Jan Peřina

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Laboratoř optiky
Vedoucí laboratoře: Prof. RNDr. Bedřich Havelka, doktor věd

SUPERPOSITION OF THERMAL AND COHERENT FIELDS

JAN PEŘINA

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1. INTRODUCTION

In the recent papers by Lachs [1] and Troup [2] some statistical properties of a mode of an electromagnetic field which is a superposition of thermal and coherent radiation have been studied. We shall extend the results of the above mentioned two papers for a mode into the field containing M modes. We shall calculate the probability distributions $P(W)$ of a „classical" quantity W of the field which can be interpreted as the integrated intensity of the field and $p(n)$ of the number of photons in the field. Further we shall calculate the moments of this distributions and the expressions for the coherent and thermal fields will be obtained as limit cases. The expressions given here for M degrees of freedom can be measured by a real photodetector with N_1 coherence areas and with resolving time $T \gg \tau_c$ (τ_c ist the coherence time). Then $M = N_1 T/\tau_c$. Further, the results obtained have the meaning for a laser operating in several modes.

The density operator $\hat{\varrho}$ of the field can be written in terms of the coherent states $|\{\alpha_\lambda\}\rangle$ [3] in the form [4]

$$\hat{\varrho} = \int \Phi(\{\alpha_\lambda\}) \, |\{\alpha_\lambda\}\rangle \langle\{\alpha_\lambda\}| \, d^2\{\alpha_\lambda\}, \tag{1}$$

where $\Phi(\{\alpha_\lambda\})$ is a weighting („probability") function, α_λ is an eigenvalue of the annihilation operator $\hat{a}_\lambda$ of a photon in the coherent state $|\alpha_\lambda\rangle$ and the integrals are taken over the whole complex plane. The probability distribution for the number of photons in the field is given with the aid of (1) [5—7]

$$p(n) = \sum_{\{m_\lambda\}} \langle\{m_\lambda\}| \, \hat{\varrho} \, |\{m_\lambda\}\rangle \, \delta_{mn} = \int \Phi(\{\alpha_\lambda\}) \frac{W^n}{n!} \exp(-W) \, d^2\{\alpha_\lambda\}. \tag{2}$$

where $|\{n_\lambda\}\rangle$ are the Fock states, $n = \sum_{\lambda=1}^{M} n_\lambda$, $W = \sum_{\lambda=1}^{M} |\alpha_\lambda|^2$ and M is a number of modes in the field. After substitution

$$P(W') = \int \Phi(\{\alpha_\lambda\}) \, \delta(W' - W) \, d^2\{\alpha_\lambda\} \tag{3}$$

we have [8]

$$p(n) = \int_0^\infty P(W) \frac{W^n}{n!} \exp(-W) \, \mathrm{d}W. \tag{4}$$

This relation may be inverted into the form [9, 10, 7]

$$P(W) = \exp W \sum_{n=0}^{\infty} (-1)^n p(n) \, \delta^{(n)}(W), \tag{5}$$

where $\delta^{(n)}(W)$ is the n-th order derivative of the Dirac δ — function.

We shall have for the coherent field in the state $|\beta_\lambda\rangle$

$$\Phi_1(\alpha_\lambda) = \delta(\alpha_\lambda - \beta_\lambda) \tag{6}$$

and for the thermal field

$$\Phi_2(\alpha_\lambda) = (\pi\langle n_{T\lambda}\rangle)^{-1} \exp\left[-\frac{|\alpha_\lambda|^2}{\langle n_{T\lambda}\rangle}\right], \tag{7}$$

where $\langle n_{T\lambda}\rangle$ represents the average number of photons in the mode λ. For a superposition of the coherent and thermal fields in the mode λ we obtain

$$\Phi(\alpha_\lambda) = \int \Phi_1(\alpha'_\lambda) \, \Phi_2(\alpha_\lambda - \alpha'_\lambda) \, \mathrm{d}^2\alpha'_\lambda = (\pi\langle n_{T\lambda}\rangle)^{-1} \exp\left[-\frac{|\alpha_\lambda - \beta_\lambda|^2}{\langle n_{T\lambda}\rangle}\right]. \tag{8}$$

