ - R^{n+1,\ldots,n+1} \, . \, R^{n+2,\ldots,n+2} \neq 0, \; ^*)$$

following specialization may be performed

$$R^{\mu\ldots\mu\nu} = 0, \qquad R^{n+1,\ldots,n+1} = R^{n+2,\ldots,n+2} \neq 0, \qquad \mu \neq \nu. \tag{8}$$

A geometric characterization of specialization (8).

From (7) we get for roots $t_1, \ldots, t_n$ following relations

$$t_1 + t_2 + \ldots + t_n = -n \frac{R^{n+1,\ldots,n+1,n+2}}{R^{n+1,\ldots,n+1}} = 0,$$

$$t_1t_2 + t_1t_3 + \ldots + t_{n-1}t_n = \binom{n}{2} \frac{R^{n+1,\ldots,n+1,n+2,n+2}}{R^{n+1,\ldots,n+1}},$$

$$t_1t_2\ldots t_n = (-1)^n \frac{R^{n+2,\ldots,n+2}}{R^{n+1,\ldots,n+1}} = (-1)^n,$$

$$t_1^{-1} + t_2^{-1} + \ldots + t_n^{-1} = -n \frac{R^{n+2,\ldots,n+2,n+1}}{R^{n+1,\ldots,n+1}} = 0.$$

We denote with Γ_ν the hyperplane $\Gamma_\nu = (\mathbf{M}, \mathbf{e}_1, \ldots, \mathbf{e}_n, \mathbf{e}_\nu)$. w_s is the double ratio

$$w_s = DV(\Gamma_{n+1}, \Gamma_{n+2}, \Gamma_1, \Gamma_{s+1}), \qquad s = 1, \ldots, n-1.$$

From the given relations between the roots of equation (7) we can find that the coordinate hyperplanes Γ_{n+1}, Γ_{n+2} are chosen so that the corresponding double ratios hold

$$w_1 + \ldots + w_{n-1} + 1 = 0, \quad w_1^{-1} + \ldots + w_{n-1}^{-1} + 1 = 0.$$

Excluding the case where a coordinate hyperplane is focal, then it results from these

---

***) In case $n = 2$ there is always $\mathbf{R} = 0$ and the specialization of this type is thus impracticable.**

relations that the hyperplane $\Gamma^* = (\mathbf{M}, \mathbf{e}_1, \dots, \mathbf{e}_n, \mathbf{e}_{n+1} + \mathbf{e}_{n+2})$ is chosen by specialization (8) so that

$$w_1^* \,.\, w_2^* \,.\, \dots \,.\, w_n^* = 1,$$

where

$$w_\alpha^* = DV(\Gamma_{n+1}, \Gamma_{n+2}, \Gamma^*, \Gamma_\alpha).$$

Let us come back to specialization (8). Thereby

$$\pi_\mu^\nu = 0, \qquad \nu \neq \mu \quad \text{and} \quad \pi_{n+1}^{n+1} = \pi_{n+2}^{n+2}.$$

The forms ω_μ^ν are the principal ones and we may write

$$\omega_\mu^\nu = R_{\mu\gamma}^\nu \omega^\gamma, \qquad \nu \neq \mu. \tag{9}$$

In a similar way as before specialization (8) and no the assumption that det $\| R_{\alpha\beta}^\nu \| = R^{\nu\dots\nu} \neq 0$ we can set

$$R_{\mu\beta}^\nu = 0, \qquad \mu \neq \nu. \tag{10}$$

(10) leads to the annihilation of forms π_μ^α and thus to

$$\omega_\mu^\alpha = R_{\mu\beta}^\alpha \omega^\beta. \tag{11}$$

A geometric singificance of specialization (10).

A hyperplane Γ_ν be given and let us look for a characteristic element of variety which represents the envelope of this hyperplane in the motion of the point M along the variety Φ_n. An arbitrary point of a characteristic element is given by

$$\mathbf{X} = \mathbf{M} + x^\alpha \mathbf{e}_\alpha + x^\lambda \mathbf{e}_\lambda.$$

As a point of this envelope it must satisfy the following two equations

$$(\mathbf{X} - \mathbf{M}, \mathbf{e}_1, \dots, \mathbf{e}_n, \mathbf{e}_\nu) = 0,$$
$$\mathrm{d}(\mathbf{X} - \mathbf{M}, \mathbf{e}_1, \dots, \mathbf{e}_n, e_\nu) = 0.$$

These two equations yield conditions

$$x^\mu = 0, \qquad x^\alpha \omega_\alpha^\mu + x^\nu \omega_\nu^\mu = 0, \qquad \mu \neq \nu.$$

After substitution (4) and (9) we obtain

$$\omega^\beta (x^\alpha R_{\alpha\beta}^\mu + x^\nu R_{\nu\beta}^\mu) = 0.$$

This system of equations must be satisfied in an arbitrary motion $\omega^1 : \omega^2 : \dots : \omega^n$ and we get for x^α, x^ν a system of n linear homogeneous equations

$$x^\alpha R_{\alpha\beta}^\mu + x^\nu R_{\nu\beta}^\mu = 0.$$

Since det $\| R_{\alpha\beta}^\mu \| \neq 0$ and (10) holds we get $x^\alpha = 0$.

