Abstracts of theses in mathematics

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ABSTRACTS OF Ph.D. THESES IN MATHEMATICS

defended recently at Charles University, Prague

DEVELOPMENT OF TEACHING COMPLEX NUMBERS AT CZECH HIGH SCHOOLS SINCE EXNER-BONITZ PROSPECT (1849)

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(February 18, 2003; supervisor J. Potůček)

The development of the educational system in our country has been well mapped as a whole. However, we do not find enough dissertations on the history of classes in single subjects (including mathematics) and about the conceptual modifications in the actual instructional units. The chief task of this work is to explore the development of teaching the complex numbers at Czech high schools since 1849. The study deals with certain historical circumstances which had impact on the educational system reform, and it regards the system itself as well as the contents. The thematic aim of the dissertation makes it possible to focus on one part of the high school mathematics. Yet to retain the situational context, some of general connections have to be mentioned. The aim of the study is to expand on the former, rather generally written works on the subject and to clarify in this way some questions connected to the context development of teaching mathematics, textbooks and didactic problems as well.

SOME EXAMPLES FROM THE CALCULUS OF VARIATIONS

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(March 11, 2003; supervisor J. Malý)

In this Doctoral Dissertation Thesis, we give three examples. The first one and the second one are examples of a functional

$$F(u) = \int_I f(u, u')\, dt$$

which is not lower semicontinuous with respect to the L^1-convergence, although the function f is nonnegative and

(i) lower semicontinuous, linearly coercive and convex in the second variable;
(ii) continuous and strictly convex in the second variable.

These examples were published in the papers

R. Černý and J. Malý, *Counterexample to lower semicontinuity in Calculus of Variations*, Math. Z. **238** (2001), 689–694,

R. Černý and J. Malý, *Another Counterexample to Lower Semicontinuity in Calculus of Variations*, Journal of Convex Analysis **9** (2002), 295–299.

The third example is connected with open problems on existence and regularity of minimal surfaces. We give a family of measures $\mu_{a,c}$ in $\mathbb{R}^3$, whose tangent measures are non-unique and non-conical. Moreover, for every $\varepsilon > 0$ there exists $\mu_{a,c}$ such that $\mu_{a,c} + \varepsilon \cdot \mathcal{H}^1 \llcorner (\{0\} \times \{0\} \times \mathbb{R})$ is a one-monotone measure and has the one-dimensional density in $\{\varepsilon\} \cup (1, 1 + 2\varepsilon)$ everywhere in its support. This result will be published in a prepared paper written together with Jan Kolář.

ROBUST CHANGE-POINT DETECTION IN THE LOCATION MODEL

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In a complex way the thesis deals with the area of statistical detection of changes in the location model. Moreover, some results go beyond the location model as they work with stochastic dominance instead of parametric description.

The following issues are particularly emphasized:

1. robustness,
2. behaviour of given statistic under various types of alternatives,
3. relation between testing statistics and change points.

Some rank and permutation tests are considered, their properties are thoroughly studied and compared with properties of selected well-known tests. Some beyond literature properties of the well-known tests are treated as well.

The theoretical part deals with the construction of appropriate statistics with regard to point 3 and with weak asymptotic properties of the statistics in question. The construction of reasonable critical values is the consecutive issue. The theoretical part is accompanied by vast simulations whose results are interpreted in view of the achieved theoretical results and of other asymptotic-type arguments. Applications to both simulated and real data are included, too. In these applications, some practical aspects such as the use of average ranks, impact of dependence, heavy tails effect, and suggesting the type of the change are studied.

Basic overview of rank and permutation tests is included in appendices. Further, one appendix collectively deals with computational aspects where testing the employed pseudo-random number generator, computational speed and S-Plus programming tricks form the main topics. In the end the thesis contains tables of critical values for the statistics in question.

