Homomorphic images of subdirectly irreducible groupoids

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Abstract. A groupoid H is a homomorphic image of a subdirectly irreducible groupoid G (over its monolith) if and only if H has a smallest ideal.

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Let $\mathcal{G}$ denote the class of all homomorphic images of subdirectly irreducible groupoids. It is easy to see that the additive semigroup of positive integers is not in $\mathcal{G}$. On the other hand, according to [3, 2.3], every groupoid possessing an absorbing element is in $\mathcal{G}$. Now, the aim of this very short note is to prove the following (somewhat surprising) result: a groupoid H is in $\mathcal{G}$ if and only if the intersection of all ideals of H is non-empty. Moreover, if $H \in \mathcal{G}$, then there exists a subdirectly irreducible groupoid G (G is finite if H is so) such that $H \simeq G/\mu$, where μ is the smallest non-trivial congruence of G.

1. Preliminaries

A *groupoid* is a non-empty set equipped with binary operation (usually denoted as multiplication). A non-empty subset I of a groupoid G is said to be an *ideal* of G if $GI \cup IG \subseteq I$ and we denote by $\mathrm{Int}(G)$ the intersection of all ideals of G. The following two lemmas are quite obvious.

1.1 Lemma. *Let G be a groupoid. Then $\mathrm{Int}(G)$ is either empty or an ideal of G. If the latter is true, then $\mathrm{Int}(G)$ is the smallest ideal of G. Moreover, G possesses an absorbing element o if and only if $\mathrm{Int}(G)$ is a one-element set; then $\mathrm{Int}(G) = \{o\}$.*

1.2 Lemma. *Let φ be a projective homomorphism of a groupoid G onto a groupoid H. If J is an ideal of H, then the inverse image $\varphi^{-1}(J)$ is an ideal of G. Consequently, $\varphi(\mathrm{Int}(G)) \subseteq \mathrm{Int}(H)$. In particular, if $\mathrm{Int}(G) \neq \emptyset$, then $\mathrm{Int}(H) \neq \emptyset$.*

A non-trivial groupoid G having a smallest non-trivial congruence μ_G is said to be *subdirectly irreducible* and μ_G is then called the *monolith* of G.

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1.3 Lemma. *Let G be a subdirectly irreducible groupoid. Then $I = \mathrm{Int}(G) \neq \emptyset$. Moreover, if $|I| \geq 2$, then $\mu_G \subseteq (I \times I) \cup \mathrm{id}_G$.*

Proof: If G contains an absorbing element o, then $\mathrm{Int}(G) = \{o\}$, and hence we will assume that G has no absorbing element. Now, being I an ideal of G, we have $|I| \geq 2$ and $\rho_I = (I \times I) \cup \mathrm{id}_G$ is a non-trivial congruence of G and so $\mu_G \subseteq \rho_I$. Consequently, if $(u,v) \in \mu_G$, $u \neq v$, then $(u,v) \in \rho_I$ and $u, v \in I$. Thus $u, v \in \mathrm{Int}(G)$. □

1.4 Corollary. *Let a groupoid H be a homomorphic image of a subdirectly irreducible groupoid G. Then $\mathrm{Int}(H) \neq \emptyset$.*

1.5 Example. If $G = \mathbb{N}(+)$ is the additive semigroup of non-negative integers, then $\mathrm{Int}(G) = \emptyset$.

1.6 Lemma. *If I, J are ideals of a groupoid G, then $IJ \cup JI \subseteq I \cap J$ and $I \cap J$ is an ideal of G.*

Proof: The result is obvious. □

1.7 Corollary. *The intersection of a non-empty finite set of ideals of G is again an ideal of G. Consequently, if the set of ideals of G is finite, then $\mathrm{Int}(G) \neq \emptyset$.*

1.8 Corollary. *If G is a finite groupoid, then $\mathrm{Int}(G) \neq \emptyset$.*

1.9 Example ([1]). Let $\mathbb{D}$ designate the set of rational numbers of the form $a/2^k$, where a, k are integers. For a positive integer n, let $\mathbb{D}_n$ be the n-th cartesian power of $\mathbb{D}$. Define an operation $\circ$ on $\mathbb{D}_n$ by $(r_1, \dots, r_n) \circ (s_1, \dots, s_n) = (\frac{1}{2}(r_1 + s_1), \dots, \frac{1}{2}(r_n + s_n))$. If H is a non-empty open convex subset of $\mathbb{D}_n$, then $H(\circ)$ is a subgroupoid of $\mathbb{D}_n(\circ)$ and $\mathrm{Int}(H(\circ)) = H$.