The characteristic of the system of a hyperplane Γ_ν looked for is the line

$$\mathbf{X} = \mathbf{M} + x^\nu \mathbf{e}_\nu \quad \text{(do not add with respect to } \nu!)$$

Thus a geometric characterization of vectors $\mathbf{e}_{n+1}$, $\mathbf{e}_{n+2}$ can be determined. Let us have the line determined by points $\mathbf{E}_{n+1} = \mathbf{M} + \mathbf{e}_{n+1}$ and $\mathbf{E}_{n+2} = \mathbf{M} + \mathbf{e}_{n+2}$. This line is intersecting the hyperplane Γ^* in a point $\mathbf{E} = (0, \ldots, 0, 1/2, 1/2)$ determining the mutual norms of vectors $\mathbf{e}_{n+1}$, $\mathbf{e}_{n+2}$.

Let us come back to the specialization of a moving frame. By exterior differentiation of (11) we get for a variation $\delta R^\alpha_{\mu\gamma}$ following relations

$$\delta R^\alpha_{\mu\gamma} - R^\alpha_{\mu\beta}\pi^\beta_\gamma + R^\beta_{\mu\gamma}\pi^\alpha_\beta - R^\alpha_{\nu\gamma}\pi^\nu_\mu = 0.$$

Let us seek a focus of a line $(\mathbf{M}\mathbf{e}_\nu)$ (ν is fixed). If

$$\mathbf{X} = \mathbf{M} + x^\nu \mathbf{e}_\nu$$

is a focus of a line $(\mathbf{M}\mathbf{e}_\nu)$, in an arbitrary motion $\omega^1 : \omega^2 : \ldots : \omega^n$ then there must hold

$$\mathrm{d}\mathbf{X} \parallel \mathbf{e}_\nu,$$

yielding a system of equations

$$\begin{aligned}
&\omega^1(1 + R^1_{\nu 1}x^\nu) + \omega^2 R^1_{\nu 2}x^\nu + \ldots + \omega^n R^1_{\nu n}x^\nu = 0,\\
&\omega^1 R^2_{\nu 1}x^\nu + \omega^2(1 + R^2_{\nu 2}x^\nu) + \ldots + \omega^n R^2_{\nu n}x^\nu = 0,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\omega^1 R^n_{\nu 1}x^\nu + \omega^2 R^n_{\nu 2}x^\nu + \ldots + \omega^n(1 + R^n_{\nu n}x^\nu) = 0.
\end{aligned}$$

Then we get for the foci

$$\begin{vmatrix}
1 + R^1_{\nu 1}x^\nu & R^1_{\nu 2}x^\nu \ldots & R^1_{\nu n}x^\nu \\
R^2_{\nu 1}x^\nu & 1 + R^2_{\nu 2}x^\nu \ldots & R^2_{\nu n}x^\nu \\
\cdots & \cdots & \cdots \\
R^n_{\nu 1}x^\nu & R^n_{\nu 2}x^\nu \ldots & 1 + R^n_{\nu n}x^\nu
\end{vmatrix} = 0.$$

We come now to an equation

$$\mathbf{A}^\nu_n (x^\nu)^n + \mathbf{A}^\nu_{n-1}(x^\nu)^{n-1} + \ldots + \mathbf{A}^\nu_1 x^\nu + \mathbf{A}^\nu_0 = 0,$$

where

$$\begin{aligned}
&\mathbf{A}^\nu_n = \det \| R^\beta_{\nu 1} R^\beta_{\nu 2} \ldots R^\beta_{\nu n} \|,\\
&\vdots\\
&\mathbf{A}^\nu_1 = R^1_{\nu 1} + R^2_{\nu 2} + \ldots + R^n_{\nu n} = R^\alpha_{\nu\alpha},\\
&\mathbf{A}^\nu_0 = 1.
\end{aligned}$$