REAL ANALYSIS METHODS IN FUNCTION SPACES

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The doctoral thesis consists of the following five research papers:

[D1] *Isometrical embeddings of separable Banach spaces into the set of nowhere approximatively differentiable and nowhere Hölder functions*, Proc. Amer. Math. Soc. **128** (2000), 3505–3511
[D2] *On the notions of absolute continuity for functions of several variables*, Fund. Math. **173** (2002), 175-189
[D3] *Mappings of finite distortion: Hausdorff measure of zero sets*, joint paper with J. Malý, Math. Ann. **324** (2002), 451-46
[D4] *Measures of non-compactness of classical embeddings of Sobolev spaces*, to appear in Math. Nachr.
[D5] *A sharp form of an embedding into exponential and double exponential spaces*, to appear in J. Funct. Anal.

The famous Banach-Mazur theorem says that every separable Banach space can be isometrically embedded into $C([0,1])$. Recently L. Rodriguez-Piazza [RP] has shown that this embedding can have the property that the image of each nonzero element is a nowhere differentiable function.

We extend this result in [D1] using a more complicated construction which uses an idea of J. Malý and L. Zajíček [MZ]. We prove that every separable Banach space can be isometrically embedded into $C([0,1])$ in a way that the image of each nonzero element is a nowhere approximatively differentiable and nowhere Hölder function.

Absolutely continuous functions of one variable are admissible transformations for the change of variables in Lebesgue integral. Recently J. Malý [M] introduced a class of n-absolutely continuous functions giving an n-dimensional analog of the notion of absolute continuity from this point of view.

In the second paper [D2] we consider a more general definition of absolutely continuous functions suggested by L. Zajíček. Analogously to [M] the functions from our class are continuous, weak differentiable with gradient in L^n, differentiable almost everywhere and the formula on change of variables is valid for transformations from this class.

The important difference between the classes is not only that our class is wider. A result of M. Csörnyei [C] states that 2-absolutely continuous functions with respect to balls in the sense of [M] are not the same as 2-absolutely continuous functions with respect to cubes. On the contrary our definition does not depend on the shape of the "ball".

Let $\Omega \subset \mathbb{R}^n$ be an open set and $f : \Omega \to \mathbb{R}^n$ be a mapping whose derivative Df in the sense of distributions is a locally integrable matrix-valued function. We denote by J_f the pointwise Jacobian of f. We suppose that there is a measurable function $K : \Omega \to [1, \infty)$ such that

$$|Df(x)|^n \le K(x) J_f(x) \quad \text{a.e. in } \Omega.$$

Then f is said to be a mapping of finite distortion.

The main goal of the theory of mappings of finite distortion is to look for weak assumptions for regularity and invertibility properties, which were achieved originally for holomorphic functions (to which the theory reduces when $n = 2$ and $K \equiv 1$) and then for quasiregular mappings (the case that K is bounded).

In the paper [D3], which is a joint work with J. Malý, we proved that for a mapping f of finite distortion $K \in L^{p/(n-p)}$, the $(n-p)$-Hausdorff measure of any point preimage is zero provided J_f is integrable, $Df \in L^s$ with $s > p$, and the multiplicity function of f is essentially bounded. As a consequence for $p = n-1$ we obtain that the mapping is then open and discrete. This partially solves a famous open problem by Iwaniec and Šverák [IŠ].

The measure of non-compactness of the bounded linear mapping $T : X \to Y$ between two Banach spaces is defined by

$$\beta(T) =$$

$$\inf\Big\{\epsilon > 0 : \text{ there are } k \in \mathbb{N} \text{ and } c_j \in Y \text{ such that } T(U_X) \subset \bigcup_{j=1}^{k} (c_j + \epsilon U_Y)\Big\},$$

where U_X and U_Y denote the unit balls in the spaces X and Y. Among the basic properties let us mention that $\beta(T) = 0$ if and only if T is compact.

In the paper [D4] we have studied the measure of non-compactness of various non-compact embeddings of Sobolev spaces. Let Ω be an open subset of $\mathbb{R}^n$. We proved that the measure of non-compactness of the Sobolev embedding $I : W_0^{k,p}(\Omega) \to L^{p^*}(\Omega)$ for $k \in \mathbb{N}$, $1 \le p < \infty$, $kp < n$ and $p^* = \frac{np}{n-kp}$ is equal to its norm. The same is true, when $\mathcal{L}_n(\Omega)$ is small enough, for the embedding of $W_0^{1,n}(\Omega)$ into the Orlicz space with Young function $\exp(t^{n/(n-1)}) - 1$. The position is different for the embedding of $W_0^{1,p}(J)$ in $C^{0,1-1/p}(\overline{J})$, $J = (0,1)$, when $p \in (1, \infty)$: in this case the measure of non-compactness is less than the norm.

