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On locally r -incomparable families of infinite-dimensional Cantor manifolds

VITALIJ A. CHATYRKO

Abstract. The notion of locally r -incomparable families of compacta was introduced by K. Borsuk [KB]. In this paper we shall construct uncountable locally r -incomparable families of different types of finite-dimensional Cantor manifolds.

Keywords: Cantor manifolds, countable-dimensional, weakly infinite-dimensional, strongly infinite-dimensional

Classification: 54F45

0. Introduction

Throughout this note we shall consider only separable metrizable spaces. By dimension we mean the covering dimension dim .

A subset L of a space X is a partition in X if there exist two non-empty open in X subsets U and V such that $L = X \setminus (U \cup V)$. We say in this case that X is separated by L .

An infinite-dimensional Cantor manifold is an infinite-dimensional compact space which cannot be separated by any finite-dimensional subspace.

There exist different types of infinite-dimensional Cantor manifolds. In particular, there exist countable-dimensional Cantor manifolds [Ch1], [O], weakly infinite-dimensional Cantor manifolds which cannot be separated by any countable-dimensional subspace (as recently showed by E. Pol [EP]) and even strongly infinite-dimensional Cantor manifolds which cannot be separated by any weakly infinite-dimensional subspace.

The last type of infinite-dimensional Cantor manifolds can be obtained as follows. It is well known that every strongly infinite-dimensional compact space contains an hereditarily strongly infinite-dimensional closed subset (see for example [R-S-W]). Every hereditarily infinite-dimensional compact space contains an infinite-dimensional Cantor manifold ([T]). Thus every strongly infinite-dimensional compact space contains hereditarily strongly infinite-dimensional Cantor manifold. Note that every hereditarily strongly infinite-dimensional Cantor manifold cannot be separated by any weakly infinite-dimensional subspace.

We shall call two compact spaces A, B injectively different if A does not embed into B and vice versa. A family \mathcal{A} of compacta is injectively different if every two different elements $A, B \in \mathcal{A}$ are injectively different.

E. Pol proved the following

Theorem 0.1 ([EP]). *There exists an injectively different family \mathcal{A} ($|\mathcal{A}| = 2^{\aleph_0}$) of hereditarily infinite-dimensional Cantor manifolds.*

Remark 0.1. The proof of Theorem 0.1 is based on the existence of hereditarily infinite-dimensional compact spaces. The existence of weakly infinite-dimensional hereditarily infinite-dimensional compact spaces is an open question ([RP1]). If we use in the proof of Theorem 0.1 an hereditarily strongly infinite-dimensional compactum (which exists) we shall obtain that the family \mathcal{A} consists of hereditarily strongly infinite-dimensional Cantor manifolds.

Two compact spaces A, B are locally r -incomparable if any non-empty open subset of A does not embed into B and vice versa. A family \mathcal{A} of compacta is locally r -incomparable if every two different elements $A, B \in \mathcal{A}$ are locally r -incomparable.

This notion was introduced by K. Borsuk. It is well known that for every $n = 1, 2, \dots$ there exists an uncountable locally r -incomparable family of n -dimensional AR -compacta (see for example [KB]). Recently this fact was used in order to define a fractional dimension function satisfying Menger's axioms in the class of finite-dimensional locally compact spaces ([T-H]).

It is clear that every locally r -incomparable family of compacta is injectively different.

In this paper we shall construct uncountable locally r -incomparable families of named above types of infinite-dimensional Cantor manifolds.

1. Terminology and notation

The necessary information about notions and notations we use can be found in [A-P] and [E].

A space X is countable-dimensional (shortly c.d.) if X can be represented as a countable union of 0-dimensional subspaces.

A Cantor $trInd$ -manifold of class α , $\alpha < \omega_1$, is a compact space which cannot be separated by any partition L with $trIndL < \alpha$.

It is known that for every $\alpha < \omega_1$ there exists a c.d. Cantor $trInd$ -manifold of class α ([Ch1], see also part 2).

A space X is A-weakly infinite-dimensional (shortly A-w.i.d.) if for each infinite sequence $(A_1, B_1), (A_2, B_2), \dots$ of pairs of disjoint closed subsets of X there exist partitions L_i between A_i and B_i in X such that $\bigcap_{i=1}^{\infty} L_i = \emptyset$.