2. CALCULATION OF THE PROBABILITY DISTRIBUTIONS $P(W)$ AND $p(n)$

After substitution from (8) to (3) we obtain the probability distribution $P(W)$ of $W = \sum_{\lambda=1}^{M} |\alpha_\lambda|^2$ in the form

$$P(W') = \prod_{\lambda=1}^{M} (\pi\langle n_{T\lambda}\rangle)^{-1} \int \exp\left[-\frac{|\alpha_\lambda - \beta_\lambda|^2}{\langle n_{T\lambda}\rangle}\right] \delta\Big(\sum_{\lambda'=1}^{M} |\alpha_{\lambda'}|^2 - W'\Big) \, \mathrm{d}^2\alpha_\lambda \tag{9}$$

under the assumption that quantities $|\alpha_\lambda|^2$ of different modes are statistically independent. If we write $\alpha_\lambda = r_\lambda \exp i\vartheta_\lambda$ and $\beta_\lambda = s_\lambda \exp i\varphi_\lambda$ we can take the integrals over $\{\vartheta_\lambda\}$ in (9) and $P(W)$ has the form

$$P(W') = \prod_{\lambda=1}^{M} \langle n_{T\lambda}\rangle^{-1} \exp\left[-\frac{s_\lambda^2}{\langle n_{T\lambda}\rangle}\right] \int_0^\infty \exp\left[-\frac{r_\lambda^2}{\langle n_{T\lambda}\rangle}\right] . \\ . \, I_0\left(2\frac{r_\lambda s_\lambda}{\langle n_{T\lambda}\rangle}\right) \delta\Big(\sum_{\lambda'=1}^{M} r_{\lambda'}^2 - W'\Big) \, \mathrm{d}r_\lambda^2, \tag{10}$$

where $I_0(z)$ is the modified Bessel function of zero order. If we represent δ-function in (10) in the integral form

$$(2\pi)^{-1} \int_{-\infty}^{+\infty} \exp\Big[i\Big(W' - \sum_{\lambda=1}^{M} r_\lambda^2\Big) \mu\Big] \, \mathrm{d}\mu$$

and if we take I_0 in the form of series we can carry out the integrations over $\{r_\lambda^2\}$ and we obtain

$$P(W) = (2\pi)^{-1} \exp\left[-\sum_{\lambda=1}^{M} \frac{s_\lambda^2}{\langle n_{T\lambda}\rangle}\right] .$$

$$. \sum_{n=0}^{\infty} \frac{1}{n!} \int_{-\infty}^{+\infty} \frac{\exp iW\mu}{\prod_{\lambda=1}^{M} (1 + i\mu\langle n_{T\lambda}\rangle)} \left[\sum_{\lambda=1}^{M} \frac{s_\lambda^2}{\langle n_{T\lambda}\rangle (1 + i\mu\langle n_{T\lambda}\rangle)}\right]^n \mathrm{d}\mu. \qquad (11)$$

Let us assume for simplicity now that $\langle n_{T\lambda}\rangle = \langle n_T\rangle/M$ where $\langle n_T\rangle$ is the average number of photons in the whole field. If we compute the integral in (11) with the aid of residuum theorem we have

$$P(W) = \frac{M}{\langle n_T\rangle} \left(\frac{W}{\langle n_C\rangle}\right)^{\frac{M-1}{2}} \exp\left[-\frac{W + \langle n_C\rangle}{\langle n_T\rangle} M\right] I_{M-1}\left(2 \frac{\sqrt{W\langle n_C\rangle}}{\langle n_T\rangle} M\right), \qquad (12)$$

where $\langle n_C\rangle = \sum_{\lambda=1}^{M} s_\lambda^2$ and $I_M(z)$ is the modified Bessel function of the M-th order.

From the relation (4) the probability distribution $p(n)$ can be determined:

$$p(n) = \frac{(n + M - 1)!}{n!(M - 1)!} \left(1 + \frac{M}{\langle n_T\rangle}\right)^{-n} \left(1 + \frac{\langle n_T\rangle}{M}\right)^{-M} .$$

$$. \exp\left[-\frac{\langle n_C\rangle M}{\langle n_T\rangle}\right] {}_1F_1\left(n + M, M; \frac{\langle n_C\rangle M^2}{\langle n_T\rangle (\langle n_T\rangle + M)}\right) =$$

$$= \frac{(n + M - 1)!}{n!(M - 1)!} \left(1 + \frac{M}{\langle n_T\rangle}\right)^{-n} \left(1 + \frac{\langle n_T\rangle}{M}\right)^{-M} .$$

$$. \exp\left[-\frac{\langle n_C\rangle M}{M + \langle n_T\rangle}\right] {}_1F_1\left(-n, M; -\frac{\langle n_C\rangle M^2}{\langle n_T\rangle (\langle n_T\rangle + M)}\right), \qquad (13)$$

where ${}_1F_1$ is a confluent hypergeometric function and the relation ${}_1F_1(\alpha, \gamma; z) = {}_1F_1(\gamma - \alpha, \gamma; -z) \exp z$ was used.