Let Ω be a bounded domain in $\mathbb{R}^n$, $n \ge 2$. In the well-known paper [Mo] Moser proved that for $K \ge n^{-(n-1)/n} \omega_{n-1}^{-1/n}$ we have

$$\sup\left\{\int_\Omega \exp((f(x)/K)^{n'}) : f \in W_0^{1,n}(\Omega), \|\nabla f\|_{L^n} \le 1\right\} < \infty$$

but that for $K < n^{-(n-1)/n}\omega_{n-1}^{-1/n}$ the integral $\int_\Omega \exp((f(x)/K)^{n'})$ can be made arbitrarily large by an appropriate choice of a function $f \in W_0^{1,n}(\Omega)$, $\|\nabla f\|_{L^n} \le 1$.

In [D5] we extended this result to the situation in which the underlying space L^n is replaced by the Zygmund space $L^n \log^\alpha L$ ($\alpha < n-1$), the corresponding space of exponential growth then being given by a Young function which behaves like $\exp(t^{n/(n-1-\alpha)}) - 1$ for large t. The case when $\alpha = n-1$, which gives rise to a space of double exponential growth, is also discussed.

References

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[Mo] Moser J., *A sharp form of an inequality by N. Trudinger*, Indiana Univ. Math. J. **20** (1971), 1077–1092.

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FORMAL CONCEPT ANALYSIS

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(September 30, 2003; supervisor L. Beran)

Formal concept analysis has been developed in the last twenty years mainly to support data analysis and knowledge processing. The mathematical foundation is grounded on the notions of a formal context, derivation operators, formal concept, subconcept–superconcept–relation and concept lattice. Given regular contexts of lattices L_1 and L_2, we describe how to construct regular context of their direct product. We also show that the regular contexts of the ordinal sum of two lattices can be constructed from the contexts determined by L_1 and L_2 in the case of the direct sum and we establish that the regular context of the ordinal product of L_1 and L_2 can be deduced from the order matrix of L_1 and from the context determined by L_2. To demonstrate the possibilities of data processing by the formal concept analysis method we have given examples from the urological praxis which concerns the prostate carcinoma, benign prostate hyperplasia and erectile dysfunction.

CONNECTIONS BETWEEN CODES, GROUPS AND LOOPS

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The thesis presents new results on loops, mostly Moufang, and on their connections to codes of high divisibility level and to classical groups.

1) Combinatorial polarization is a process similar the the principle of inclusion and exclusion. It can be used to construct a class of Moufang loops, called code loops, starting with an arbitrary doubly even code. The process is generalized to codes of higher divisibility level.

2) Many small Moufang loops are of type $M(G, 2)$, where G is a nonabelian subgroup of index 2. We show that the construction $M(G, 2)$ is unique, up to symmetry, in the sense that no Bol (and hence Moufang) loops are obtained by similar constructions. We also derive presentations for loops $M(G, 2)$ when G is generated by 2 elements. A visual description of the smallest Moufang loop is obtained.

3) All finite nonassociative simple Moufang loops stem from Zorn vector matrix algebras over finite fields. There are thus naturally many connections between these loops and classical groups, especially the Chevalley groups $D_4(q)$ and $G_2(q)$. We show that the automorphism group of the unique nonassociative simple Moufang loop over $GF(q)$ is isomorphic to the semidirect product of $G_2(q)$ and $\mathrm{Aut}(GF(q))$. We also show that every such loop is generated by 3 elements; an interesting sequel to the fact that every finite simple group is generated by 2 elements.

4) Two constructions due to Drápal yield all Moufang 2-loops up to order 32 by repeatedly modifying exactly one quarter of a single multiplication table for each order and each type of the associator subloop. Several thousands of Moufang loops of order 64 are obtained in this way by GAP, a first significant step toward the classification of these loops.