A space X is hereditarily A-w.i.d. if every subspace of X is A-w.i.d.

A space X is A-strongly infinite-dimensional (shortly A-s.i.d.) if it is not A-w.i.d.

Remind that each c.d. space is A-w.i.d. Moreover, a space which is the union of countably many c.d. (A-w.i.d.) subspaces is c.d. (A-w.i.d.).

If a space X is compact then one say that X is weakly infinite-dimensional (shortly w.i.d.) or strongly infinite-dimensional (shortly s.i.d.) respectively.

It is known that there exists a w.i.d. compact space P which cannot be separated by any hereditarily A-w.i.d. subspace ([EP]). Note that P cannot be separated by any countable-dimensional subspace. In particular P is not c.d. Remind that the first example of a w.i.d. compactum which is not c.d. was given by R. Pol [RP2].

A compact space X is hereditarily infinite-dimensional, shortly h.i.d. (hereditarily strongly infinite-dimensional, shortly h.s.i.d.), if each nonempty closed subset of X is either 0-dimensional or infinite-dimensional (strongly infinite-dimensional).

The first example of h.i.d. compactum was given by D. Henderson [H1].

In [H2] D. Henderson has constructed a c.d. AR-compactum H^α with $trIndH^\alpha = \alpha$ for every $\alpha < \omega_1$. Remind this construction.

Let $H^I = I = [0, 1], p_1 = \{0\}$. Assume that for every $\beta < \alpha$ the compacta H^β and the points $p_\beta \in H^\beta$ have already been defined. If $\alpha = \beta + 1$, then we set $H^{\beta+1} = H^\beta \times I$ and $p_{\beta+1} = (p_\beta, 0)$. If α is a limit ordinal, then K_β is the union of the H^β and a half-open arc A_β such that $A_\beta \cap H^\beta = \{p_\beta\} = \{\text{endpoint of the arc } A_\beta\}$, $\beta < \alpha$. Let us define H^α as the one-point compactification of the free sum $\bigoplus_{\beta < \alpha} K_\beta$ and let p_α be the compactification point.

It is well known that every ordinal α may be represented in the form $\alpha = p(\alpha) + n(\alpha)$, where $p(\alpha)$ is a limit ordinal and $n(\alpha) < \omega$.

Note that the compactum H^α , where $n(\alpha) \geq 1, \alpha < \omega_1$, cannot be separated by a point.

A dimension function d is monotone if for any space X and any subset $A \subset X$ closed in X , $dA \leq dX$.

2. Variation of Fedorchuk’s construction

Let R be the real line, $Q \subset R$ be the rational numbers, $Irr \subset R$ be the irrational numbers and $I = [0, 1]$. The notation $Z \simeq Y$ will mean that spaces Z and Y are homeomorphic.

We shall follow [Ch2] as a variation of [F1], [F2]. Remind some definitions.

A continuous mapping $f : X \rightarrow Y$ is called fully closed if for any point $y \in Y$ and any finite covering $\{U_i : i = 1, 2, \dots, s\}$ of $f^{-1}y$ by sets open in X , the set $\{y\} \cup (\bigcup_{i=1}^s f^\#U_i)$ is open in Y . Here $f^\#U = Y \setminus f(X \setminus U)$.

A continuous mapping $f : X \rightarrow Y$ is called ring-like if for any point $x \in X$ and arbitrary neighbourhoods Ox and Ofx , the set $f^\#Ox$ contains a partition between the point $f x$ and the set $Y \setminus Ofx$ in the space Y .

A continuous mapping $f : X \rightarrow Y$ is called monotone if for any point $y \in Y$ the set $f^{-1}y$ is connected.

A continuous mapping $f : X \rightarrow Y$ is called irreducible if for any non-empty open subset $O \subset X$ we have $f^\#O \neq \emptyset$.

Consider a continuum Y with a countable everywhere dense subset $P = \{a_1, a_2, a_3, \dots\} \subset Y$ and fix an embedding $Y \subset I^\infty$. Define a mapping $f : (0, 1] \rightarrow I^\infty$ as follows. Namely

$f|_{[1/(i+1), 1/i]}: [1/(i+1), 1/i] \rightarrow I^\infty$ is a path between the points a_{i+1} and a_i in $1/i$ -neighborhood of Y , $i = 1, 2, \dots$.