Substituting (13) in (5), expressing $\delta^{(n)}(W)$ in the integral form and carrying out summation and integration with the use of the residuum theorem the probability distribution (12) is obtained again.

We can easily verify that $\int_0^\infty P(W)\,\mathrm{d}W = 1$ and $\sum_{n=0}^{\infty} p(n) = 1$.

If we put in (12) and (13) $\langle n_C\rangle = 0$ (i.e. $|\beta_\lambda|^2 = \langle n_{C\lambda}\rangle = 0$ for all λ) we obtain the expressions for the thermal field [11, 12] (cf. also Eq. (11) simplified for this special case)

$$P(W) = \frac{M^M W^{M-1}}{(M - 1)!\, \langle n_T\rangle^M} \exp\left[-\frac{WM}{\langle n_T\rangle}\right] \qquad (14)$$

and

$$p(n) = \frac{(n + M - 1)!}{n!(M - 1)!} \left(1 + \frac{M}{\langle n_T\rangle}\right)^{-n} \left(1 + \frac{\langle n_T\rangle}{M}\right)^{-M} . \qquad (15)$$

Putting $M = 1$ we obtain the equations for a mode given by Lachs [1] and Troup [2]

$$P(W) = \langle n_T \rangle^{-1} \exp\left[-\frac{W + \langle n_C \rangle}{\langle n_T \rangle}\right] I_0 \left(2\frac{\sqrt{W\langle n_C \rangle}}{\langle n_T \rangle}\right) \tag{16}$$

and

$$p(n) = (1 + \langle n_T \rangle)^{-1} \left(1 + \frac{1}{\langle n_T \rangle}\right)^{-n} \exp\left[-\frac{\langle n_C \rangle}{1 + \langle n_T \rangle}\right] . \\ . {}_1F_1 \left(-n, 1; -\frac{\langle n_C \rangle}{\langle n_T \rangle (1 + \langle n_T \rangle)}\right), \tag{17}$$

where $W = |\alpha_\lambda|^2 = r_\lambda^2$ and $\langle n_{C\lambda} \rangle = |\beta_\lambda|^2 = s_\lambda^2$.

3. CALCULATION OF THE MOMENTS

Now we can compute the moments of the distributions (12) and (13). For the distribution (12) it is valid

$$\langle W^k \rangle = \frac{(k + M - 1)!}{(M - 1)!} \frac{\langle n_T \rangle^k}{M^k} \exp\left[-\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right] {}_1F_1 \left(k + M, M; \frac{\langle n_C \rangle M}{\langle n_T \rangle}\right) = \\ = \frac{(k + M - 1)!}{(M - 1)!} \frac{\langle n_T \rangle^k}{M^k} {}_1F_1 \left(-k, M; -\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right). \tag{18}$$

Further, if we express ${}_1F_1$ in the form of series

$$\langle W^{k+1} \rangle = \frac{\langle n_T \rangle^{k+1}}{M^{k+1}} \exp\left[-\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right] \sum_{m=0}^{\infty} \frac{(m + k + M)!}{m!\,(m + M - 1)!} \left(\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right)^m = \\ = (k + M) \frac{\langle n_T \rangle}{M} \langle W^k \rangle + \\ + \frac{\langle n_T \rangle^{k+1}}{M^{k+1}} \exp\left[-\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right] \sum_{m=0}^{\infty} m \frac{(m + k + M - 1)!}{m!\,(m + M - 1)!} \left(\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right)^m \tag{19}$$

and if we compute

$$\frac{\partial}{\partial \langle n_C \rangle} \langle W^k \rangle = -\frac{M}{\langle n_T \rangle} \langle W^k \rangle + \\ + \frac{1}{\langle n_C \rangle} \frac{\langle n_T \rangle^k}{M^k} \exp\left[-\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right] \sum_{m=0}^{\infty} m \frac{(m + k + M - 1)!}{m!\,(m + M - 1)!} \left(\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right)^m \tag{20}$$

we obtain combining Eqs. (19) and (20)

$$\langle W^{k+1} \rangle = \left(\langle n_T \rangle + \langle n_C \rangle + k\frac{\langle n_T \rangle}{M}\right) \langle W^k \rangle + \frac{\langle n_T \rangle \langle n_C \rangle}{M} \frac{\partial \langle W^k \rangle}{\partial \langle n_C \rangle}. \tag{21}$$