OPTIMIZATION IN CENTRAL BANK'S FOREIGN RESERVE MANAGEMENT

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(September 9, 2003; supervisor J. Dupačová)

The goal of the thesis is to compare different approaches of risk management of the foreign reserve portfolio of the central bank. The central bank holds its foreign reserve portfolio in order to practise its monetary policy through OTC operations. The purpose of managing such a portfolio is rather to conserve its value than maximize profit. Market risk of the portfolio can be managed in

different ways. The studied approaches are constant structure of the portfolio, risk limit given in duration, risk limit given in Value at Risk and risk limit given in Conditional Value at Risk. The models described in the thesis are designed to find optimal values for individual limits and also to evaluate performance of all studied approaches.

Another goal of the thesis is to develop software for performing all the calculations and also to facilitate further research of these methods.

INTEGRAL REPRESENTATION IN CONVEX AND STOCHASTIC ANALYSIS

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(September 9, 2003; supervisor J. Štěpán)

The thesis is devoted to two different questions. Both of them are focused on the possibility to represent some element as an integral. In the first case, we ask when we can represent an element x of some locally convex space E as the barycenter of some Radon probability measure ν concentrated on the extreme boundary of a given set C containing x. This question was first solved by G. Choquet in case when C is a compact convex metrizable set.

The basic concept of this part is Choquet order and the basic question is when a chain of Radon probability measures on C has an upper bound in this order concentrated on C. Any measure convex set C with this property is called a Choquet compact set. On these sets, we can use Tzorn lemma to obtain a boundary measure with a given barycenter $x \in C$. Special cases of chains are increasing nets. They have a special "martingale" type of convergence. In this connection, we define a measure affine map as a Lusin measurable map that commutes with the barycenter map and we show that measure affine mappings preserve this type of convergence. This is the main result of the first part. Further, we show that this theory generalizes the well-known non-compact Choquet theory.

The second part is related to the question, under which condition we can represent a centered random variable that can be observed from the history of some local martingale as an integral w.r.t. this process. This question has a positive answer in case that the process is a unique (in law) solution to the Engelbert-Schmidt equation. Then Girsanov theorem enables us to extend this result to the processes that are unique (in law) solutions to the generalized Black-Scholes equation

$$(1) \quad dX_i(t) = X_i(t)\mu(t,X)\,dt + X_i(t)\sigma(t,X)\,dW(t), \quad X_i(0) = x_i > 0, \quad i \le n,$$

where μ, σ are supposed to be bounded n or $n \times n$-dimensional progressive measurable coefficients, respectively. In this part, we generalize the well-known theory of option pricing started by Black and Scholes.

SOME PROBLEMS IN SMOOTHNESS AND RENORMINGS IN BANACH SPACES

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(October 29, 2003; supervisor P. Hájek)

This thesis consists of the following four papers: *Approximation of Lipschitz mappings*, Serdica Math. J. **29** (2003), no. 2, 141–148; *Smooth approximations without critical points*, joint work with P. Hájek, Central European J. Math. **1** (2003), no. 3, 284–291; *Characterization of reflexivity by equivalent renorming*, joint work with P. Hájek, to appear in J. Funct. Anal.; and *On harmonic behaviour*, joint work with P. Hájek, preprint.

The first paper deals with approximation of Lipschitz mappings by uniformly Gâteaux smooth Lipschitz mappings. Bogachev and Shkarin showed that any L-Lipschitz mapping $f\colon X \to Y$, where X is a separable Banach space and Y is a Banach space with the Radon-Nikodym property (RNP), can be approximated uniformly on X by a Gâteaux differentiable L-Lipschitz mapping. Their proof is rather simple (albeit not elementary) and it uses the infinite-dimensional Rademacher's theorem. By a different approach, we are able to show that the assumption for Y to have the RNP can be dropped and moreover we can make the resulting approximation uniformly Gâteaux differentiable. Our proof uses the integral convolution with smooth real bump functions in a countable dense set of directions. (This method was introduced by Fabian, Whitfield and Zizler.)