The mapping f satisfies the following conditions:

- (a) for every open neighborhood O of the continuum Y in I^∞ there exists a natural number n such that $f(0, 1/n] \subset O$;
- (b) for every non-empty open subset $U \subset Y$ and every natural number n there exists a number $m \geq n$ such that $f(1/m) \in U$.

2.a Particular case

Define a mapping $g : [-1, 1] \setminus \{0\} \rightarrow I^\infty$ by $g(x) = f(|x|)$ and mappings

$$g_t : [-1 + t, 1 + t] \setminus \{t\} \rightarrow I^\infty \text{ by } g_t(x) = g(x - t), \text{ where } t \in R.$$

Consider the disjoint union $B = \cup\{Y_t : t \in R\}$, where Y_t is a point, if $t \in R \setminus Q$, and $Y_t \simeq Y$ if $t \in Q$.

Let $p_t : Y \rightarrow Y_t$ be the homeomorphism above, where $t \in Q$.

Define the mapping $\pi : B \rightarrow R$ as follows, $\pi(y) = t$, if $y \in Y_t$.

Let $\{V_n\}_{n=1}^\infty$ be a base in R , and $\{U_k\}_{k=1}^\infty$ be a base in I^∞ .

The topology τ on the set B we define as follows.

We take all sets $\pi^{-1}V_n, n = 1, 2, \dots$, and $O(U_k, t, V_n) = p_t(U_k \cap Y) \cup \pi^{-1}(g_t^{-1}U_k \cap V_n)$, where $t \in Q \cap V_n$ and $m, n = 1, 2, \dots$, as the basis sets of the topology on B .

Note that in the case the mapping π is fully closed, ring-like, irreducible and monotone.

Denote the subspace $\pi^{-1}[0, 1]$ of B via $F(Y)$.

Some properties of $F(Y)$.

- (a) FY is a continuum which is the disjoint union of continua $Y_t, t \in [0, 1]$.
- (b) $F(Y) \setminus \cup\{Y_t : t \in Q\} \simeq Irr \cap I$.
- (c) every non-empty open subset of $F(Y)$ contains a copy of Y .
- (d) every subcontinuum of $F(Y)$ either embeds in Y or is equal to $\pi^{-1}[a, b]$, where $0 \leq a < b \leq 1$.
- (e) $F(Y)$ is c.d. (w.i.d., h.s.i.d.) if Y is c.d. (w.i.d., h.s.i.d.).

Example of c.d. Cantor *trInd*-manifold of class $(\alpha + 1), \alpha < \omega_1$.

Consider the path-connected compactum $Z = F(H^\alpha) \times I/F(H^\alpha) \times \{0\}$.

Denote the compactum Z^2 via $A(H^\alpha)$. It is clear that $A(H^\alpha)$ is c.d. and every non-empty open subset of Z contains $H^{\alpha+1}$. One can prove (see [Ch1]) that for every partition L in $A(H^\alpha)$ we have $trIndL \geq \alpha + 1$. Hence the continuum $A(H^\alpha)$ is a Cantor *trInd*-manifold of class $(\alpha + 1)$.

2.b General case

Consider a continuum X and a countable subset L of X . Fix a point $x \in L$ and a sequence $\{L_i^x\}_{i=1}^\infty$ of partitions in X such that

- (a) $L_i^x = X \setminus (U_i^x \cup V_i^x)$, where U_i^x, V_i^x are disjoint non-empty open subsets of the continuum X and $x \in U_i^x$ for every i ;
- (b) $U_i^x \cup L_i^x \subset U_{i-1}^x, i = 2, 3, \dots$;
- (c) $\{U_i^x\}_{i=1}^\infty$ is a base in the point x .

Note that all partitions $L_i^x, i = 1, 2, \dots$ are non-empty.

Define a mapping $h_x : V_1^x \cup \bigcup_{i=1}^\infty L_i^x \rightarrow (0, 1]$ as follows

- (a) $h_x(X \setminus U_1^x) = 1$;
- (b) $h_x(L_i^x) = 1/i, i = 2, 3, \dots$.

