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Transformace některých nelineárních diferenčních rovnic na nehomogenní lineární diferenční rovnici 1. řádu

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TRANSFORMACE NĚKTERÝCH NELINEÁRNÍCH DIFERENČNÍCH ROVNIC NA NEHOMOGENNÍ LINEÁRNÍ DIFERENČNÍ ROVNICI 1. ŘÁDU

VLADIMÍR VLČEK

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Dokazuje se věta o existenci a tvaru obecného řešení nelineární diferenční rovnice $\prod_{j=0}^{n} X(x+j) = P(x)$, $P(x) > O$, $n \in \mathbf{N}$ (pomocí její transformace na lineární nehomogenní diferenční rovnici n-tého řádu). Jsou uvedeny věty o transformaci a existenci obecného řešení jistých typů nelineárních diferenčních rovnic na lineární nehomogenní rovnici 1. řádu, kdy substituční funkcí příslušné transformace je převážně složená funkce výrazu $\prod_{j=0}^{n} X(x+j)$ resp. jeho diference.

Poznámka o užité symbolice: $\mathbf{N}$ značí množinu všech přirozených čísel, $\mathbf{N}^* = \mathbf{N} \cup (0)$; prvky množiny $\mathbf{N}^*$ ozn. x. $\mathbf{R}$ resp. $\mathbf{K}$ označuje obor všech reálných resp. komplexních čísel, $\mathbf{P}$ je prostor všech reálných funkcí definovaných na $\mathbf{N}^*$ (stacionární funkci zapisujeme místo $C(x)$ jen C, kde $C \in \mathbf{R}$, speciálně triviální funkci $O(x)$ též jen O). Jednoparametrický prostor všech funkcí $F(x)$ primitivních k $P(x)$ vyznačujeme $\rfloor P(x) = F(x) + C$, $C \in \mathbf{R}$, kde pro $F(x)$ platí: $\Delta F(x) = F(x+1) - F(x) = P(x)$. VĚTY jsou většinou formulovány pomocí znaků obvykle užívaných v logice: symbol $\wedge$ resp. $\vee$ znamená konjunkci resp. disjunkci výroků, $\Rightarrow$ resp. $\Leftrightarrow$ implikaci resp. ekvivalenci mezi výroky. $\exists$ resp. $\forall$ je existenční resp. obecný kvantifikátor; znakem $\rightarrow$ rozumíme rčení „tak (-ový, -ová, -ové), že“, dvojtečka ve výrokové funkci značí „platí“.

Pomocná věta:

Buď $P(x) \in \mathbf{P}$, $x \in \mathbf{N}^*$, taková, že pro $\forall (x \in \mathbf{N}^*)$: $[P(x) > 0]$.
Pak obecným řešením nelineární diferenční rovnice

$$\prod_{j=0}^{n} X(x+j) = P(x), \qquad n \in \mathbf{N}$$

je n-parametrický prostor $X(x, C_1, \ldots, C_n) \subset \mathbf{P}$, $C_i \in \mathbf{R}$, $i = 1, \ldots, n$, všech kladných funkcí tvaru

$$X(x, C_1, \ldots, C_n) = \exp \sum_{j=1}^{n} Z_j(x)\, Y_j(x),$$

kde $Y_j(x)$, $j = 1, \ldots, n$, tvoří bázi prostoru všech řešení lineární homogenní diferenční rovnice $\sum_{i=0}^{n} \binom{n+1}{i} \Delta^{n-i} Y_j(x) = 0$ a kde $Z_j(x) = Z_j(x, C_1, \ldots, C_n) \subset \subset \mathbf{P}$, $C_i \in \mathbf{R}$, $i = 1, \ldots, n$, vyhovují lineární nehomogenní diferenční rovnici n-tého řádu

$$\sum_{i=0}^{n} \binom{n+1}{i} \Delta^{n-i} [\sum_{j=1}^{n} Z_j(x)\, Y_j(x)] = \ln P(x).$$

Důkaz: Označíme-li $Y(x) = \ln X(x)$, pak $\left\{\prod_{i=0}^{n} X(x+i) = P(x)\right\} \Leftrightarrow \left\{\sum_{i=0}^{n} Y(x+i) = = \ln P(x)\right\} \Leftrightarrow \left\{\sum_{i=0}^{n} \binom{n+1}{i} \Delta^{n-i} Y(x) = \ln P(x)\right\}$, neboť $\left\{\left(\sum_{i=0}^{n} Y(x+i) = Y(x+n) + + Y(x+n-1) + \ldots + Y(x)\right) \wedge \left(\Delta^n Y(x) = Y(x+n) - \binom{n}{1} Y(x+n-1) + + \binom{n}{2} Y(x+n-2) - \ldots + (-1)^n Y(x)\right) \wedge \left(\Delta^{n-1} Y(x) = Y(x+n-1) - - \binom{n-1}{1} Y(x+n-2) + \binom{n-1}{2} Y(x+n-3) - \ldots + (-1)^{n-1} Y(x)\right) \wedge \left(\Delta^{n-2} Y(x) = = Y(x+n-2) - \binom{n-2}{1} Y(x+n-3) + \binom{n-2}{2} Y(x+n-4) - \ldots + + (-1)^{n-2} Y(x)\right) \wedge \ldots \wedge \left(\Delta Y(x) = Y(x+1) - Y(x)\right)\right\} \Rightarrow \left\{\sum_{i=0}^{n} Y(x+i) = \Delta^n Y(x) + + \binom{n}{1} Y(x+n-1) - \binom{n}{2} Y(x+n-2) + \ldots - (-1)^n Y(x) + \Delta^{n-1} Y(x) + + \binom{n-1}{1} Y(x+n-2) - \binom{n-1}{2} Y(x+n-3) + \ldots - (-1)^{n-1} Y(x) + + \Delta^{n-2} Y(x) + \binom{n-2}{1} Y(x+n-3) - \binom{n-2}{2} Y(x+n-4) + \ldots - - (-1)^{n-2} Y(x) + \ldots + \Delta Y(x) + Y(x) + Y(x) = \Delta^n Y(x) + \binom{n+1}{1} \Delta^{n-1} Y(x) + + \binom{n+1}{2} \Delta^{n-2} Y(x) + \ldots + \binom{n+1}{n} Y(x) = \sum_{i=0}^{n} \binom{n+1}{i} \Delta^{n-i} Y(x)\right\}$ a platí:

$\forall \left(\sum_{i=0}^{n} \binom{n+1}{i} \Delta^{n-i} Y(x) = 0\right) \exists \left(\sum_{j=0}^{n} z^j = 0 \wedge z \in \mathbf{K} \wedge z \neq 0 \wedge z \neq 1\right) \rightarrow \rightarrow \forall \left(z \in \mathbf{K} \wedge \sum_{j=0}^{n} z^j = 0\right) : [z^{n+1} - 1 = 0 \wedge |z| = 1]$, neboť pro $\forall (n \in \mathbf{N})$: $\left[z^{n+1} - 1 = = (z-1) \sum_{j=0}^{n} z^j\right] \wedge \forall (k = 0, 1, \ldots, n-1 \wedge n > 1 \wedge n \in \mathbf{N})$: $\left[z_k = \cos \frac{k+1}{n} 2\pi + + i \sin \frac{k+1}{n} 2\pi \text{ kromě } z = 1\right]$; pro $n = 1$ je $z = -1$.