In the second paper we prove the statement that any continuous function on a separable Banach space admitting a C^k-smooth bump and containing c_0 can be uniformly approximated by a C^k-smooth function such that the image of its derivative is of the first category and can avoid any prescribed countable set. In our approach to the problem we exploit the fact that smooth functions that locally depend on finitely many coordinates on c_0 form a sufficiently rich structure. However, our method fails for the spaces with the RNP (in particular Hilbert, or even reflexive spaces).

In the third paper we characterize reflexive Banach spaces as those admitting an equivalent W2R norm. A norm $\|\cdot\|$ on a Banach space X is W2R if for every $\{x_n\} \subset B_X$ such that $\lim_{m,n\to\infty} \|x_m + x_n\| = 2$ there is an $x \in X$ such that $\lim_{n\to\infty} x_n = x$ in the weak topology.

The last paper focuses on the structure of Banach spaces Y such that all sufficiently smooth operators from B_X into Y, where $X = \ell_p$, have a property that we call "harmonic behaviour", that is $T(B_X) \subset \overline{T(\partial B_X)}$. The main result is that if $1 \le p < \infty$, p is not an even integer and $1 \ge \alpha > p - [p]$, then every $C^{[p],\alpha}$-smooth operator $T\colon B_{\ell_p} \to Y$ has a harmonic behaviour unless $\ell_{\frac{p}{K}} \subset Y$ for some $K \in \mathbb{N}$.

MATHEMATICAL MODELS FOR ACCOUNTING ACCORDING TO THE INTERNATIONAL ACCOUNTING STANDARDS AND FOR THE MIDDLE TERM PLANNING IN LIFE INSURANCE

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The topic of the thesis is middle term planning in the life insurance company and using of such plan for calculation of the profit & loss account according to the international account standards.

The complete formalization is the first step with the formulae for the net profit and for the cash flow as the results. The model is developed on monthly basis and for the thesis, insurance company offering only capital life insurance with the same sum assured for the case of death and for the case of survival the policy duration is taken into account. However, this factor is not too limiting and the thesis can be used as a guide for a company with a wide spectrum of life insurance products.

The second step is change of view on two input variables number of new policies sold and probability of policy canceling. They are considered as random and under some assumptions, the expectation and variance of cash flow are calculated.

The following part is dedicated to the above mentioned calculation of one part (profit of the new business) of the profit & loss account according to the new accounting standards.

In the last chapter, the numerical illustrations are made. Results were calculated for a simplified situation modeled in MS Excel.

FINITE ELEMENT APPROXIMATION OF A NONLINEAR PARABOLIC HEAT CONDUCTION PROBLEM AND A POSTERIORI ERROR ESTIMATORS

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The thesis is devoted to a nonlinear parabolic heat conduction problem, to its finite element approximation and especially to *a posteriori* error estimates. We consider the following nonlinear parabolic equation

$$c\varrho\partial_t u - \nabla\cdot(\mathcal{A}(x,t,u)\nabla u) = f \quad \text{in } \Omega\times(0,T), \tag{1}$$

which together with initial and Newton boundary conditions

$$\begin{aligned} u &= u_0 \quad \text{in } \Omega \text{ and for } t=0,\\ \alpha u + \mathcal{A}\nabla u\cdot\nu &= g \quad \text{on } \partial\Omega\times[0,T] \end{aligned}$$

describes the heat conduction in nonhomogeneous and anisotropic media.

The thesis consists of three parts. The fundamental results of each part have been published as scientific papers and are included in the thesis. The first part is connected to the existence of a weak solution of (1) and deals with the theory of monotone operators. It is shown that the associated operator to equation (1) is not monotone (except for the linear case) and therefore, the theory of monotone operators generally cannot be used for the existence proof. In the second part, the comparison principle for problem (1) is proven. The third part is devoted to *a posteriori* error estimates. It surveys all main strategies with emphasis to a clear and brief description of the fundamental ideas. The author's contribution to the topic is the proof of asymptotic exactness of a hierarchical *a posteriori* error estimator for a nonlinear parabolic problem in one dimension. Finally, a description of a computer implementation of different *a posteriori* error estimators together with results of numerical experiments is given.