By $q_x : X \setminus \{x\} \rightarrow (0, 1]$ we denote an extension of h_x on $X \setminus \{x\}$ such that $q_x((U_i^x \cup L_i^x) \setminus U_{i+1}^x) \subset [1/(i+1), 1/i], i = 1, 2, \dots$.

Put $g_x = f \circ q_x$. The mapping g_x satisfies the following conditions:

- (a) for every open neighborhood O of the continuum Y in I^∞ there exists a natural number n such that $g_x U_n^x \subset O$;
- (b) for every non-empty open subset $U \subset Y$ and every natural number n there exists a number $m \geq n$ such that $g_x(L_m^x) \subset U$.

Consider the disjoint union $B(X, Y, L) = \cup\{Y_x : x \in X\}$, where Y_x is a point if $x \in X \setminus L$ and $Y_x \simeq Y$ if $x \in L$.

Let $p_x : Y \rightarrow Y_x$ be the homeomorphism above, where $x \in L$.

Define the mapping $\pi : B(X, Y, L) \rightarrow X$ by $\pi(y) = x$ if $y \in Y_x$.

Let $\{V_n\}_{n=1}^\infty$ be a base in X , and $\{U_k\}_{k=1}^\infty$ be a base in I^∞ .

We define the topology τ on the set $B(X, Y, L)$ as follows.

We take all sets $\pi^{-1}V_n, n = 1, 2, \dots$, and $O(U_k, x, V_n) = p_x(U_k \cap Y) \cup \pi^{-1}(g_x^{-1}U_k \cap V_n)$, where $x \in L \cap V_n$ and $m, n = 1, 2, \dots$, as the basis sets of the topology on $B(X, Y, L)$.

Note that in this case the mapping π is fully closed, ring-like, irreducible and monotone.

Note some properties of $B(X, Y, L)$.

Proposition 2.1. (a) $B(X, Y, L)$ is a continuum which is the disjoint union of continua $Y_x, x \in X$.

(b) $B(X, Y, L) \setminus \cup\{Y_x : x \in L\} \simeq X \setminus L$.

(c) Every non-empty open subset of $B(X, Y, L)$ contains a copy of Y if L is an everywhere dense subset of X .

(d) Every subcontinuum C of $B(X, Y, L)$ either embeds in Y or is equal to $\pi^{-1}\pi C = B(\pi C, Y, L \cap \pi C)$. Moreover in the last case either C lies in $X \setminus L$ if $L \cap \pi C = \emptyset$ or C contains a copy of Y if $L \cap \pi C \neq \emptyset$.

(e) $B(X, Y, L)$ is c.d. (w.i.d., h.s.i.d.) if X, Y are c.d. (w.i.d., h.s.i.d.).

(f) Let C be a partition in $B(X, Y, L)$. Then there exists a partition C_1 in X such that for each subspace Z of C_1 the subspace $Z \setminus L$ embeds into C . In particular, if X is an infinite-dimensional Cantor manifold then $B(X, Y, L)$ is the same.

PROOF: (a)–(d) follow from the construction and the properties of π .

(e) We shall prove only that the continuum $B(X, Y, L)$ is w.i.d. if the continua X, Y are w.i.d. Consider a countable family $\{(A_j^i, B_j^i) : i = 0, 1, \dots; j = 1, 2, \dots\}$ of pairs of disjoint closed subsets of $B(X, Y, L)$. Let $L = \{l_1, l_2, \dots\}$. For every $i = 1, 2, \dots$ there exist partitions L_j^i between A_j^i and B_j^i in $B(X, Y, L)$ such that $(\bigcap_{j=1}^\infty L_j^i) \cap Y_{l_i} = \emptyset$. Denote the compactum $\bigcap_{i=1}^\infty (\bigcap_{j=1}^\infty L_j^i)$ via A . Note that $A \subset B(X, Y, L) \setminus \cup\{Y_x : t \in L\} \simeq X \setminus L$ and hence A is w.i.d. There exist partitions L_j^0 between A_j^0 and B_j^0 in $B(X, Y, L)$ such that $(\bigcap_{j=1}^\infty L_j^0) \cap A = \bigcap_{i=0}^\infty (\bigcap_{j=1}^\infty L_j^i) = \emptyset$. Hence the compactum $B(X, Y, L)$ is w.i.d.