a) Je-li $n = 2m \wedge m \in \mathbf{N}$, pak pro $\Big\{\forall(i = 1, \ldots, n) : \Big[\sum_{j=0}^{n} z_i^j = 0 \wedge z_i \in \mathbf{K} \wedge z_1 =$ $= a_1 + ib_1 \wedge z_2 = a_1 - ib_1 \wedge z_3 = a_2 + ib_2 \wedge z_4 = a_2 - ib_2 \wedge \ldots \wedge z_{2m-1} = a_m +$ $+ ib_m \wedge z_{2m} = a_m - ib_m \wedge (a_1 \wedge \ldots \wedge a_m \wedge b_1 \wedge \ldots \wedge b_m \in \mathbf{R})\Big] \wedge \exists\Big(Y_1(x) = \cos\frac{2\pi}{n}x \wedge$ $\wedge\ Y_2(x) = \sin\frac{2\pi}{n}x \wedge Y_3(x) = \cos\frac{4\pi}{n}x \wedge Y_4(x) = \sin\frac{4\pi}{n}x \wedge \ldots \wedge Y_{2m-1}(x) =$ $= \cos\frac{n-1}{n}2\pi x \wedge Y_{2m}(x) = \sin\frac{n-1}{n}2\pi x\Big) \rightarrow \forall(j = 1, \ldots, 2m) \wedge \forall(x \in \mathbf{N}^*) :$ $: \Big[\sum_{i=0}^{2m}\binom{2m+1}{i}\Delta^{2m-i}Y_j(x) \equiv 0\Big]\Big\}$

b) Je-li $n = 2m + 1 \wedge m \in \mathbf{N}$, pak $\Big\{\exists\Big(Y_j(x) \in \mathbf{P} \wedge j = 1, \ldots, 2m+1 \wedge Y_1(x) =$ $= \cos\frac{2\pi}{n}x \wedge Y_2(x) = \sin\frac{2\pi}{n}x \wedge \ldots \wedge Y_{2m-1}(x) = \cos\frac{n-1}{n}2\pi x \wedge Y_{2m}(x) =$ $= \sin\frac{n-1}{n}2\pi x \wedge Y_{2m+1} = (-1)^x\Big) \rightarrow \forall(x \in \mathbf{N}^*) \wedge \forall(j = 1, \ldots, 2m+1) :$ $: \Big[\sum_{i=0}^{2m+1}\binom{2m+2}{i}\Delta^{2m+1-i}Y_j(x) \equiv 0\Big]\Big\}$

$\Big\{\forall(n \in \mathbf{N} \wedge n > 1) \wedge \forall(Y_j(x) \in \mathbf{P} \wedge j = 1, \ldots, n) \wedge \forall(x \in \mathbf{N}^*) : [W[Y_1(x+1), \ldots,$ $Y_n(x+1)] \neq 0]\Big\} \Rightarrow \Big\{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n)\ \exists\Big(Y(x) \in \mathbf{P} \wedge Y(x) = \sum_{j=1}^{n} Z_j(x)\,Y_j(x) \wedge$ $\wedge\ Z_j(x) = Z_j(x, C_j) = \int \frac{(-1)^{n+j}\ln P(x)\,W_j[Y_1(x+1), \ldots, Y_n(x+1)]}{W[Y_1(x+1), \ldots, Y_n(x+1)]} +$ $+ C_j \wedge W_j[Y_1(x+1), \ldots, Y_n(x+1)] =$

$$= \begin{vmatrix} Y_1(x+1) & \ldots Y_{j-1}(x+1) & Y_{j+1}(x+1) & \ldots Y_n(x+1) \\ \Delta Y_1(x+1) & \ldots \Delta Y_{j-1}(x+1) & \Delta Y_{j+1}(x+1) & \ldots \Delta Y_n(x+1) \\ \vdots & & & \\ \Delta^{n-2}Y_1(x+1) & \ldots \Delta^{n-2}Y_{j-1}(x+1) & \Delta^{n-2}Y_{j+1}(x+1) & \ldots \Delta^{n-2}Y_n(x+1) \end{vmatrix}\Bigg)$$

$\rightarrow \forall(x \in \mathbf{N}^*) : \Big[\sum_{i=0}^{n}\binom{n+1}{i}\Delta^{n-i}Y(x) \equiv \ln P(x)\Big] \wedge \exists\Big(X(x) \in \mathbf{P} \wedge X(x) =$ $= X(x, C_1, \ldots, C_n) = \exp\sum_{j=1}^{n} Z_j(x)\,Y_j(x)\Big) \rightarrow \forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n) \wedge \forall(n \in \mathbf{N}) \wedge$ $\wedge\ \forall(x \in \mathbf{N}^*) : \Big[\prod_{j=0}^{n} X(x+j) \equiv P(x)\Big]\Big\}$

Speciálně pro

1. $n = 1$:

$$\Big\{\exists(P(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) > 0] \wedge \exists\Big(\mathrm{J}\,\frac{\ln P(x)}{(-1)^{x+1}} \in \mathbf{P}\Big)\Big\} \Rightarrow \Big\{\forall(C \in \mathbf{R})\, \exists\Big(X(x) \in$$

$$\in \mathbf{P} \wedge X(x) = X(x, C) = \exp\Big\{\Big[\mathrm{J}\,\frac{\ln P(x)}{(-1)^{x+1}} + C\Big](-1)^x\Big\}\Big) \to \forall(x \in \mathbf{N}^*) :$$

$$: \Big[\prod_{i=0}^{1} X(x+i) \equiv P(x)\Big]\Big\}$$

2. $n = 2$:

$$\Big\{\exists(P(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) > 0] \wedge \exists\Big(\mathrm{J}\Big(\ln P(x) \,.\, \sin\frac{(x+1)2\pi}{3}\Big) \in \mathbf{P} \wedge$$

$$\wedge\, \mathrm{J}\Big(\ln P(x) \,.\, \cos\frac{(x+1)2\pi}{3}\Big) \in \mathbf{P}\Big)\Big\} \Rightarrow \Big\{\forall(C_i \in \mathbf{R} \wedge i = 1, 2)\, \exists\Big(X(x) \in \mathbf{P} \wedge X(x) =$$

$$= X(x, C_1, C_2) = \exp\Big[-\frac{2}{\sqrt{3}}\,\mathrm{J}\Big(\ln P(x) \,.\, \sin\frac{(x+1)2\pi}{3}\Big) + C_1\Big]\cos\frac{2\pi x}{3} +$$

$$+ \Big[\frac{2}{\sqrt{3}}\,\mathrm{J}\Big(\ln P(x) \,.\, \cos\frac{(x+1)2\pi}{3}\Big) + C_2\Big]\sin\frac{2\pi x}{3}\Big) \to \forall(C_i \in \mathbf{R} \wedge i = 1, 2) \wedge$$

$$\wedge\, \forall(x \in \mathbf{N}^*) : \Big[\prod_{i=0}^{2} X(x+i) \equiv P(x)\Big]\Big\}$$

3. $n = 3$:

$$\Big\{\exists(P(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) > 0] \wedge \exists\Big(\mathrm{J}\,\frac{\ln P(x)}{(-1)^{x+1}} \in \mathbf{P} \wedge \mathrm{J}\Big(\ln P(x) \,.\, \sin\frac{2x+3}{4}\pi\Big) \in$$

$$\in \mathbf{P} \wedge \mathrm{J}\Big(\ln P(x) \,.\, \cos\frac{2x+3}{4}\pi\Big) \in \mathbf{P}\Big)\Big\} \Rightarrow \Big\{\forall(C_j \in \mathbf{R} \wedge j = 1, 2, 3)\, \exists\Big(X(x) \in \mathbf{P} \wedge$$

$$\wedge\, X(x) = X(x, C_1, C_2, C_3) = \exp\Big\{\Big[\frac{1}{2}\,\mathrm{J}\,\frac{\ln P(x)}{(-1)^{x+1}} + C_1\Big](-1)^x + \Big[-\frac{\sqrt{2}}{2}\,\mathrm{J}\Big(\ln P(x) \,.$$

$$.\, \sin\frac{2x+3}{4}\pi\Big) + C_2\Big]\cos\frac{\pi x}{2} + \Big[\frac{\sqrt{2}}{2}\,\mathrm{J}\Big(\ln P(x) \,.\, \cos\frac{2x+3}{4}\pi\Big) + C_3\Big]\sin\frac{\pi x}{2}\Big\}\Big) \to$$

$$\to \forall(x \in \mathbf{N}^*) \wedge \forall(C_j \in \mathbf{R} \wedge j = 1, 2, 3) : \Big[\sum_{i=0}^{3} X(x+i) \equiv P(x)\Big]\Big\}$$

atd.