As $\langle W \rangle = \langle n_T \rangle + \langle n_C \rangle$ we have for $k = 1$

$$\langle W^2 \rangle = \langle n_T \rangle^2 \frac{M + 1}{M} + 2\langle n_T \rangle \langle n_C \rangle \frac{M + 1}{M} + \langle n_C \rangle^2. \tag{22a}$$

i.e.

$$\langle(\Delta W)^2\rangle = \langle W^2\rangle - \langle W\rangle^2 = \frac{\langle n_T\rangle^2 + 2\langle n_T\rangle\langle n_C\rangle}{M}. \tag{22b}$$

If we put $\langle n_C\rangle = 0$ we obtain the moments of (14) and for $M = 1$ we obtain the corresponding expressions for a mode. From (18) we have for $\langle n_C\rangle = 0$ $\langle W^k\rangle = \langle W\rangle^k M^{-k}(k + M - 1)!/(M - 1)!$ while for the coherent field we have $\langle W^k\rangle = \langle W\rangle^k$ [13, 7].

Let us compute the moments of the distribution (13). If we denote $b = \langle n_T\rangle/M$ we have

$$\langle n^k\rangle = \sum_{n=0}^{\infty} \frac{n^k}{(1+b)^M}\left(\frac{b}{1+b}\right)^n \frac{(n+M-1)!}{n!\,(M-1)!}$$
$$\exp\left[-\frac{\langle n_C\rangle}{1+b}\right] {}_1F_1\left(-n, M; -\frac{\langle n_C\rangle}{b(1+b)}\right). \tag{23}$$

Further we obtain

$$\frac{\partial\langle n^k\rangle}{\partial b} = -\frac{M}{1+b}\langle n^k\rangle + \frac{1}{b(1+b)}\langle n^{k+1}\rangle + \frac{\langle n_C\rangle}{(1+b)^2}\langle n^k\rangle -$$
$$-\frac{(2b+1)\langle n_C\rangle}{b^2(1+b)^2 M}\cdot\sum_{n=0}^{\infty}\frac{n^{k+1}}{(1+b)^M}\left(\frac{b}{1+b}\right)^n\frac{(n+M-1)!}{n!\,(M-1)!}$$
$$\exp\left[-\frac{\langle n_C\rangle}{1+b}\right]{}_1F_1\left(-n+1, M+1; -\frac{\langle n_C\rangle}{b(1+b)}\right) \tag{24}$$

and

$$\frac{\partial\langle n^k\rangle}{\partial\langle n_C\rangle} = -\frac{1}{1+b}\langle n^k\rangle + \frac{1}{b(1+b)M}\sum_{n=0}^{\infty}\frac{n^{k+1}}{(1+b)^M}\left(\frac{b}{1+b}\right)^n\cdot$$
$$\cdot\frac{(n+M-1)!}{n!\,(M-1)!}\exp\left[-\frac{\langle n_C\rangle}{1+b}\right]{}_1F_1\left(-n+1, M+1; -\frac{\langle n_C\rangle}{b(1+b)}\right) \tag{25}$$

where we have utilized

$$\frac{\mathrm{d}}{\mathrm{d}z}\,{}_1F_1(\alpha,\gamma;z) = \frac{\alpha}{\gamma}\,{}_1F_1(\alpha+1, \gamma+1; z).$$

Combining (24) and (25) we obtain

$$\langle n^{k+1}\rangle = (\langle n_T\rangle + \langle n_C\rangle)\langle n^k\rangle +$$
$$+\langle n_T\rangle\left(1 + \frac{\langle n_T\rangle}{M}\right)\frac{\partial\langle n^k\rangle}{\partial\langle n_T\rangle} + \langle n_C\rangle\left(1 + 2\frac{\langle n_T\rangle}{M}\right)\frac{\partial\langle n^k\rangle}{\partial\langle n_c\rangle}. \tag{26}$$