2. Main result

2.1 Construction. Let $n \geq 3$ be an odd number. We define a groupoid $Z_n(*)$ on the set $\{0, 1, \dots, n-1\}$ in the following way:

- $0 * m = 0$ for every $0 \leq m \leq n-1$;
- $k * 0 = 0$ for every odd $1 \leq k \leq n-2$;
- $l * 0 = 1$ for every even $2 \leq l \leq n-1$;
- $1 * m = m$ for every $1 \leq m \leq n-1$;
- $m * m = m$ for every $2 \leq m \leq n-1$;
- $k * l = k+1$ for all $2 \leq k \leq n-2$, $1 \leq l \leq n-1$, $k \neq l$;
- $(n-1) * l = 0$ for every $1 \leq l \leq n-2$.

It is easy to check that $Z_n(*)$ is a simple idempotent groupoid and that no right translation of this groupoid is a permutation. Moreover, 0 is a left absorbing element and 1 is a left neutral element.

2.2 Construction. Let $n \geq 4$ be an even number. We define a groupoid $Z_n(*)$ on the set $\{0, 1, \ldots, n-1\}$ in the following way:

- $0 * m = 0$ for every $0 \leq m \leq n-1$;
- $k * 0 = 1$ for every odd $1 \leq k \leq n-1$;
- $l * 0 = 0$ for every even $2 \leq l \leq n-2$;
- $1 * m = m$ for every $1 \leq m \leq n-1$;
- $m * m = m$ for every $2 \leq m \leq n-1$;
- $k * l = k+1$ for all $2 \leq k \leq n-2$, $1 \leq l \leq n-1$, $k \neq l$;
- $(n-1) * l = 0$ for every $1 \leq l \leq n-1$.

Again, $Z_n(*)$ is a simple idempotent groupoid whose no right translation is a permutation. The element 0 is left absorbing.

The groupoid $Z_2(*)$ on the set $\{0, 1\}$ is defined in the following way: $1 * 0 = 0, 0 * 0 = 0 * 1 = 1 * 1 = 1$.

2.3 Construction. Let κ be an infinite cardinal number. We define a groupoid $Z_\kappa(*)$ on the set κ in the following way:

- $0 * \alpha = 0$ for every $0 \leq \alpha < \kappa$;
- $(\beta + k) * 0 = 1$ for all limit ordinals $\beta < \kappa$ and finite even numbers $k \geq 0$, $\beta + k \neq 0$;
- $(\beta + l) * 0 = 0$ for all limit ordinals $\beta < \kappa$ and finite odd numbers $l \geq 1$;
- $1 * \alpha = \alpha$ for every $1 \leq \alpha < \kappa$;
- $\alpha * \alpha = \alpha$ for every $2 \leq \alpha < \kappa$;
- $\alpha * \beta = \alpha + 1$ for all $2 \leq \alpha < \kappa$ and $1 \leq \beta < \kappa$.

The groupoid $Z_\kappa(*)$ is simple and idempotent and none of its right translations is a permutation. The element 0 is left absorbing and the element 1 is left neutral.

2.4 Theorem. *The following conditions are equivalent for a groupoid H:*

(i) *H is a homomorphic image of a subdirectly irreducible groupoid;*
(ii) $\mathrm{Int}(H) \neq \emptyset$*;*
(iii) *H possesses a smallest ideal;*
(iv) *H is isomorphic to G/μ_G for a subdirectly irreducible groupoid G.*

PROOF: In view of 1.4, it is enough to show that (iii) implies (iv). Hence, let $I = \mathrm{Int}(H)$, $K = H \setminus I$ and $\kappa = \max(|I|, |K|)$.

