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OPTICAL SYNTHESIS OF HOLOGRAMS

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To 65th Birthday of Professor B. Havelka

Introduction

The superposition of holograms in one photographic plate was proposed by Leith and Upatnieks [1], who utilized different orientations of the reference beam at the individual exposures with the aim to separately reconstruct each of the images. Gabor et al. [2] carried out the superposition of holograms with one reference beam permitting the holographic interferometry and the image synthesis.

Under the term optical synthesis of holograms [3—5] we understand a process which makes it possible to get a hologram of an object by superposition of component holograms of the individual points (generally parts) of an object. As we want to get an image of the entire object in the reconstruction, we must utilize the same reference beam in each exposition. In the photographic plate we make the incoherent superposition of component holograms. In principle, the method of the optical synthesis of holograms is similar to the incoherent holography [6], where each point of the object creates its own fringe pattern and these patterns are superposed incoherently.

1. Theory of the optical synthesis of holograms

Let us suppose that the object whose hologram we want to get is composed of N points. Similarly as at the computer synthesis of holograms [7] it is not necessary that the object exist physically, its mathematical form is sufficient. A point of the object is represented by the aperture consisting of the lens-pinhole assembly. One position of the aperture corresponds then to one point of the object and the movement of the aperture may be controlled by the computer. Reference beam is the same for all the exposures. A schematic recording system is shown in Fig. 1. The wave coming from the n-th object point is described by the complex amplitude of the electric field vector $E_n(x, y, z)\, e^{i\omega t}$. Temporal factor $e^{i\omega t}$ may be omitted, because it is unobservable in optical experiments. So it is sufficient to write only a phasor $E_n(x, y, z)$. Similarly the reference wave is described by the phasor $E_r(x, y, z)$. Photographic plate placed in the plane H records the intensity

$$I_n(x, y, z) = (\mathbf{E}_r + \mathbf{E}_n)(\mathbf{E}_r + \mathbf{E}_n)^\star . \tag{1}$$

This is the intensity distribution representing a component hologram of the n-th object point. Carrying out such an exposure for every object point we obtain the resultant intensity by superposition of intensities due to the individual object points, i. e.

$$I(x, y, z) = \sum_{n=1}^{N} |\mathbf{E}_n|^2 + N |\mathbf{E}_r|^2 + \mathbf{E}_r^* \sum_{n=1}^{N} \mathbf{E}_n + \mathbf{E}_r \sum_{n=1}^{N} \mathbf{E}_n^* . \tag{2}$$

For simplification the exposure time $t_n = 1$ was supposed in all exposures. Equations (1) and (2) apply to all the known types of holograms. From Eq. (2) one can easily see that if one illuminates the hologram with a wave $\mathbf{E}_r$, one will simultaneously reconstruct from its third term the sum $\Sigma \mathbf{E}_n$ of the waves corresponding to the individual component object points or elements, thus reconstructing the entire object.

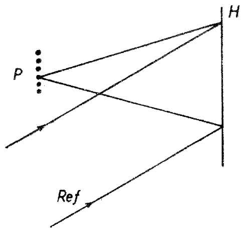

Fig. 1

Recording all the object points simultaneously, we obtain for the intensity distribution at the hologram

$$I(x, y, z) = |\mathbf{E}_r|^2 + \sum_{n=1}^{N} |\mathbf{E}_n|^2 + \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m \neq n}}^{N} \mathbf{E}_n^* \mathbf{E}_m + \mathbf{E}_r^* \sum_{n=1}^{N} \mathbf{E}_n + \mathbf{E}_r \sum_{n=1}^{N} \mathbf{E}_n^* . \tag{3}$$

By comparison of expressions (2) and (3) we can see that in the simultaneous exposure case we obtain in the zeroth maximum one more term arising as a result of the mutual interference between the individual object points. In the successive exposure case this term is absent, because the waves from the individual object points are superposed incoherently. The zeroth maximum will have in this case much smaller spatial extent, which makes it possible to place the object nearer to the reference point.