As $\langle n\rangle = \langle W\rangle = \langle n_T\rangle + \langle n_C\rangle$ (we have the same result from (26) for $k = 0$) we have

$$\langle n^2\rangle = \langle n_T\rangle + \langle n_C\rangle + \langle n_T\rangle^2\frac{M+1}{M} + 2\langle n_T\rangle\langle n_C\rangle\frac{M+1}{M} + \langle n_C\rangle^2 =$$
$$= \langle W\rangle + \langle W^2\rangle, \tag{27a}$$

i.e.

$$\langle(\Delta n)^2\rangle = \langle n^2\rangle - \langle n\rangle^2 = \langle n_T\rangle + \langle n_C\rangle + \frac{\langle n_T\rangle^2 + 2\langle n_T\rangle\langle n_C\rangle}{M} =$$
$$= \langle W\rangle + \langle(\Delta W)^2\rangle. \tag{27b}$$

If we put $\langle n_C\rangle = 0$ we obtain the relations for the moments of the distribution (15) and for $M = 1$ Eq. (26) represents Eq. (A8) of the paper by Lachs [1]. From (27b) we can see that the quantity $\langle(\Delta n)^2\rangle$ for the whole field is a sum of the same quantities for single modes. The first and the second terms in (27b) correspond to the Poisson distribution, the third term represents the photon bunching of the thermal field and the fourth term represents the photon bunching due to interference effects.

The method given above enables us to calculate the probability distributions $P(W)$ and $p(n)$ in the case that the average numbers of photons $\langle n_{T\lambda}\rangle$ in individual modes are not the same. (See Eq. (11).) Nevertheless, the expressions are more complicated in this case.

4. COHERENT FIELD AS LIMIT CASE

From (8) we can see that in the limit $\langle n_{T\lambda}\rangle \to 0$ we obtain the function (6) for the coherent field. Therefore the expressions given above must converge in the limit $\langle n_{T\lambda}\rangle \to 0$ to the corresponding expressions for the coherent field.

If we use the asymptotic expressions for the functions $I_M(z)$ and ${}_1F_1(\alpha, \gamma; z)$, i.e. [14]

$$I_M(z) \underset{z\to\infty}{\cong} (2\pi z)^{-1/2} \exp z \tag{28}$$

and (see (A5), too)

$${}_1F_1(\alpha, \gamma; z) \underset{z\to\infty}{\cong} \frac{\Gamma(\gamma)}{\Gamma(\alpha)} z^{\alpha-\gamma} \exp z \tag{29}$$

we obtain from (12)

$$P(W) = \lim_{\langle n_T\rangle\to 0} \left(\frac{W}{\langle n_C\rangle}\right)^{\frac{M-1}{2}} \frac{1}{(\langle n_C\rangle W)^{1/4}} \frac{\exp\left[-\frac{(\sqrt{W} - \sqrt{\langle n_C\rangle})^2}{\langle n_T\rangle} M\right]}{2\sqrt{\pi}\sqrt{\frac{\langle n_T\rangle}{M}}} =$$
$$= \frac{\delta(\sqrt{W} - \sqrt{\langle n_C\rangle})}{2\sqrt{\langle n_C\rangle}} = \delta(W - \langle n_C\rangle), \tag{30}$$

where the term $\delta(\sqrt{W} + \sqrt{\langle n_C\rangle})/2\sqrt{\langle n_C\rangle}$, the value of which is zero ($\langle n_C\rangle > 0$), was added and from (13) we have

$$p(n) = \frac{\langle n_C\rangle^n}{n!} \exp[-\langle n_C\rangle]. \tag{31}$$

From (18) or (21) we obtain for $\langle n_T\rangle \to 0$: $\langle W^k\rangle = \langle n_C\rangle^k = \langle W\rangle^k$ and from (26):

$$\langle n^{k+1}\rangle = \langle n_C\rangle\langle n^k\rangle + \langle n_C\rangle \frac{\partial\langle n^k\rangle}{\partial\langle n_C\rangle}, \tag{32}$$

i.e. $\langle n^2 \rangle = \langle n_C \rangle^2 + \langle n_C \rangle$ and $\langle (\Delta n)^2 \rangle = \langle n \rangle$. These expressions correspond to the Poisson distribution (31).