If $\kappa = 1$, then $|H| \leq 2$. If $|H| = 1$, then G can be chosen to be any simple groupoid. If $|H| = 2$, then $|I| = 1$, i.e. $I = \{o\}$, where o is an absorbing element of $H = \{o, a\}$. We take $G = H \cup \{b\}$, $b \notin H$, and put $u \circ v = uv$ for all $u, v \in H$, $a \circ b = b$, $o \circ b = b \circ o = b \circ a = b \circ b = o$. Clearly, $G(\circ)$ is a subdirectly irreducible groupoid and $H \simeq G/\mu_G$.

Now, assume that $\kappa \geq 2$ and $K \neq \emptyset$.

2.4.1 Lemma. *There exist permutations $\pi_{a,u} \in \kappa!$ for all $a \in I, u \in K$, such that the following two conditions are satisfied:*

(A) $\pi_{a,u} \neq \pi_{a,v}$ *for all $a \in I, u, v \in K$;*

(B) *for all* $a, b \in I$, $a \neq b$, *and all* $0 \leq \alpha < \kappa$, *there exists some* $u \in K$ *with* $\pi_{a,u}(\alpha) \neq \pi_{b,u}(\alpha)$.

Proof: (a) Let $|I| = \kappa$. Choose $w \in K$, a bijection $\xi : I \to \kappa$ and a quasigroup $Q(\diamond)$ defined on κ. Put $\pi_{a,w}(\alpha) = \alpha \diamond \xi(a)$ for all $a \in I$ and $\alpha < \kappa$. Now it is easy to find the remaining permutations $\pi_{a,u}$, $a \in I$, $u \in K \setminus \{w\}$.

(b) Let $|I| < \kappa, 4 \leq \kappa$. Choose a permutation $\rho \in \kappa!$ without fix points, an injective mapping $\xi : I \to K$ and permutations $\pi_u \in \kappa!$, $u \in K$ such that $\rho\pi_u \neq \pi_v \neq \pi_u$ for all $u, v \in K$, $u \neq v$. Now, define $\pi_{a,u} = \pi_u$ for all $a \in I$, $u \in K$ such that $\xi(a) \neq u$ and $\pi_{b,\xi(b)} = \rho\pi_{\xi(b)}$ for every $b \in I$.

(c) Let $|I| < \kappa \leq 3$. This case is easy. □

Put $G = (I \times \kappa) \cup K$ and define an operation $\circ$ on G in the following way:

- $u \circ v = uv$ for all $u, v \in K$ such that $uv \in K$;
- $u \circ v = (uv, 0)$ for all $u, v \in K$ such that $uv \in I$;
- $(a, \alpha) \circ (b, \beta) = (ab, \alpha * \beta)$ for all $a, b \in I$, $0 \leq \alpha, \beta < \kappa$ (the operation $*$ on κ is defined in 2.1, 2.2 and 2.3);
- $u \circ (a, \alpha) = (ua, \pi_{a,u}(\alpha))$ for all $a \in I, u \in K, 0 \leq \alpha < \kappa$;
- $(a, \alpha) \circ u = (au, \pi_{a,u}(\alpha))$ for all $a \in I, u \in K, 0 \leq \alpha < \kappa$.

For $a \in I$, let $I_a = \{a\} \times \kappa \subseteq I \times \kappa$.

2.4.2 Lemma. *The groupoid* $G(\circ)$ *is subdirectly irreducible and*

$$\mu_{G(\circ)} = \bigcup_{a \in I} (I_a \times I_a) \cup \mathrm{id}_G\,.$$

Proof: It is clear that $\mu = \bigcup_{a \in I} (I_a \times I_a) \cup \mathrm{id}_G$ is a non-trivial congruence of G and we have to show that $\mu \subseteq \nu$ for any non-trivial congruence of $G(\circ)$. For this purpose, put $J = \{a \in I : I_a \times I_a \subseteq \nu\}$. If $\kappa = 2$, then obviously $JI \cup IJ \subseteq J$. If $\kappa \geq 3$, $a \in J$, $b \in I$ and $0 \leq \alpha < \kappa$, then $((a, 0), (a, \alpha)) \in \nu$, and therefore $((ab, 0), (ab, \alpha)) = ((a, 0) \circ (b, \alpha), (a, \alpha) \circ (b, \alpha)) \in \nu$. From this $ab \in J$ and, quite similarly, $ba \in J$. Thus $JI \cup IJ \subseteq J$. If $a \in J$, $u \in K$ and $0 \leq \alpha, \beta < \kappa$, then $((au, \pi_{a,u}(\alpha)), (au, \pi_{a,u}(\beta))) = ((a, \alpha) \circ u, (a, \beta) \circ u) \in \nu$. Since $\pi_{a,u}$ is a permutation, we get $au \in J$. Quite similarly, $ua \in J$, and we conclude that J is an ideal of G, $J = I$ and $\mu \subseteq \nu$, provided that $J \neq \emptyset$. Consequently, it remains to show that J is nonempty. This will be done in next five steps.