An important factor determining the luminance of the reconstructed image is the diffraction efficiency of holograms. The diffraction efficiency is proportional to the square of visibility of the holographic interference pattern, which is given by the following expression

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} . \tag{4}$$

For the successive exposure case we obtain the visibility of the n-th component from Eq. (2)

$$V_n = \frac{2K\,|\mathbf{E}_r|\,|\mathbf{E}_n|}{N\,|\mathbf{E}_r|^2 + \sum\limits_{n=1}^{N} |\mathbf{E}_n|^2}\,, \tag{5}$$

where K is a constant including the influence of the photographic film. For N component holograms the diffraction efficiency is proportional to

$$V^2 = \frac{4\,NK^2\,|\mathbf{E}_r|^2\,|\mathbf{E}_n|^2}{\left(N\,|\mathbf{E}_r|^2 + \sum\limits_{n=1}^{N}|\mathbf{E}_n|^2\right)^2}. \tag{6}$$

Choosing $|\mathbf{E}_r| > |\mathbf{E}_n|$ we can see that V^2 decreases with the increasing number of component holograms N. For the simultaneous recording the diffraction efficiency is independent of N.

2. Possibilities of increasing the diffraction efficiency of multiply exposed holograms

As shown in the preceding chapter, the diffraction efficiency of multiply exposed holograms decreases with the number of component exposures of the photographic plate, supposing the beam balance ratio the same as that recommended for single exposure holography to assure a linear recording. In this way the diffraction efficiency limits the number of component holograms superposed at one photographic plate. Stroke et al. [4] report a successful reconstruction from the hologram recorded by superposition of 1000 component exposures. Similarly Caulfield et al. [8] report the superposition of 4000 component exposures. Unfortunately, the authors do not mention either the diffraction efficiency of these holograms or the beam balance ratio. A recontruction of image from the hologram obtained by superposition of 500 component holograms of digital numbers is shown in ref. [9]. The authors report the diffraction efficiency 0,001, but they do not mention the beam balance ratio in the recording.

The diffraction efficiency of multiply exposed hologram can be increased if we expose only a part of the photographic plate (e. g. M-th part) at the individual exposures. A suitable random mask may be placed before the hologram and displaced after every component exposure. The diffraction efficiency of such a hologram will be increased M-times. This is the same as the recording of information in blocks of M points.

Another method of increasing the diffraction efficiency of multiply exposed holograms is based on the hologram copying. The real image is used for construction of a new hologram, which will be of the focused image type. This hologram will have the diffraction efficiency independent of the original hologram. Moreover, it can be reconstructed in white light. For the reconstruction all the spectral band of white light can be utilized, not only one spectral component as in other white light holograms.

Further method of increasing the diffraction efficiency of multiply exposed holograms makes use of the suitable choice of the beam balance ratio at the individual exposures. Stroke et al. [4] found a value 5:1 as a most suitable for the optical synthesis of holograms. The same value is recommended for recordng of single exposure holograms. A detailed theoretical analysis [10] shows, however, that this is not the optimum choice for the multiple exposure holography. For spherical object waves the value of beam balance ratio of one makes it possible to record multiple exposure holograms with diffraction efficiency equal to that of single exposure holograms.

3. Methods of the optical synthesis of holograms

A general arrangement for the optical synthesis of holograms was proposed by Spitz and Werts [3] The authors worked out the theoretical principles of the method with the aim of visualization of functions of three variables.

Stroke et al. [4] proposed a set-up for the optical synthesis of Fourier holograms which is shown in Fig. 2. Laser beam is expanded by the telescopic system. A combination of microscope lens M_1 and pinhole is used to produce the reference wave. A microscope lens M_2 is placed is such a way that it can be

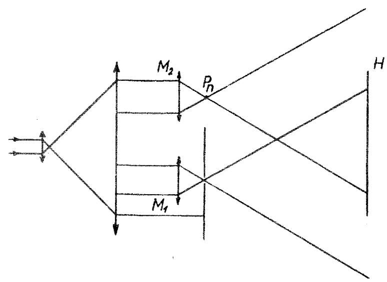

Fig. 2

movable in three perpendicular directions, allowing the realization of the individual object points. Every component exposure for given position of microscope lens M_2 then represents the individual point of the object. The arrangement introduced has one drawback, i. e. a considerable part of the light energy is lost. It is more suitable to use fiber optics to produce a movable object point, as it was shown in ref. [11].

Analog to the construction of the hologram of an object composed of points, we can synthesize the hologram of an object consisting of continuous lines, as shown in ref. [12]. In the hologram recording the aperture was used, which moved along the lines composing the object.