Appendix

The function ${}_1F_1(\alpha, \gamma; z)$ can be expressed for $\alpha \geqq \gamma, \gamma \geqq 1$ (α, γ are integers) in the form

$$ {}_1F_1(\alpha, \gamma; z) = \frac{(\gamma - 1)!\,(\alpha - \gamma)!}{[(\alpha - 1)!]^2} L_{\alpha-\gamma}^{\gamma-1}(-z) \exp z, \tag{A1} $$

where $L_{\alpha-\gamma}^{\gamma-1}(z)$ is the Laguerre polynomial.

Putting $\alpha = \gamma + \beta$ ($\beta \geqq 0$) we have

$$ {}_1F_1(\gamma + \beta, \gamma; z) = \frac{(\gamma - 1)!}{(\gamma + \beta - 1)!} \sum_{k=0}^{\infty} \frac{(\gamma + \beta + k - 1)!}{k!\,(\gamma + k - 1)!} z^k, \tag{A2} $$

i.e.

$$ {}_1F_1(\gamma + \beta, \gamma; z) = \frac{(\gamma - 1)!}{(\gamma + \beta - 1)!} \sum_{k=0}^{\infty} \frac{(\gamma + k)(\gamma + k + 1)\ldots(\gamma + k + \beta - 1)}{k!} z^k. \tag{A3} $$

Now

$$ \frac{1}{z^{\gamma-1}} \left(\frac{\mathrm{d}}{\mathrm{d}z}\right)^{\beta} z^{k+\gamma+\beta-1} = (\gamma + k)(\gamma + k + 1)\ldots(\gamma + k + \beta - 1)\, z^k \tag{A4} $$

is valid and we obtain

$$ \begin{aligned} {}_1F_1(\gamma + \beta, \gamma; z) &= \frac{(\gamma - 1)!}{(\gamma + \beta - 1)!\, z^{\gamma-1}} \left(\frac{\mathrm{d}}{\mathrm{d}z}\right)^{\beta} \{z^{\beta+\gamma-1} \exp z\} = \\ &= \frac{(\gamma - 1)!\,\beta!}{[(\gamma + \beta - 1)!]^2} L_{\beta}^{\gamma-1}(-z) \exp z = \\ &= (\gamma - 1)!\,\beta! \exp z \,.\, \sum_{m=0}^{\beta} \frac{z^m}{m!\,(\beta - m)!\,(\gamma + m - 1)!}\,. \end{aligned} \tag{A5} $$

For $p(n)$ and $\langle W^k \rangle$ we have, with the aid of (A5),

$$ \begin{aligned} p(n) &= \frac{1}{(n + M - 1)!} \left(1 + \frac{M}{\langle n_T \rangle}\right)^{-n} \left(1 + \frac{\langle n_T \rangle}{M}\right)^{-M} . \\ &. \exp\left[-\frac{\langle n_C \rangle M}{\langle n_T \rangle + M}\right] L_n^{M-1}\left(-\frac{\langle n_C \rangle M^2}{\langle n_T \rangle (\langle n_T \rangle + M)}\right) \end{aligned} \tag{A6} $$

and

$$ \begin{aligned} \langle W^k \rangle &= \left\langle \frac{n!}{(n - k)!} \right\rangle = \frac{k!}{(k + M - 1)!} \frac{\langle n_T \rangle^k}{M^k} L_k^{M-1}\left(-\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right) = \\ &= k!\,(k + M - 1)! \frac{\langle n_T \rangle^k}{M^k} \sum_{m=0}^{k} \frac{1}{m!(k - m)!\,(m + M - 1)!} \left(\frac{\langle n_C \rangle M}{\langle n_T \rangle}\right)^m . \end{aligned} \tag{A7} $$

Expressing the moments $\langle n^k \rangle$ by means of $\langle W^j \rangle$ ($j = 1, 2, \ldots, k$) on the basis of (4) [8] we can easily compute the moments $\langle n^k \rangle$ for arbitrary k.

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Shrnutí

SUPERPOSICE TERMÁLNÍCH A KOHERENTNÍCH POLÍ

Jan Peřina

Výsledky obdržené Lachsem a Troupem pro superposici jednomodového termálního a koherentního pole jsou zobecněny na případ superposice M modových polí. Jsou vypočteny rozdělení pravděpodobnosti $P(W)$ integrované intenzity W a $p(n)$ počtu registrovaných fotonů (emitovaných fotoelektronů) n a jejich momenty. Výrazy pro koherentní a termální pole jsou obdrženy jako limitní případy.