Poznámka:

$$\Big\{\Big[\prod_{i=0}^{k} X(x+n-i) = P(x)\Big] \wedge (n \wedge k \in \mathbf{N}) \wedge k \leqq n\Big\} \Leftrightarrow \Big\{\Big[\prod_{i=0}^{k} Y(x+i) = P(x)\Big] \wedge [Y(x) =$$

$$= X(x+n-k)\Big]\Big\}$$

Věta 1.:

$$\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall)x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists(\int \ln|1 - P(x)| \in$$
$$\in \mathbf{P} \wedge \int \frac{Q(x)\exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\mathrm{sgn}[1 - P(x)]\}^x} \in \mathbf{P}) \wedge \exists(U(x) = U(x, C) = \{\mathrm{sgn}[1 -$$
$$- P(x)]\}^x \exp \int \ln|1 - P(x)| [\int \frac{Q(x)\exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\mathrm{sgn}[1 - P(x)]\}^x} + C] \in \mathbf{P} \wedge$$
$$\wedge C \in \mathbf{R}) \to \forall(x \in \mathbf{N}^*) : [U(x) > 0]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in$$
$$\in \mathbf{N}) \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \ldots, C_n)) \to \forall(x \in \mathbf{N}^*) : [X(x + n + 1) +$$
$$+ (P(x) - 1) X(x) \equiv \frac{Q(x)}{\prod_{i=1}^{n} X(x + i)}]\}$$

Důkaz:

$$\{\exists(P(x) \wedge Q(x) \wedge X(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0 \wedge X(x) \neq 0]\} \Rightarrow$$
$$\Rightarrow \{(X(x + n + 1) + [P(x) - 1] X(x) = \frac{Q(x)}{\prod_{i=1}^{n} X(x + i)}) \Leftrightarrow (X(x + n + 1) X(x + n) .$$
$$. X(x + n - 1) \ldots X(x + 1) - X(x + n) X(x + n - 1) \ldots X(x + 1) X(x) [1 - P(x)] =$$
$$= Q(x)) \Leftrightarrow (\Delta \prod_{i=0}^{n} X(x + i) + P(x) \prod_{i=0}^{n} X(x + i) = Q(x)) \Leftrightarrow (\Delta U(x) + P(x) U(x) =$$
$$= Q(x) \wedge U(x) = \prod_{i=0}^{n} X(x + i))\} ;$$

$$\forall(C \in \mathbf{R}) \exists(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = \{\mathrm{sgn}[1 - P(x)]\}^x \exp \int \ln|1 - P(x)| .$$
$$. [\int \frac{Q(x)\exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\mathrm{sgn}[1 - P(x)]\}^x} + C]) \to \forall(x \in \mathbf{N}^*) : [\Delta U(x) + P(x) U(x) \equiv Q(x)];$$

$$\{\exists(C \in \mathbf{R}) \to \forall(x \in \mathbf{N}^*) : [U(x, C) > 0 \wedge \prod_{i=0}^{n} X(x + i) = U(x, C)]\} \Rightarrow \{\exists(Y_j(x) \in \mathbf{P} \wedge$$
$$\wedge j = 1, \ldots, n \wedge n \in \mathbf{N}) \to \forall(x \in \mathbf{N}^*) \wedge \forall(j = 1, \ldots, n) : [\sum_{i=0}^{n} \binom{n + 1}{i} \Delta^{n-i} Y_j(x) \equiv$$
$$\equiv 0 \wedge W[Y_1(x + 1), \ldots, Y_n(x + 1)] \neq 0] \wedge \forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n) \exists(Z_j(x) = Z_j(x,$$

$$C_1, \ldots, C_n) \in \mathbf{P}) \rightarrow \forall (x \in \mathbf{N}^*) : \left[\sum_{i=0}^{n} \left[\binom{n+1}{i} \Delta^{n-i} \left(\sum_{j=1}^{n} Z_j(x)\, Y_j(x) \right) \right] \equiv \ln U(x, C) \right] \wedge$$

$$\wedge\, \exists \left(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \ldots, C_n) = \exp \sum_{j=1}^{n} Z_j(x)\, Y_j(x) \right) \rightarrow \forall (x \in \mathbf{N}^*) :$$

$$: \left[X(x+n+1) + [P(x) - 1]\, X(x) \equiv \frac{Q(x)}{\prod_{i=1}^{n} X(x+i)} \right] \Bigg\} .$$

Poznámky:

1. $\{\exists (P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall (x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) \neq 0] \wedge \exists (\mathrm{J} Q(x) \in \mathbf{P}) \wedge$
$\wedge\, \exists (U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = \mathrm{J} Q(x) + C \wedge C \in \mathbf{R}) \rightarrow \forall (x \in \mathbf{N}^*) : [U(x, C) > 0]\} \Rightarrow$

$$\Rightarrow \Bigg\{ \forall (C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists (X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \ldots, C_n)) \rightarrow$$

$$\rightarrow \forall (x \in \mathbf{N}^*) : \left[X(x+n+1) - X(x) \equiv \frac{Q(x)}{\prod_{i=0}^{n} X(x+i)} \right] \Bigg\}$$

2. $\{\exists (P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall (x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) > 0]\} \Rightarrow \Big\{ \forall (C_j \in \mathbf{R} \wedge j =$

$$= 1, \ldots, n \wedge n \in \mathbf{N})\, \exists (X(x) \in \mathbf{P} \wedge X(x+1) = X(x+1, C_1, \ldots, C_n)) \rightarrow \forall (x \in \mathbf{N}^*) :$$

$$: \left[X(x+n+1) \equiv \frac{Q(x)}{\prod_{i=1}^{n} X(x+i)} \right] \Bigg\}$$

Věta 2.:

$$\Big\{ \exists (P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall (x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists \Big(\mathrm{J} \ln |1 - P(x)| \in$$

$$\in \mathbf{P} \wedge \mathrm{J} \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\, \{\operatorname{sgn} [1 - P(x)]\}^x} \in \mathbf{P} \Big) \wedge \exists \Big(U(x) \in \mathbf{P}) \wedge U(x) = \{\operatorname{sgn} [1 -$$

$$- P(x)]\}^x \exp \mathrm{J} \ln |1 - P(x)| \left[\mathrm{J} \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\, \{\operatorname{sgn} [1 - P(x)]\}^x} + C \right] \wedge C \in \mathbf{R} \Big) \rightarrow$$

$$\rightarrow \forall (x \in \mathbf{N}^*) : [U(x) = U(x, C) > 0] \Big\} \Rightarrow \Big\{ \exists (Y_j(x) \in \mathbf{P} \wedge j = 1, \ldots, n-1 \wedge$$

$$\wedge\, n \in \mathbf{N} \wedge n > 1) \rightarrow \forall (j = 1, \ldots, n-1) \wedge \forall (x \in \mathbf{N}^*) : \left[\sum_{i=0}^{n-1} \binom{n}{i} \Delta^{n-1-i} Y_j(x) \equiv \right.$$

$$\left. \equiv 0 \wedge W[Y_1(x+1), \ldots, Y_{n-1}(x+1)] \neq 0 \right] \wedge \forall (C_j \in \mathbf{R} \wedge j = 1, \ldots, n -$$

$$- 1)\, \exists (Z_j(x) = Z_j(x, C_1, \ldots, C_{n-1}) \in \mathbf{P}) \rightarrow \forall (x \in \mathbf{N}^*) : \left[\sum_{i=0}^{n-1} \binom{n}{i} \Delta^{n-1-i} \right.$$

$$\left(\sum_{j=0}^{n-1} Z_j(x)\, Y_j(x)\right) \equiv \ln \frac{1}{U(x)}\Bigg] \wedge \exists\big(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \dots, C_{n-1}) =$$
$$= \exp \sum_{j=1}^{n-1} Z_j(x)\, Y_j(x)\big) \rightarrow \forall(x \in \mathbf{N}^*) : [X(x) + [P(x) - 1]\, X(x+n) \equiv$$
$$\equiv Q(x) \prod_{i=0}^{n} X(x+i)]\Big\}$$