(1) Assume that $((a, \alpha), (a, \beta)) \in \nu$ for some $a \in I$ and $0 \leq \alpha < \beta < \kappa$.
If $\alpha = 0$ and $\beta = 1$, then, using the right translation by (a, γ) for all $0 \leq \gamma < \kappa$, we get $aa \in J$.
If $\alpha * 0 \neq \beta * 0$, then using the right translation by $(a, 0)$, we get $((aa, 0), (aa, 1)) \in \nu$, and hence $aa \cdot aa \in J$.
Finally, if $\alpha * 0 = \beta * 0$ and $2 \leq \beta$, then using the right translation by (a, α) for $\alpha \neq 0$ and by $(a, 1)$ for $\alpha = 0$, we get $((aa, \alpha), (aa, \beta^{\oplus})) \in \nu$

(here $\beta^{\oplus} = \beta + 1$ for κ infinite or κ finite and $\beta \leq \kappa - 2$, and $\beta^{\oplus} = 0$ for κ finite and $\beta = \kappa - 1$). According to the preceding part of the proof, we have $(aa \cdot aa)(aa \cdot aa) \in J$.

(2) Assume that $((a, \alpha), (b, \beta)) \in \nu$ for some $a, b \in I$, $a \neq b$, $0 \leq \alpha < \beta < \kappa$ and take an arbitrary $c \in I$.
If $2 \leq \alpha$, then, applying the right translations by $(c, 1)$ and (c, α), we get $((ac, \alpha^{\oplus}), (bc, \beta^{\oplus})) \in \nu$ and $((bc, \beta^{\oplus}), (ac, \alpha)) \in \nu$. So $((ac, \alpha^{\oplus}), (ac, \alpha)) \in \nu$ and our result follows from (1).
If $\alpha = 1$, then, applying the right translations by (c, γ) and $(c, 1)$, where $2 \leq \gamma \neq \beta$, we get $((ac, \gamma), (bc, \beta^{\oplus})) \in \nu$ and $((bc, \beta^{\oplus}), (ac, 1)) \in \nu$. Thus $((ac, \gamma), (ac, 1)) \in \nu$ and (1) applies again.
If $\alpha = 0$ and $2 \leq \beta$, then, using the right translations by (c, γ) and (c, β), where $1 \leq \gamma \neq \beta$, we get $((ac, 0), (bc, \beta^{\oplus})) \in \nu$ and $((ac, 0), (bc, \beta)) \in \nu$. That is $((bc, \beta^{\oplus}), (bc, \beta)) \in \nu$ and (1) takes place.
If $\alpha = 0$, $\beta = 1$ and $3 \leq \kappa$, then, because of the right translations by $(c, 2)$ and $(c, 1)$, we get $((ac, 0), (bc, 2)) \in \nu$ and $((ac, 0), (bc, 1)) \in \nu$. It follows that $((bc, 2), (bc, 1)) \in \nu$ and (1) makes the job.
Finally, if $\alpha = 0$, $\beta = 1$ and $\kappa = 2$, then, because of the right translations by $(c, 1)$ and $(c, 0)$, we get $((ac, 1), (bc, 1)) \in \nu$ and $((ac, 1), (bc, 0)) \in \nu$. So $((bc, 1), (bc, 0)) \in \nu$ and (1) works.

(3) Assume that $((a, \alpha), (b, \alpha)) \in \nu$ for some $a, b \in I$, $a \neq b$ and $0 \leq \alpha < \kappa$. Then, by (B) (see 2.4.1), there is $u \in K$ such that $\beta = \pi_{a,u}(\alpha) \neq \pi_{b,u}(\alpha) = \gamma$. Thus, using the right translation by u, we get $((au, \beta), (bu, \gamma)) \in \nu$. Now, either (1) or (2) can be used.