Thus for the variations of these coefficients we get

$$\begin{aligned}
&\delta\mathbf{A}^\nu_n = n\mathbf{A}^\nu_n \pi^\nu_\nu,\\
&\vdots\\
&\delta\mathbf{A}^\nu_1 = \mathbf{A}^\nu_1 \pi^\nu_\nu. \qquad \text{(do not add with respect to } \nu!)
\end{aligned}$$

Now the following specialization may be performed

$$\mathbf{A}_n^{n+1} \,.\, \mathbf{A}_n^{n+2} = 1. \tag{13}$$

By this we attain to

$$\pi_{n+1}^{n+1} = \pi_{n+2}^{n+2} = 0. \tag{14}$$

The expression $(-1)^n$. $1/\mathbf{A}_n^\nu$ is a product of coordinates of the foci on a line $(\mathbf{M}\mathbf{e}_\nu)$. In our specialization the norm of vectors $\mathbf{e}_{n+1}$, $\mathbf{e}_{n+2}$ is chosen such that the product of coordinates of the foci be equal to 1 on both lines.

At this stage the moving frame is dependent on n-l secondary not prominent parameters. Now, we shall fix these parameters as well.

Let us have a hyperplane determined by a point $\mathbf{E}_\beta$ where $\mathbf{E}_\beta = \mathbf{M} + \mathbf{e}_\beta$ and the vectors $\mathbf{e}_{\gamma_1}, \mathbf{e}_{\gamma_2}, \ldots, \mathbf{e}_{\gamma_{n-2}}, \mathbf{e}_\nu, \mathbf{e}_\mu$, $d(\mathbf{e}_\nu + \mathbf{e}_\mu)$, where $\beta, \gamma_1, \ldots, \gamma_{n-2}, \nu, \mu$ are mutually different indexes.

Let us look for the point of intersection of this hyperplane with a line $(\mathbf{M}\mathbf{e}_\alpha)$ in the motion

$$\omega^\alpha = \omega^\beta = \omega^{\gamma_1} = \ldots = \omega^{\gamma_{n-3}} = 0,$$

where α is different from all the given indexes.

If $\mathbf{X} = \mathbf{M} + t\mathbf{e}_\alpha$ is the point of intersection looked for, then t can be evaluated from the equation

$$t(R^\beta_{\nu\gamma_{n-2}} + R^\beta_{\mu\gamma_{n-2}}) + (R^\alpha_{\nu\gamma_{n-2}} + R^\alpha_{\mu\gamma_{n-2}}) = 0.$$

Now we set

$$R^\beta_{\nu\gamma_{n-2}} + R^\beta_{\mu\gamma_{n-2}} = R^\alpha_{\nu\gamma_{n-2}} + R^\alpha_{\mu\gamma_{n-2}},$$

for the following series of values

α	β	γ_1	γ_2		γ_{n-2}
n	1	2	3	....	$n-1$
$n-1$	n	1	2	....	$n-2$
$n-2$	$n-1$	n	1	....	$n-3$
......	......	......	......	....	......
3	4	5	6	....	2
2	3	4	5	............n	1

Then the point of intersection is

$$\mathbf{X} = \mathbf{M} - \mathbf{e}_\alpha \qquad \alpha = 2, 3, \ldots, n.$$

By specialization (15) it may be shown that the forms $\pi_1^1, \ldots, \pi_n^n$ may be expressed as a linear combination of forms π_β^α, $\alpha \neq \beta$ having coefficients $R^\alpha_{\mu\beta}$. Our specialization is thus completed.

The semicanonical moving frame of variety Φ_n in $\mathbf{A}^{n+2}$ is given by the following system of differential equations

$$\mathrm{d}m = \omega^\alpha \mathbf{e}_\alpha, \qquad d\mathbf{e}_i = \omega_i^j \mathbf{e}_j,$$
$$\omega^\mu = 0, \qquad \omega_\alpha^\mu = R_{\alpha\beta}^\mu \omega^\beta, \qquad \omega_\nu^\mu = R_{\nu\alpha}^\mu \omega^\alpha,$$

where

$$R_{\alpha\beta}^\mu = R_{\beta\alpha}^\mu, \qquad R^{\mu\mu\ldots\mu\nu} = 0, \qquad R^{n+1,\ldots,n+1} = R^{n+2,\ldots,n+2},$$
$$R_{\mu\alpha}^\nu = 0, \qquad R_{\alpha\beta}^\nu R_{\mu\gamma}^\alpha - R_{\alpha\gamma}^\nu R_{\mu\beta}^\alpha = 0 \quad \text{for} \quad \nu \neq \mu,\ \beta \neq \gamma$$
$$\mathbf{A}_n^{n+1} \,.\, \mathbf{A}_n^{n+2} = 1$$

and

$$R_{\nu\gamma_{n-2}}^\beta + R_{\mu\gamma_{n-2}}^\beta = R_{\nu\gamma_{n-2}}^\alpha + R_{\mu\gamma_{n-2}}^\alpha,$$

for series values in (15).