(f) Let $C = B(X, Y, L) \setminus (U \cup V)$ where U, V are disjoint non-empty open subsets of $B(X, Y, L)$. Note that the subsets $\pi^\#U, \pi^\#V$ of X are disjoint non-empty open and the subset $C_1 = X \setminus (\pi^\#U \cup \pi^\#V)$ is a partition in X . It is clear that for each subspace Z of C_1 the subspace $Z \setminus L$ embeds into C . Suppose that X is an infinite-dimensional Cantor manifold and the partition C is finite-dimensional. Therefore the subspace $C_1 \setminus L$ is finite-dimensional and hence the partition C_1 is finite-dimensional too. It is a contradiction. \square

Proposition 2.2. *Let L be an everywhere dense subset of X and Y_1, Y_2 be injectively different continua, which do not embed into X . Then continua $B(X, Y_1, L), B(X, Y_2, L)$ are locally r -incomparable.*

PROOF: Let U be an open non-empty subset of $B(X, Y_1, L)$. Suppose that $g : U \rightarrow B(X, Y_2, L)$ is an embedding. By Proposition 2.1 (c) U contains a copy of Y_1 . By Proposition 2.1 (d) the image $g(Y_1)$ of the copy of Y_1 either embeds into Y_2 (it is a contradiction) or is equal to $\pi^{-1}\pi g(Y_1)$. In the last case $g(Y_1)$ either lies in $X \setminus L \subset X$ or contains a copy of Y_2 . It is a contradiction too. \square

3. On E. Pol's proposition

The following statement in fact was proved in [EP].

Proposition 3.1. *Let A, B be two c.d. continua which cannot be separated by a point and which are injectively different. Then there exists an injectively different family $\{L_a : a \in \{0, 1\}^\infty\}$ of c.d. continua such that for every $a \in \{0, 1\}^\infty, L_a$ contains copies of A and B .*

We repeat here the description from [EP].

Choose two pairs of different points $a_1, a_2 \in A$ and $b_1, b_2 \in B$. Let X_1, X_2, \dots be a sequence of spaces such that X_i is a copy of A or B and $x_j^i = a_j$ if $X_i = A$ and $x_j^i = b_j$ if $X_i = B$, for $j = 1, 2$. Consider the equivalence relation E on the free sum $X = \bigoplus_{i=1}^{\infty} X_i$ such that xEy iff $x = y$ or $x = x_2^i$ and $y = x_1^{i+1}$ for some $i \in N$.

Let $Y = X/E$ be the quotient space and $Z = Z(X_1, X_2, \dots)$ be the one-point compactification of Y . Then Z is a c.d. continuum. Let K be the class of all spaces $Z(X_1, X_2, \dots)$ obtained in this way.

It was shown in [EP] that K contains an injectively different uncountable family $\{L_a : a \in \{0, 1\}^{\infty}\}$. Namely, for $a = \{\alpha_k\}_{k=1}^{\infty} \in \{0, 1\}^{\infty}$, $L_a = Z(X_1^a, X_2^a, \dots)$, where $X_1^a = A, X_2^a = B$ and for $k = 1, 2, \dots$:

if $a_k = 0$ then X_{5k-3+l}^a is A , for $l = 1, 2$; and it is B , for $l = 3, 4, 5$;

if $a_k = 1$ then X_{5k-3+l}^a is A , for $l = 1, 2, 3$; and it is B , for $l = 4, 5$;

4. Two c.d. injectively different infinite-dimensional continua which cannot be separated by a point

Let γ be an infinite ordinal with $n(\gamma) \geq 1$. Remind that the compactum $A(H^\gamma)$ is a c.d. Cantor $trInd$ -manifold of class $(\gamma + 1)$. Put $\beta = trIndA(H^\gamma) + 1 < \omega_1$. Note that $n(\beta) \geq 1$. Continua $A(H^\gamma)$ and H^β cannot be separated by a point. Since $trIndH^\beta = \beta > trIndA(H^\gamma)$, H^β does not embed into $A(H^\gamma)$.

We shall prove that $A(H^\gamma)$ does not embed into H^β .