In some cases it is possible to use the following procedure. Successively the planes of the object given in the mathematical form are constructed and photographed at the individual slides. These slides are then recorded on the hologram by the method of multiple exposures. The method gives high quality images for spherical and parabolic surfaces.

Redman et al. [13] utilized the optical synthesis of holograms for the reconstruction of three-dimensional object from two-dimensional X-ray pictures. Every picture represents a certain perspective of the object and the recording on the hologram is done with the corresponding orientation of the photographic plate.

King et al. [14] proposed a similar method for three- dimensional visualization, as a modification of computer generated holography. The object to be displayed by holography is stored in the memory of a computer. The computer is used for calculation of different perspectives of the object, which are recorded on the individual slides. These slides are recorded on the hologram in that way, that every slide is recorded only on one vertical stripe of the photographic plate. The resulting hologram will have only a horizontal parallax, which is sufficient for the visual observation, because the observer seldom moves his head vertically. This method allows a considerable reduction of the computer time in comparison with the complete calculation of the hologram on the computer.

4. The use of birefringent elements for the optical synthesis of holograms

Ingelstam and Perrin [15] utilized the Wollaston prism for the generation of Young's fringes of various spatial frequencies. Besides, Lohmann [16] has shown how the birefringent elements can be employed to obtain holograms in spatially incoherent light. The use of birefringent elements for the optical synthesis of holograms was proposed in ref. [17]. The principle of the method is illustrated in Fig. 3. The point source S illuminates a photographic plate H

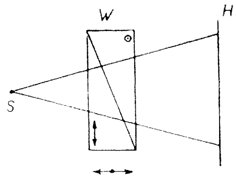

Fig. 3

through a Wollaston prism W. This element is placed between two polarizers which are fixed to it. These are not shown in Fig. 3. The exposure is made under these conditions. The source S being split into two by the Wollaston W, the plate H records Young's fringes since the two images of S are coherent. In order to assure a linear holographic recording it suffices that the two intensities be different and this is achieved by suitably orienting the two polarizers. Let us displace the Wollastion in a direction normal to H. The separation between the two images is no longer the same and the fringe frequency on H is modified. The plate is developed and is illuminated by a point source. One observes through the negative a central luminous point O (zeroth order) and two first order spectra formed of two points P_1 and P_2, as is shown in Fig. 4.

Consider the same experiment with the Wollaston prism having been rotated between two exposures. After developing, there is a point source seen through the negative. One observes the central point O (zeroth order) and the two spectra each composed of two luminous points P_1 and P_2 situated on a circumference with O as centre (Fig. 5). It is now evident that if one combines the longitudinal displacement and rotation of Wollaston, any type of plane object can be reconstructed by successive exposures. The source S being generally a laser, it is preferable to use a circular polarizer so that the intensities of the two images of S remain constant irrespective of Wollaston orientation. The circular polarizer is not necessary if we rotate the photographic plate instead of Wollaston.

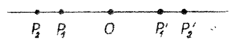

Fig. 4

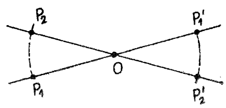

Fig. 5

For splitting the source S, the Wollaston prism may be replaced by the calcite-glass prism. The Rochon prism or its modification described in ref. [18] are suitable, too.

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SHRNUTÍ

OPTICKÁ SYNTÉZA HOLOGRAMŮ

JAROSLAV KVAPIL

V práci je odvozena teorie optické syntézy hologramů a jsou navrženy různé metody zvýšení difrakční účinnosti hologramů získaných uvedenou metodou. Dále jsou diskutovány různé metody optické syntézy hologramů. Nakonec je popsáno uspořádání pro optickou syntézu hologramů, využívající dvojlomných elementů jako je Wollastonův nebo Rochonův hranol.

SUMMARY

OPTICAL SYNTHESIS OF HOLOGRAMS

JAROSLAV KVAPIL

A theory of the optical synthesis of holograms is derived in this work and diverse methods of increasing the diffraction efficiency of holograms synthesized by the introduced method are proposed. We further discuss various methods of the optical synthesis of holograms. Finally, an arrangement for the optical synthesis of holograms is described, utilizing the birefringent elements such as Wollaston or Rochon prism.