Důkaz:

$$\{\exists(P(x) \wedge Q(x) \wedge X(x) \in \mathbf{P})\ \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0 \wedge X(x) \neq 0]\} \Rightarrow$$
$$\Rightarrow \Bigg\{\Big(X(x) + [P(x) - 1]\, X(x+n) = Q(x) \prod_{i=0}^{n} X(x+i)\Big) \Leftrightarrow \big(X(x) - X(x+n) +$$
$$+ P(x)\, X(x+n) = Q(x) \prod_{i=0}^{n} X(x+i)\big) \Leftrightarrow \left(\frac{1}{\prod\limits_{i=1}^{n} X(x+i)} + \frac{1}{\prod\limits_{i=0}^{n-1} X(x+i)} + P(x)\,.\right.$$
$$\left.\cdot \frac{1}{\prod\limits_{i=0}^{n-1} X(x+i)} = Q(x)\right) \Leftrightarrow \left(\Delta \frac{1}{\prod\limits_{i=0}^{n-1} X(x+i)} - P(x) \frac{1}{\prod\limits_{i=0}^{n-1} X(x+i)} = Q(x)\right) \Leftrightarrow \Bigg(\Delta U(x) +$$
$$+ P(x)\, U(x) = Q(x) \wedge U(x) = \frac{1}{\prod\limits_{i=0}^{n-1} X(x+i)}\Bigg)\Bigg\}\ ;$$

$$\forall(C \in \mathbf{R})\ \exists\Big(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = \{\operatorname{sgn}[1 - P(x)]\}^x \exp \int \ln |1 - P(x)|\,.$$
$$.\left[\int \frac{Q(x) \exp(-\int \ln |1 - P(x)|)}{[1 - P(x)]\, \{\operatorname{sgn}[1 - P(x)]\}^x} + C\right]\Big) \rightarrow \forall(x \in \mathbf{N}^*) : [\Delta U(x) + P(x)\, U(x) \equiv Q(x)];$$

$$\Big\{\exists(C \in \mathbf{R}) \rightarrow \forall(x \in \mathbf{N}^*) : [U(x, C) > 0] \wedge \prod_{i=0}^{n-1} X(x+i) = \frac{1}{U(x, C)}\Big\} \Rightarrow \Big\{\exists(Y_j(x) \in$$
$$\in \mathbf{P} \wedge j = 1, \dots, n-1 \wedge n \in \mathbf{N} \wedge n > 1) \rightarrow \forall(x \in \mathbf{N}^*) \wedge \forall(j = 1, \dots, n-1) :$$
$$: \left[\sum_{i=0}^{n-1} \binom{n}{i} \Delta^{n-1-i} Y_j(x) \equiv 0 \wedge W[Y_1(x+1), \dots, Y_{n-1}(x+1)] \neq 0\right] \wedge \forall(C_j \in \mathbf{R} \wedge j =$$
$$= 1, \dots, n-1)\ \exists(Z_j(x) = Z_j(x, C_1, \dots, C_{n-1}) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : \Bigg[\sum_{i=0}^{n-1} \binom{n}{i} \Delta^{n-1-i}$$
$$\left(\sum_{j=1}^{n-1} Z_j(x)\, Y_j(x)\right) \equiv \ln \frac{1}{U(x, C)}\Bigg] \wedge \exists\big(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \dots, C_{n-1}) =$$
$$= \exp \sum_{j=1}^{n-1} Z_j(x)\, Y_j(x)\big) \rightarrow \forall(x \in \mathbf{N}^*) : [X(x) + [P(x) - 1]\, X(x+n) \equiv Q(x) \prod_{i=0}^{n} X(x+i)]\Big\}$$

Poznámky:

1. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) \neq 0] \wedge \exists(\mathrm{J}Q(x) \in \mathbf{P}) \wedge$
$\wedge \exists(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = \mathrm{J}Q(x) + C \wedge C \in \mathbf{R}) \rightarrow \forall(x \in \mathbf{N}^*) : [U(x, C) > 0]\} \Rightarrow$
$\Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n - 1 \wedge n \in \mathbf{N} \wedge n > 1)\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \ldots,$
$C_{n-1})) \rightarrow \forall(x \in \mathbf{N}^*) : [X(x) - X(x + n) \equiv Q(x) \prod_{i=0}^{n} X(x + i)]\}$

2. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) > 0]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge$
$\wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x + 1) = X(x + 1, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) \wedge$
$\wedge \forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n) : [X(x) \equiv Q(x) \prod_{i=0}^{n} X(x + i)]\}$

Věta 3.:

$$\Big\{\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists\Big(\mathrm{J} \ln |1 - P(x)| \in$$
$$\in \mathbf{P} \wedge \mathrm{J} \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\{\operatorname{sgn}[1 - P(x)]\}^x} \in \mathbf{P}\Big) \wedge \exists\Big(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) =$$
$$= \{\operatorname{sgn}[1 - P(x)]\}^x \exp \mathrm{J} \ln |1 - P(x)| \left[\mathrm{J} \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\{\operatorname{sgn}[1 - P(x)]\}^x} + C\right] \wedge$$
$$\wedge C \in \mathbf{R}\Big) \rightarrow \forall(x \in \mathbf{N}^*) : [U(x) = U(x, C) \neq 0] \wedge \exists(\mathrm{J} \ln |U(x)| \in \mathbf{P}) \wedge \exists(V(x) \in$$
$$\in \mathbf{P} \wedge V(x) = V(x, \bar{C}) = \bar{C}\{\operatorname{sgn} U(x)\}^x \exp \mathrm{J} \ln |U(x)| \wedge \bar{C} \in \mathbf{R} \wedge \bar{C} \neq 0) \rightarrow$$
$$\rightarrow \forall(x \in \mathbf{N}^*) : [V(x) = V(x, C, \bar{C}) > 0]\Big\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n - 1 \wedge n \in$$
$$\in \mathbf{N} \wedge n > 1)\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_{n-1})) \rightarrow \forall(x \in \mathbf{N}^*) :$$
$$: [X(x + n + 1) X(x) + [P(x) - 1] X(x + n) X(x + 1) \equiv Q(x) X(x) X(x + 1)]\}$$

Důkaz:

$$\{\exists(P(x) \wedge Q(x) \wedge X(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0 \wedge X(x) \neq 0]\} \Rightarrow$$
$$\Rightarrow \Big\{\forall(n \in \mathbf{N} \wedge n > 1) : \Big[(X(x + n + 1) X(x) + [P(x) - 1] X(x + n) X(x + 1) =$$
$$= Q(x) X(x) X(x + 1)) \Leftrightarrow \left(\frac{X(x + n + 1)}{X(x + 1)} + [P(x) - 1] \frac{X(x + n)}{X(x)} = Q(x)\right) \Leftrightarrow$$
$$\Leftrightarrow \left(\Delta \frac{X(x + n)}{X(x)} + P(x) \frac{X(x + n)}{X(x)} = Q(x)\right) \Leftrightarrow \Big(\Delta U(x) + P(x) U(x) = Q(x) \wedge U(x) =$$
$$= \frac{X(x + n)}{X(x)}\Big)\Big]\Big\};$$

$$\forall(C \in \mathbf{R})\, \exists\Big(U(x) \in \mathbf{P} \wedge U(x, C) = \{\operatorname{sgn}[1 - P(x)]\}^x \exp \mathrm{J} \ln |1 - P(x)| \,.$$

$$\cdot\left[\mathrm{J}\,\frac{Q(x)\exp\left(-\mathrm{J}\ln|1-P(x)|\right)}{[1-P(x)]\,\{\mathrm{sgn}\,[1-P(x)]\}^{x}}+C\right]\Big)\rightarrow\forall(x\in\mathbf{N}^{*}):[\Delta U(x)+P(x)\,U(x)\equiv Q(x)];$$