(4) Assume that $((a, \alpha), u) \in \nu$ for some $a \in I$, $u \in K$ and $0 \leq \alpha < \kappa$.
If $ba \neq bu$ for some $b \in I$, then, using the left translation by $(b, 0)$, we get $((ba, 0 * \alpha), (bu, \pi_{u,b}(0)))$ and either (2) or (3) can be used.
If $\kappa \geq 3$ and $ca = cu$ for some $c \in I$, then, using the facts that $\pi_{c,u}$ is a permutation of κ, but no right translation of $Z_\kappa(*)$ is a permutation, we find $0 \leq \beta < \kappa$ such that $\beta * \alpha \neq \pi_{c,u}(\beta) = \gamma$. We apply the left translation by (c, β) and we get $((ca, \beta * \alpha), (cu, \gamma)) \in \nu$. Thus (1) takes place.
The case $\kappa = 2$ is clear.

(5) Assume that $(u, v) \in \nu$ for some $u, v \in K$, $u \neq v$, and take arbitrary $a \in I$. By (A) (see 2.4.1), there is $0 \leq \alpha < \kappa$ such that $\beta = \pi_{a,u}(\alpha) \neq \pi_{a,v}(\alpha) = \gamma$. Now, applying the left translation by (a, α), we get $((au, \beta), (av, \gamma)) \in \nu$. Thus at least one of (1) and (2) can be used.

□

2.4.3 Lemma. $G(\circ)/\mu_{G(\circ)} \simeq H$.

Proof: Easy to see. □

Now, we will discuss the case $K = \emptyset$, i.e. H is an ideal-free groupoid and $I = \operatorname{Int} H = H$. Put $G = H \times \kappa$ and define an operation $\circ$ on H by $(a,\alpha)\circ(a,\beta) = (aa, \beta * \alpha)$ and $(a,\alpha)\circ(b,\beta) = (ab, \alpha * \beta)$ for all $a, b \in H$, $a \neq b$, $0 \leq \alpha, \beta < \kappa$. For $a \in H$, let $H_a = \{a\} \times \kappa$.

2.4.4 Lemma. *The groupoid $G(\circ)$ is subdirectly irreducible and*

$$\mu_{G(\circ)} = \bigcup_{a \in H} (H_a \times H_a).$$

Proof: We have to show that $\mu = \bigcup_{a \in I} (H_a \times H_a) \subseteq \nu$ for any non-trivial congruence ν of $G(\circ)$. Proceeding similarly as in the proof of 2.4.2, it is sufficient to check that $H_a \times H_a \subseteq \nu$ for at least one $a \in H$. This will be done in the next three steps.

(1) Assume that $((a,\alpha),(a,\beta)) \in \nu$ for some $a \in H$ and $0 \leq \alpha < \beta < \kappa$. Now, using left translations instead of the right ones, we can proceed similarly as in 2.4.2 (1).
(2) Assume that $((a,\alpha),(b,\beta)) \in \nu$ for some $a, b \in H$, $a \neq b$, and $0 \leq \alpha < \beta < \kappa$. If $\kappa \geq 3$, then we can proceed similarly as in 2.4.2 (2); we have to choose $a \neq c \neq b$. The case $\kappa = 2$ is clear.
(3) Assume that $((a,\alpha),(b,\alpha)) \in \nu$ for some $a, b \in H$, $a \neq b$ and $0 \leq \alpha < \kappa$. There is $0 \leq \beta < \kappa$ such that $\alpha * \beta \neq \beta * \alpha$, and hence, using the right translation by (a,β), we get $((aa, \beta * \alpha),(ba, \alpha * \beta)) \in \nu$. Now, either (1) or (2) takes place.

□

2.4.5 Lemma. $G(\circ)/\mu_{G(\circ)} \simeq H$.

Proof: Easy to see. □

The proof of Theorem 2.4 is completed. □

2.5 Corollary. *Let H be a finite groupoid, $|H| = n$ and $|\operatorname{Int}(H)| = m$. Then there exists a finite subdirectly irreducible groupoid G such that $G/\mu_G \simeq H$. Moreover, G can be chosen in such a way that*

(1) $|G| = 2$ *if* $n = 1$;
(2) $|G| = 3$ *if* $n = 2$;
(3) $|G| = m^2 + (n - m)$ *if* $n \geq 3$ *and* $n \leq 2m$;
(4) $|G| = (m + 1)(n - m)$ *if* $n \geq 3$ *and* $n > 2m$.