The solution of this system is dependent on $n^2 - n + 2$ function of n arguments.

REFERENCES:

[1] *Marková Libuše:* Reporáž systému anholonomních subvariet trojrozměrné variety v pětirozměrném ekviafinním prostoru. Matematický časopis 22 (1972), No. 1.

[2] *Ивлев Е. Т., Лучинин А. А.:* О репераже поверхности Φ_n в P^{n+2} ($n \geqq 2$), Известия высших учебных заведений, мат. № 9 64, 1967.

[3] *Щербаков Р. Н.:* Курс аффиной и проективной дифференциальной геометрии, Томск 1960.

[4] *Kolář I:* Užití Cartanových metod ke studiu obecné sítě křivek na ploše v trojrozměrném projektivním prostoru. Rozpravy ČSAV, 77, 1967.

[5] *Svoboda K., Havel V., Kolář I.:* La métode du repérage des systémes de sous-variétés, Comm. Math. Univ. Carolinae 5 (1964) str. 183—201.

SOUHRN

REPERÁŽ SYSTÉMU ANHOLONOMNÍCH SUBVARIET N-ROZMĚRNÉ VARIETY V N+2-ROZMĚRNÉM EKVIAFINNÍM PROSTORU

LIBUŠE MARKOVÁ

V článku se uvádí konstrukce polokanonického reperu soustavy subvariet dané variety Φ_n v ekviafinním prostoru $\mathbf{A}^{n+2}$. Konstrukce se provádí Cartanovou metodou. Geometricky je reper charakterizován takto. Vektory $\mathbf{e}_1, \ldots, \mathbf{e}_n$ patří do zaměření tečné n-roviny v bodě **M** variety Φ_n. Souřadné nadroviny $\Gamma_\mu = (\mathbf{M}, \mathbf{e}_1, \ldots, \mathbf{e}_n, \mathbf{e}_\mu)$, $\mu = n + 1, n + 2$ jsou zvoleny tak, aby byly splněny rovnice

$$w_1 + \ldots + w_{n-1} + 1 = 0, \qquad w_1^{-1} + \ldots + w_{n-1}^{-1} + 1 = 0,$$

kde w_s je dvojpoměr nadrovin $\Gamma_{n+1}, \Gamma_{n+2}, \Gamma_1, \Gamma_{s+1}$ v daném pořadí a Γ_α, $\alpha = 1, \ldots, n$ je fokální nadrovina. Vektor $\mathbf{e}_{n+1}$, resp. $\mathbf{e}_{n+2}$ určuje směr charakteristiky obálky nadroviny Γ_{n+1}, resp. Γ_{n+2} při libovolném pohybu po varietě.

РЕЗЮМЕ

О РЕПЕРАЖЕ СИСТЕМ НЕГОЛОНОМНЫХ ПОДМНОГООБРАЗИЙ n-МЕРНОЙ ПОВЕРХНОСТИ В $n+2$-МЕРНОМ ЭКВИАФФИННОМ ПРОСТРАНСТВЕ

ЛИБУШЕ МАРКОВА

В статье приводится конструкция полуканонического пепера системи подмногообразий данного многообразия Φ_n в эквиаффинном пространстве $\mathbf{A}^{n+2}$. Конструкция построена методом Картана. Геометрически этот репер характеризуется следующим способом. Векторы $\mathbf{e}_1, \ldots, \mathbf{e}_n$ направлены по касательной n — плоскости в данной точке **M** многообразия Φ_n. Координатные гиперплоскости $\Gamma_n = (\mathbf{M}, \mathbf{e}_1, \ldots, \mathbf{e}_n, \mathbf{e}_\mu)$, $n = n + 1, n + 2$ выбраны таким образом, чтобы имели место уравнения

$$w_1 + \ldots + w_{n-1} + 1 = 0, \qquad w_1^{-1} + \ldots + w_{n-1}^{-1} + 1 = 0,$$

где w_s — двойное отношение гиперплоскостей $\Gamma_{n+1}, \Gamma_{n+2}, \Gamma_1, \Gamma_{s+1}$ в данном порядке и Γ_α, $\alpha = 1, \ldots, n$ является фокальной гиперплоскостью. Векторы $\mathbf{e}_{n+1}, \mathbf{e}_{n+2}$ определяют направление характеристик огибающих гиперплоскостей $\Gamma_{n+1}, \Gamma_{n+2}$ при любом движении по многообразии.