$$\left\{U(x)=\frac{X(x+n)}{X(x)}=\frac{X(x+n)\,X(x+n-1)\dots X(x+1)}{X(x+n-1)\dots X(x+1)\,X(x)}=\frac{\prod\limits_{i=1}^{n}X(x+i)}{\prod\limits_{i=0}^{n-1}X(x+i)}=\right.$$

$$\left.=\frac{V(x+1)}{V(x)}\wedge V(x)=\prod_{i=0}^{n-1}X(x+i)\right\}\Rightarrow\{(V(x+1)=U(x)\,V(x))\Leftrightarrow(\Delta V(x)+$$

$$+[1-U(x)]\,V(x)=0)\};$$

$\{\exists(C\in\mathbf{R})\rightarrow\forall(x\in\mathbf{N}^{*}):[U(x,C)\neq 0]\}\Rightarrow\{\forall(\bar{C}\in\mathbf{R})\,\exists(V(x)\in\mathbf{P}\wedge V(x)=V(x,\bar{C})=$ $=\bar{C}\,\{\mathrm{sgn}\,U(x)\}^{x}\exp\mathrm{J}\ln|U(x)|)\rightarrow\forall(x\in\mathbf{N}^{*}):[\Delta V(x)+[1-U(x)]\,V(x)\equiv 0]\};$

$\{\exists(\bar{C}\in\mathbf{R}\wedge\bar{C}\neq 0)\rightarrow\forall(x\in\mathbf{N}^{*}):[V(x)=V(x,\bar{C})>0]\}\Rightarrow\{\forall(C_{j}\in\mathbf{R}\wedge j=1,\dots,$ $n-1\wedge n\in\mathbf{N}\wedge n>1)\,\exists(X(x)\in\mathbf{P}\wedge X(x)=X(x,C,\bar{C},C_{1},\dots,C_{n-1}))\rightarrow\forall(x\in\mathbf{N}^{*}):$ $:[X(x+n+1)\,X(x)+[P(x)-1]\,X(x+n)\,X(x+1)\equiv Q(x)\,X(x)\,X(x+1)]\}$

Poznámky:

1. $\{\exists(P(x)\wedge Q(x)\in\mathbf{P})\rightarrow\forall(x\in\mathbf{N}^{*}):[P(x)\neq 1\wedge Q(x)=0]\wedge\exists(\mathrm{J}\ln|1-P(x)|\in$ $\in\mathbf{P})\wedge\exists(U(x)\in\mathbf{P}\wedge U(x)=U(x,C)=C\,\{\mathrm{sgn}\,[1-P(x)]\}^{x}\exp\mathrm{J}\ln|1-P(x)|\wedge$ $\wedge C\in\mathbf{R})\rightarrow\forall(x\in\mathbf{N}^{*}):[U(x)=U(x,C)\neq 0]\wedge\exists(\mathrm{J}\ln|U(x)|\in\mathbf{P})\wedge\exists(V(x)\in\mathbf{P}\wedge$ $\wedge V(x)=V(x,\bar{C})=\bar{C}\,\{\mathrm{sgn}\,U(x)\}^{x}\exp\mathrm{J}\ln|U(x)|\wedge\bar{C}\in\mathbf{R})\rightarrow\forall(x\in\mathbf{N}^{*}):V(x)=$ $=V(x,C,\bar{C})>0]\}\Rightarrow\{\forall(C_{j}\in\mathbf{R}\wedge j=1,\dots,n-1\wedge n\in\mathbf{N}\wedge n>1)\,\exists(X(x)\in\mathbf{P}\wedge$ $\wedge X(x)=X(x,C,\bar{C},C_{1},\dots,C_{n-1}))\rightarrow\forall(x\in\mathbf{N}^{*}):[X(x+n+1)\,X(x)+[P(x)-$ $-1]\,X(x+n)\equiv 0]\}$

2. $\{\exists(P(x)\wedge Q(x)\in\mathbf{P})\rightarrow\forall(x\in\mathbf{N}^{*}):[P(x)=0\wedge Q(x)\neq 0]\wedge\exists(\mathrm{J}Q(x)\in\mathbf{P})\rightarrow$ $\rightarrow\forall(x\in\mathbf{N}^{*}):[U(x)=U(x,C)=\mathrm{J}Q(x)+C\neq 0\wedge C\in\mathbf{R}]\wedge\exists(\mathrm{J}\ln|U(x)|\in\mathbf{P})\wedge$ $\wedge\exists(V(x)\in\mathbf{P}\wedge V(x)=V(x,\bar{C})=\bar{C}\,\{\mathrm{sgn}\,U(x)\}^{x}\exp\mathrm{J}\ln|U(x)|\wedge\bar{C}\in\mathbf{R})\rightarrow\forall(x\in\mathbf{N}^{*}):$ $:[V(x)=V(x,C,\bar{C})>0]\}\Rightarrow\{\forall(C_{j}\in\mathbf{R}\wedge j=1,\dots,n-1\wedge n\in\mathbf{N}\wedge n>1)\,\exists(X(x)\in$ $\in\mathbf{P}\wedge X(x)=X(x,C,\bar{C},C_{1},\dots,C_{n-1}))\rightarrow\forall(x\in\mathbf{N}^{*}):[X(x+n+1)\,X(x)-$ $-X(x+n)\,X(x+1)\equiv Q(x)\,X(x)\,X(x+1)]\}$

3. $\{\exists(P(x)\wedge Q(x)\in\mathbf{P})\rightarrow\forall(x\in\mathbf{N}^{*}):[P(x)=0\wedge Q(x)=0]\wedge\exists(C\in\mathbf{R}\wedge C\neq 0)\rightarrow$ $\rightarrow\forall(x\in\mathbf{N}^{*}):[\bar{C}\,\{\mathrm{sgn}\,C\}^{x}\,C^{x}>0\wedge\bar{C}\in\mathbf{R}]\}\Rightarrow\{\forall(C_{j}\in\mathbf{R}\wedge j=1,\dots,n-1\wedge n\in$ $\in\mathbf{N}\wedge>1)\,\exists(X(x)\in\mathbf{P}\wedge X(x)=X(x,C,\bar{C},C_{1},\dots,C_{n-1}))\rightarrow\forall(x\in\mathbf{N}^{*}):[X(x+n+1)\,.$ $.\,X(x)-X(x+n)\,X(x+1)\equiv 0]\}$

Pomocné tvrzení:

$$\forall(P(x)\wedge Q(x)\in\mathbf{P})\wedge\forall(C_{i}\in\mathbf{R}\wedge i=1,\dots,n\wedge n\in\mathbf{N})\,\exists(X(x)\in\mathbf{P}\wedge X(x)=$$

$$=X(x,C_{1},\dots,C_{n})=\sum_{i=1}^{n}C_{i}x^{n-i})\rightarrow\forall(x\in\mathbf{N}^{*}):[\Delta^{n}X(x)+[P(x)-1]\,.\,\Delta^{n}X(x+1)\equiv$$

$$\equiv Q(x)\,\Delta^{n}X(x)\,\Delta^{n}X(x+1)]$$

Poznámka (definice):

$$\forall(n \in \mathbf{N}) : [\int^n P(x) = \int[\int^{n-1} P(x)] \wedge \int^0 P(x) = P(x)]$$