2.6 Remark. The results can be easily strengthened to all algebras with at least one at least binary operation. Given algebra H of signature Σ (with all symbols of finite arity) with operations $(o_\sigma : \sigma \in \Sigma)$, a non-empty subset I of H is said to be an *ideal* of H if for every symbol $\sigma \in \Sigma$ of arity $n \geq 1$ and for every

$x_1, \dots, x_n \in H$ it holds $o_\sigma(x_1, \dots, x_n) \in I$ whenever $x_i \in I$ for at least one i. We denote by $\mathrm{Int}(H)$ the intersection of all ideals of H.

Now, Theorem 2.4 and Corollary 2.5 hold also for all algebras with signature containing at least one symbol of arity at least 2. Obviously the statements 1.1–1.4 work for such algebras. So it remains to construct the subdirectly irreducible algebra G with operations $(g_\sigma : \sigma \in \Sigma)$ satisfying condition 2.4 (iv). It is defined on the same set $(I \times \kappa) \cup K$ in the following way (for every symbol $\sigma \in \Sigma$):

- $g_\sigma(u_1, \dots, u_n) = o_\sigma(u_1, \dots, u_n) = v$ for all $u_1, \dots, u_n \in K$ such that $v \in K$;
- $g_\sigma(u_1, \dots, u_n) = (o_\sigma(u_1, \dots, u_n), 0)$ for all $u_1, \dots, u_n \in K$ not satisfying previous condition;
- if σ is unary, then $g_\sigma((a, \alpha)) = (o_\sigma(a), \alpha)$ for every $a \in I$, $0 \le \alpha < \kappa$;
- if σ is at least binary, then

$$g_\sigma(u_1, \dots, u_k, (a, \alpha), u_{k+1}, \dots, u_{n-1}) = \\ = (o_\sigma(u_1, \dots, u_k, a, u_{k+1}, \dots, u_{n-1}), \pi_{a,u_1}(\alpha))$$

  for all $a \in I$, $u_1, \dots, u_{n-1} \in K$, $0 \le \alpha < \kappa$, $k = 0, \dots, n-1$;
- if σ is at least binary, then $g_\sigma(x_1, \dots, x_n) = (o_\sigma(\xi x_1, \dots, \xi x_n), \alpha_1 * (\alpha_2 \odot \dots \odot \alpha_k))$ for all $x_1, \dots, x_n \in G$ such that $x_{i_1}, \dots, x_{i_k} \in I$, $k \ge 2$, denoting $x_{i_j} = (a_j, \alpha_j)$, $j = 1, \dots, k$, and $\xi : G \to H$, $\xi|_K = \mathrm{id}_K$, $\xi(a, \alpha) = a$ for all $a \in I$, $0 \le \alpha < \kappa$, and $\odot$ some group operation on κ.

It is easy to see that in the case of groupoids (i.e. Σ contains one binary operation) this definition gives precisely the same subdirectly irreducible groupoid as in the proof of Theorem 2.4. In fact, the proof of property 2.4 (iv) in the general case can be done easily following the proof of this theorem. We omit this proof because of its technical difficulty and absence of any new ideas.

On the other hand, if the signature of the given algebra contains only unary operations, this result does not work anymore. Any suitable characterization is not known yet.

2.7 Remark. Let $\mathcal{F}$ denote the class of finite groupoids and $\mathcal{H}$ the class of all groupoids $H \in \mathcal{F}$ such that $H \simeq G/\mu_G$ for a finite subdirectly irreducible groupoid G with $|G| = |H| + 1$. According to [2, 4.11] and [3, 2.4] (see also [4]), the following groupoids belong to $\mathcal{H}$:

(1) finite groupoids with zero multiplication,
(2) finite quasigroups,
(3) finite simple groupoids,
(4) finite subdirectly irreducible groupoids H such that the monolith contains at least one pair (a, b) with $aa = a \neq b$.

On the other hand, any "reasonable" characterization of $\mathcal{H}$ seems to be an open problem.

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