Věta 4.:

$$\Big\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists(X(x) \in \mathbf{P}) \to$$
$$\to \forall(C_i \in \mathbf{R} \wedge i = 1, \ldots, n \wedge n \in \mathbf{N}) : [X(x) \neq \sum_{i=0}^{n} C_i x^{n-i}] \wedge \exists\Big(\int \ln|1 - P(x)| \in$$
$$\in \mathbf{P} \wedge \int \frac{Q(x) \exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\mathrm{sgn}[1 - P(x)]\}^x} \in \mathbf{P} \wedge \int^n \frac{1}{Y(x)} \in \mathbf{P} \wedge Y(x) = \{\mathrm{sgn}[1 -$$
$$- P(x)]\}^x \exp \int \ln|1 - P(x)| \left[\int \frac{Q(x) \exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\mathrm{sgn}[1 - P(x)]\}^x} + C\right] \neq 0 \wedge$$
$$\wedge C \in \mathbf{R}\Big)\Big\} \Rightarrow \Big\{\forall(C_i \in \mathbf{R} \wedge i = 1, \ldots, n) \exists\Big(X(x) \in \mathbf{P} \wedge X(x) = X(x, C_1, \ldots, C_n) =$$
$$= \int^n \frac{1}{Y(x)} + \sum_{i=0}^{n} C_i x^{n-i}\Big) \to \forall(x \in \mathbf{N}^*) : [\Delta^n X(x) + [P(x) - 1]\,\Delta^n X(x + 1) \equiv$$
$$\equiv Q(x)\,\Delta^n X(x) \,.\, \Delta^n X(x + 1)]\Big\}$$

Důkaz:

$$\{\exists P((x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists(X(x) \in \mathbf{P}) \to \forall(C_i \in \mathbf{R} \wedge$$
$$\wedge i = 1, \ldots, n \wedge n \in \mathbf{N}) : [X(x) \neq \sum_{i=1}^{n} C_i x^{n-i}]\} \Rightarrow \Big\{(\Delta^n X(x) + [P(x) - 1]\,\Delta^n X(x + 1) =$$
$$= Q(x)\,\Delta^n X(x)\,\Delta^n X(x + 1)] \Leftrightarrow \left(\frac{1}{\Delta^n X(x + 1)} + \frac{P(x) - 1}{\Delta^n X(x)} = Q(x)\right) \Leftrightarrow \left(\Delta \frac{1}{\Delta^n X(x)} +\right.$$
$$\left.+ P(x) \frac{1}{\Delta^n X(x)} = Q(x)\right) \Leftrightarrow \left(\Delta Y(x) + P(x)\,Y(x) = Q(x) \wedge Y(x) = \frac{1}{\Delta^n X(x)}\right)\Big\} \,.$$
$$.\ \forall(C \in \mathbf{R}) \exists\Big(Y(x) \in \mathbf{P} \wedge Y(x) = Y(x, C) = \{\mathrm{sgn}[1 - P(x)]\}^x \exp \int \ln|1 - P(x)| \,.$$
$$.\left[\int \frac{Q(x) \exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\mathrm{sgn}[1 - P(x)]\}^x} + C\right] \neq 0\Big) \to \forall(x \in \mathbf{N}^*) : [\Delta Y(x) + P(x)\,Y(x) \equiv$$
$$\equiv Q(x)];$$

$$\left\{\Delta^n X(x) = \frac{1}{Y(x)}\right\} \Rightarrow \Big\{\forall(C_i \in \mathbf{R} \wedge i = 1, \ldots, n \wedge n \in \mathbf{N}) \exists\Big(X(x) = X(x, C, C_1, \ldots,$$
$$C_n) = \int^n \frac{1}{Y(x)} + \sum_{i=0}^{n} C_i x^{n-i}\Big) \to \forall(x \in \mathbf{N}^*) : [\Delta^n X(x) + [P(x) - 1]\,\Delta^n X(x + 1) \equiv$$
$$\equiv Q(x)\,\Delta^n X(x)\,\Delta^n X(x + 1)]\Big\}$$

Poznámky:

1. $$\Big\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) \neq 0] \wedge \exists(X(x) \in \mathbf{P}) \to$$

$$\to \forall(C_i \in \mathbf{R} \wedge i = 1, \dots, n \wedge n \in \mathbf{N}) : [X(x) \neq \sum_{i=0}^{n} C_i x^{n-i}] \wedge \exists(\mathrm{J}Q(x) \in \mathbf{P} \wedge \mathrm{J}Q(x) +$$

$$+ C \neq 0 \wedge C \in \mathbf{R}) \wedge \exists\left(\mathrm{J}^n \frac{1}{\mathrm{J}Q(x) + C} \in \mathbf{P}\right)\Big\} \Rightarrow \Big\{\forall(C_i \in \mathbf{R} \wedge i = 1, \dots, n)\, \exists\Big(X(x) \in$$

$$\in \mathbf{P} \wedge X(x) = X(x, C, C_1, \dots, C_n) = \mathrm{J}^n \frac{1}{\mathrm{J}Q(x) + C} + \sum_{i=1}^{n} C_i x^{n-i}\Big) \to \forall(x \in \mathbf{N}^*) :$$

$$: [\Delta^n X(x) - \Delta^n X(x+1) \equiv Q(x)\, \Delta^n X(x)\, \Delta^n X(x+1)]\Big\}$$

2. $$\Big\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) \neq 0] \wedge \exists\Big(\mathrm{J}^n \frac{1}{Q(x)} \in \mathbf{P} \wedge$$

$$\wedge\, n \in \mathbf{N}\Big)\Big\} \Rightarrow \Big\{\forall(C_i \in \mathbf{R} \wedge i = 1, \dots, n)\, \exists\Big(X(x) \in \mathbf{P} \wedge X(x) = X(x, C_1, \dots, C_n) =$$

$$= \sum_{i=1}^{n} C_i x^{n-i} \vee X(x+1) = X(x+1, C_1, \dots, C_n) = \mathrm{J}^n \frac{1}{Q(x)} + \sum_{i=1}^{n} C_i x^{n-i}\Big) \to$$

$$\to \forall(x \in \mathbf{N}^*) : [\Delta^n X(x) \equiv Q(x)\, \Delta^n X(x)\, \Delta^n X(x+1)]\Big\}$$

Věta 5.:

$$\Big\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists\Big(\mathrm{J} \ln |1 - P(x)| \in$$

$$\in \mathbf{P} \wedge \mathrm{J}\, \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\, \{\mathrm{sgn}\, [1 - P(x)]\}^x} \in \mathbf{P}\Big) \wedge \exists\Big(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) =$$

$$= \{\mathrm{sgn}\, [1 - P(x)]\}^x \exp \mathrm{J} \ln |1 - P(x)| \left[\mathrm{J}\, \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\, \{\mathrm{sgn}\, [1 - P(x)]\}^x} + C\right] \wedge$$

$$\wedge\, C \in \mathbf{R}\Big) \wedge \exists(\mathrm{J}U(x, C) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [\mathrm{J}U(x, C) + \bar{C} > 0 \wedge \bar{C} \in R]\Big\} \Rightarrow$$

$$\Rightarrow \Big\{\forall(C_j \in \mathbf{R} \wedge j = 1, \dots, n \wedge n \in \mathbf{N} \wedge n > 1)\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C},$$

$$C_1, \dots, C_n)) \to \forall(x \in \mathbf{N}^*) : \Big[\big(X(x+n+2) - X(x+1)\big)\, X(x+n+1) +$$

$$+ [P(x) - 1]\big(X(x+n+1) - X(x)\big)\, X(x+1) \equiv \frac{Q(x)}{\prod_{i=1}^{n-1} X(x+1+i)}\Big]\Big\}$$

Důkaz:

$$\{\exists(P(x) \wedge Q(x) \wedge X(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0 \wedge X(x) \neq 0]\} \Rightarrow$$

$$\Rightarrow \Bigg\{ \forall(n \in \mathbf{N} \wedge n > 1) : \Bigg[\Bigg((X(x+n+2) - X(x+1))\, X(x+n+1) + [P(x) - 1] \,.$$

$$.\, (X(x+n+1) - X(x))\, X(x+1) = \frac{Q(x)}{\prod_{i=1}^{n-1} X(x+1+i)} \Bigg) \Leftrightarrow \big((X(x+n+2) -$$

$$- X(x+1))\, X(x+n+1)\, X(x+n)\, X(x+n-1) \ldots X(x+2) + [P(x) - 1] \,.$$

$$.\, (X(x+n+1) - X(x))\, X(x+n) \ldots X(x+2)\, X(x+1) = Q(x)\big) \Leftrightarrow \Big(\Delta \prod_{i=0}^{n} X(x+1+i) +$$

$$+ [P(x) - 1]\, \Delta \prod_{i=0}^{n} X(x+i) = Q(x)\Big) \Leftrightarrow \Big(\Delta\Big[\Delta \prod_{i=0}^{n} X(x+i) + P(x)\, \Delta \prod_{i=0}^{n} X(x+i) =$$

$$= Q(x)\Big) \Leftrightarrow \Big(\Delta U(x) + P(x)\, U(x) = Q(x) \wedge U(x) = \Delta \prod_{i=0}^{n} X(x+i)\Big) \Bigg]\Bigg\} \, ;$$

$$\forall(C \in \mathbf{R})\, \exists\Bigg(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = \{\operatorname{sgn}[1 - P(x)]\}^x \exp \text{J} \ln |1 - P(x)| \,.$$

$$.\, \left[\text{J}\, \frac{Q(x) \exp(-\text{J} \ln |1 - P(x)|)}{[1 - P(x)]\, \{\operatorname{sgn}[1 - P(x)]\}^x} + C\right]\Bigg) \rightarrow \forall(x \in \mathbf{N}^*) : [\Delta U(x) + P(x)\, U(x) \equiv Q(x)];$$

$$\{\exists(\text{J} U(x, C) \in \mathbf{P}) \wedge \exists(\bar{C} \in \mathbf{R}) \rightarrow \forall(x \in \mathbf{N}^*) : [\text{J} U(x, C) + \bar{C} > 0 \wedge \prod_{i=0}^{n} X(x+i) =$$

$$= \text{J} U(x, C) + \bar{C} > 0]\} \Rightarrow \Bigg\{ \forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) =$$

$$= X(x, C, \bar{C}, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : \Bigg[(X(x+n+2) - X(x+1))\, X(x+n+1) +$$

$$+ [P(x) - 1]\, (X(x+n+1) - X(x))\, X(x+1) \equiv \frac{Q(x)}{\prod_{i=1}^{n-1} X(x+1+i)} \Bigg]\Bigg\}$$

Poznámky:

1. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) = 0] \wedge \exists(\text{J} \ln |1 - P(x)| \in \mathbf{P}) \wedge \exists(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = C\, \{\operatorname{sgn}[1 - P(x)]\}^x \exp \text{J} \ln |1 - P(x)| \wedge C \in \mathbf{R}) \wedge \exists(U(x, C) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [\text{J} U(x, C) + \bar{C} > 0 \wedge \bar{C} \in \mathbf{R}]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : [(X(x+n+2) - X(x+1))\, X(x+n+1) + [P(x) - 1]\, (X(x+n+1) - X(x))\, X(x+1) \equiv 0]\}$

2. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) \neq 0] \wedge \exists(\mathrm{J}Q(x) \in \mathbf{P} \wedge \mathrm{J}^2 Q(x) \in \mathbf{P}) \wedge \exists(C \wedge \bar{C} \in \mathbf{R}) \to \forall(x \in \mathbf{N}^*) : [\mathrm{J}^2 Q(x) + Cx + \bar{C} > 0]\} \Rightarrow \Big\{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N} \wedge n > 1)\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \to \forall(x \in \mathbf{N}^*) : \Big[(X(x + n + 2) - X(x + 1))\, X(x + n + 1) - (X(x + n + 1) - X(x)) \,.$

$$. X(x + 1) \equiv \frac{Q(x)}{\prod_{i=1}^{n-1} X(x + 1 + i)} \quad \text{i.e. } \Delta^2 \prod_{i=0}^{n} X(x + i) \equiv Q(x)\Big]\Big\}$$

3. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) = 0] \wedge \exists(C \wedge \bar{C} \in \mathbf{R}) \to \forall(x \in \mathbf{N}^*) : [Cx + \bar{C} > 0]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \to \forall(x \in \mathbf{N}^*) : [(X(x + n + 2) - X(x + 1))\, X(x + n + 1) - (X(x + n + 1) - X(x))\, X(x + 1) \equiv 0]\}$

4. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) \neq 0] \wedge \exists(\mathrm{J}Q(x) \in \mathbf{P}) \wedge \exists(C \in \mathbf{R}) \to \forall(x \in N^*) : [\mathrm{J}Q(x) + C > 0]\} \Rightarrow \Big\{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N} \wedge n > 1)\, \exists(X(x) \in \mathbf{P} \wedge X(x + 1) = X(x + 1, C, C_1, \ldots, C_n)) \to \forall(x \in \mathbf{N}^*) : \Big[(X(x + n + 2) - X(x + 1))\, X(x + n + 1) \equiv \frac{Q(x)}{\prod_{i=1}^{n-1} X(x + 1 + i)}$ i.e. $\prod_{i=0}^{n} X(x + 1 + i) \equiv \mathrm{J}Q(x) + C\Big]\Big\}$

5. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) = 0]\} \Rightarrow \{\forall(C \in \mathbf{R} \wedge C > 0 \wedge C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, C_1, \ldots, C_n)) \to \forall(x \in \mathbf{N}^*) : [(X(x + n + 2) - X(x + 1))\, X(x + n + 1) \equiv 0$ i.e. $\prod_{i=0}^{n} X(x + 1 + i) \equiv C]\}$

Věta 6.:

$\Big\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) \neq 0] \wedge \exists\Big(\mathrm{J} \ln |1 - P(x)| \in \mathbf{P} \wedge \mathrm{J} \frac{Q(x) \exp(-\mathrm{J} \ln |1 - P(x)|)}{[1 - P(x)]\{\operatorname{sgn}[1 - P(x)]\}^x} \in \mathbf{P}\Big) \wedge \exists\Big(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) =$

$$= \{\operatorname{sgn}[1 - P(x)]\}^x \exp \textstyle\int \ln|1 - P(x)| \left[\int \frac{Q(x)\exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\operatorname{sgn}[1 - P(x)]\}^x} + \right.$$

$$\left. + C\right] \neq 0 \wedge C \in \mathbf{R}\Big) \wedge \exists \left(\int \frac{1}{U(x, C)} \in \mathbf{P}\right) \to \forall(x \in \mathbf{N}^*) : \left[\int \frac{1}{U(x, C)} + \bar{C} > 0 \wedge \right.$$

$$\left.\wedge\ \bar{C} \in \mathbf{R}\right]\Big\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\ \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C,$$

$$\bar{C}, C_1, \ldots, C_n)) \to \forall(x \in \mathbf{N}^*) : [(X(x + n + 1) - X(x))\, X(x + 1) + [P(x) -$$

$$- 1](X(x + n + 2) - X(x + 1))\, X(x + n + 1) \equiv Q(x)(X(x + n + 2) -$$

$$- X(x + 1))(X(x + n + 1) - X(x)) \prod_{i=0}^{n} X(x + 1 + i)]\}$$

Důkaz:

$$\{\exists(P(x) \wedge Q(x) \wedge X(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) \wedge \forall(n \in \mathbf{N}) : [P(x) \neq 1 \wedge Q(x) \neq 0 \wedge X(x +$$

$$+ n + 1) \neq X(x)]\} \Rightarrow \Big\{((X(x + n + 1) - X(x))\, X(x + 1) + [P(x) - 1](X(x + n +$$

$$+ 2) - X(x + 1))\, X(x + n + 1) = Q(x)(X(x + n + 2) - X(x + 1))(X(x + n +$$

$$+ 1) - X(x)) \prod_{i=0}^{n} X(x + i + 1)) \Leftrightarrow \left(\frac{X(x + 1)}{X(x + n + 2) - X(x + 1)} + [P(x) - 1]\,.\right.$$

$$\left.\cdot \frac{X(x + n + 1)}{X(x + n + 1) - X(x)} = Q(x) \prod_{i=0}^{n} X(x + i + 1)\right) \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{1}{(X(x + n + 2) - X(x + 1))\, X(x + n + 1) \ldots X(x + 2)} + [P(x) - 1]\,.\right.$$

$$\left.\cdot \frac{1}{(X(x + n + 1) - X(x))\, X(x + n) \ldots X(x + 2)\, X(x + 1)} = Q(x)\right) \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{1}{\Delta \prod_{i=0}^{n} X(x + i + 1)} + [P(x) - 1] \frac{1}{\Delta \prod_{i=0}^{n} X(x + i)} = Q(x)\right) \Leftrightarrow \left(\Delta \frac{1}{\Delta \prod_{i=0}^{n} X(x + i)} + \right.$$

$$\left. + P(x) \frac{1}{\Delta \prod_{i=0}^{n} X(x + i)} = Q(x)\right) \Leftrightarrow \left(\Delta U(x) + P(x)\, U(x) = Q(x) \wedge U(x) = \right.$$

$$\left. = \frac{1}{\Delta \prod_{i=0}^{n} X(x + i)}\right)\Big\}\,;$$

$$\forall(C \in \mathbf{R})\ \exists\Big(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = \{\operatorname{sgn}[1 - P(x)]\}^x \exp \textstyle\int \ln|1 - P(x)|\,.$$

$$\left.\cdot\left[\int \frac{Q(x)\exp(-\int \ln|1 - P(x)|)}{[1 - P(x)]\{\operatorname{sgn}[1 - P(x)]\}^x} + C\right] \neq 0\right) \to \forall(x \in \mathbf{N}^*) : [\Delta U(x) + P(x)\, U(x) \equiv$$

$$\equiv Q(x)];$$

$$\left\{\Delta \prod_{i=0}^{n} X(x+i) = \frac{1}{U(x, C)}\right\} \Rightarrow \left\{\forall(\bar{C} \in \mathbf{R}) : [\prod_{i=0}^{n} X(x+i) = \mathrm{J}\,\frac{1}{U(x, C)} + \bar{C}\right\};$$

$$\left\{\exists(\bar{C} \in \mathbf{R}) \rightarrow \forall(x \in \mathbf{N}^*) : \left[\mathrm{J}\,\frac{1}{U(x, C)} + \bar{C} > 0\right]\right\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : [(X(x+n+1) - X(x))\, X(x+1) + [P(x) - 1]\, (X(x+n+2) - X(x+1))\, X(x+n+1) \equiv Q(x)\, (X(x+n+2) - X(x+1))\, (X(x+n+1) - X(x)) \prod_{i=0}^{n} X(x+i+1)]\}$$

Poznámky:

1. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) \neq 1 \wedge Q(x) = 0] \wedge \exists(\mathrm{J} \ln |1 - P(x)| \in \mathbf{P}) \wedge \exists(U(x) \in \mathbf{P} \wedge U(x) = U(x, C) = C\, \{\operatorname{sgn} [1 - P(x)]\}^x \exp \mathrm{J} \ln |1 - P(x)| \wedge C \in \mathbf{R} \wedge C \neq 0) \wedge \exists\left(\mathrm{J}\,\frac{1}{U(x, C)} \in \mathbf{P}\right) \rightarrow \forall(x \in \mathbf{N}^*) : \left[\mathrm{J}\,\frac{1}{U(x, C)} + \bar{C} > 0 \wedge \bar{C} \in \mathbf{R}\right]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : [(X(x+n+1) - X(x))\, X(x+1) + [P(x) - 1]\, (X(x+n+2) - X(x+1))\, X(x+n+1) \equiv 0]\}$

2. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) \neq 0] \wedge \exists(\mathrm{J}Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [\mathrm{J}Q(x) + C \neq 0 \wedge C \in \mathbf{R}] \wedge \exists\left(\mathrm{J}\,\frac{1}{\mathrm{J}Q(x) + C} \in \mathbf{P}\right) \rightarrow \forall(x \in \mathbf{N}^*) : \left[\mathrm{J}\,\frac{1}{\mathrm{J}Q(x) + C} + \bar{C} > 0 \wedge \bar{C} \in \mathbf{R}\right]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : [(X(x+n+1) - X(x))\, X(x+1) - (X(x+n+2) - X(x+1))\, X(x+n+1) \equiv Q(x)\, (X(x+n+2) - X(x+1)) \,.\, (X(x+n+1) - X(x)) \,.\, \prod_{i=0}^{n} X(x+i+1)]\}$

3. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) = 0 \wedge Q(x) = 0] \wedge \exists(C \wedge \bar{C} \in \mathbf{R} \wedge C \neq 0) \rightarrow \forall(x \in \mathbf{N}^*) : \left[\frac{x}{C} + \bar{C} > 0\right]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N})\, \exists(X(x) \in \mathbf{P} \wedge X(x) = X(x, C, \bar{C}, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : [(X(x+n+1) - X(x))\, X(x+1) - (X(x+n+2) - X(x+1))\, X(x+n+1) \equiv 0]\}$

4. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \rightarrow \forall(x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) \neq 0] \wedge \exists\left(\mathrm{J}\,\frac{1}{Q(x)} \in \mathbf{P}\right) \rightarrow \forall(x \in \mathbf{N}^*) : \left[\mathrm{J}\,\frac{1}{Q(x)} + C > 0 \wedge C \in \mathbf{R}\right]\} \Rightarrow \{\forall(C_j \in \mathbf{R} \wedge j = 1, \ldots, n \wedge n \in \mathbf{N}) \,.\, \exists(X(x) \in \mathbf{P} \wedge X(x+1) = X(x+1, C, C_1, \ldots, C_n)) \rightarrow \forall(x \in \mathbf{N}^*) : [(X(x+n+1) -$

$- X(x))\, X(x + 1) \equiv Q(x)\, \big(X(x + n + 2) - X(x + 1)\big)\, \big(X(x + n + 1) -$
$- X(x))\prod_{i=0}^{n} X(x + i + 1)$ i.e. $\prod_{i=0}^{n} X(x + i + 1) \equiv \int \frac{1}{Q(x)} + C\Big]\Big\}$

5. $\{\exists(P(x) \wedge Q(x) \in \mathbf{P}) \to \forall(x \in \mathbf{N}^*) : [P(x) = 1 \wedge Q(x) = 0]\} \Rightarrow \Big\{\exists\Big(X(x) \in \mathbf{P} \wedge$
$\wedge \sum_{i=0}^{n} \binom{n+1}{i} \Delta^{n-i} X(x) = C \wedge C \in \mathbf{R} \wedge n \in \mathbf{N}\Big) \to \forall(x \in \mathbf{N}^*) \wedge \forall(n \in \mathbf{N}) : [(X(x + n +$
$+ 1) - X(x))\, X(x + 1) \equiv 0]\Big\}$

Резюме

ПРЕОБРАЗОВАНИЕ НЕКОТОРЫХ НЕЛИНЕЙНЫХ УРАВНЕНИЙ В КОНЕЧНЫХ РАЗНОСТЬЯХ В НЕОДНОРОДНОЕ ЛИНЕЙНОЕ УРАВНЕНИЕ В КОНЕЧНЫХ РАЗНОСТЬЯХ 1-ГО ПОРЯДКА

ВЛАДИМИР ВЛЧЕК

В работе прежде всего доказывается теорема об существовании и форме общего решения нелинейного уравнения в конечных разностьях типа $\prod_{j=0}^{n} X(x + j) = P(x), P(x) > 0, n \in \mathbf{N}$ (при помощи его трансформации в линейное неоднородное разноростное уравнение n-того порядка).

В дальнейшем предоставлены теоремы о преобразованиях и существовании общего решения некоторых типов нелинейных уравнений в конечных разностьях в линейное неоднородное разноростное уравнение 1-го порядка, когда подстановочной функцией присущей трансформации главным образом является сложная функция выражения $\prod_{j=0}^{n} X(x + j)$ или его конечная разность.