Remind that H^β is the union of countably many finite-dimensional compacta. Assume that $A(H^\gamma)$ embeds into H^β . Hence $A(H^\gamma)$ is the union of countably many finite-dimensional compacta at least one of which contains a non-empty open subset of $A(H^\gamma)$. But every non-empty open subset of $A(H^\gamma)$ contains a copy of H^γ with $trIndH^\gamma = \gamma \geq \omega$. It is a contradiction. Hence $A(H^\gamma)$ does not embed into H^β .

Note that both compacta $A(H^\gamma)$ and H^β contain H^γ . Now with help of Proposition 3.1 the following statement is evident.

Proposition 4.1. *For every ordinal $\gamma < \omega_1$ there exists an injectively different family $\{L_a : a \in \{0, 1\}^{\infty}\}$ of c.d. continua such that for every $a \in \{0, 1\}^{\infty}$, L_a contains a copy of H^γ .*

5. Main results

Here we shall construct uncountable locally r -incomparable families of named in the introduction types of infinite-dimensional Cantor manifolds.

First we need the following evident (see the separation theorem for dimension 0 ([E, p. 11]))

Lemma 5.1. *Let A be a 0-dimensional subset of a compactum Z . Assume that $trIndQ < \alpha$ for every compactum $Q \subset Z \setminus A$. Then $trIndZ \leq \alpha$.*

In particular, if $\text{trInd}Z \geq \beta + 1$, then there exists a compactum $Q \subset Z \setminus A$ such that $\text{trInd}Q = \beta$.

Theorem 5.1. For every $\alpha < \omega_1$ there exists a locally r -incomparable family \mathcal{A} ($|\mathcal{A}| = 2^{\aleph_0}$) of c.d. Cantor trInd -manifolds of class α .

PROOF: Fix an ordinal $\alpha < \omega_1$. Denote $A(H^\alpha)$ via X . Note that X is a c.d. Cantor trInd -manifold of class $(\alpha + 1)$. Let $\gamma = \text{trInd}X + 1$. By Proposition 4.1 there exists an injectively different family $\{L_a : a \in \{0, 1\}^\infty\}$ of c.d. continua such that for every $a \in \{0, 1\}^\infty$, L_a contains a copy of H^γ . Remind that $\text{trInd}H^\gamma = \gamma$ ([H2]) and the dimension trInd is monotone. Hence for every $a \in \{0, 1\}^\infty$, L_a does not embed into X .

Let L be an everywhere dense countable subset of X .

By Propositions 2.1 (e), (f), 2.2 and Lemma 5.1 the family $\{B(X, L_a, L) : a \in \{0, 1\}^\infty\}$ is locally r -incomparable and it consists of c.d. Cantor trInd -manifolds of class α . \square

Now we need the following evident

Lemma 5.2. Let X be a A -s.i.d. space and Y be a 0-dimensional subspace of X . Then the subspace $X \setminus Y$ is A -s.i.d.

Theorem 5.2. There exists a locally r -incomparable family \mathcal{A} ($|\mathcal{A}| = 2^{\aleph_0}$) of w.i.d. Cantor manifolds which cannot be separated by any hereditarily A -w.i.d. subspace.

PROOF: Denote the w.i.d. compactum P from part 1 via X . Let $\text{dim}_w X = \alpha < \omega_1$, where dim_w is Borst's transfinite extension of the covering dimension dim ([PB]). Put $\gamma = \alpha + 1$. By Proposition 4.1 there exists an injectively different family $\{L_a : a \in \{0, 1\}^\infty\}$ of c.d. continua such that for every $a \in \{0, 1\}^\infty$, L_a contains a copy of H^γ . Remind that $\text{dim}_w H^\gamma = \gamma$ ([PB]) and the dimension dim_w is monotone. Hence for every $a \in \{0, 1\}^\infty$, L_a does not embed into X . Let L be an everywhere dense countable subset of X . By Propositions 2.1 (e), (f), 2.2 and Lemma 5.2, the family $\{B(X, L_a, L) : a \in \{0, 1\}^\infty\}$ is locally r -incomparable and it consists of w.i.d. Cantor manifolds which cannot be separated by any hereditarily A -w.i.d. subspace. \square

Theorem 5.3. There exists a locally r -incomparable family \mathcal{A} ($|\mathcal{A}| = 2^{\aleph_0}$) of h.s.i.d. Cantor manifolds.

PROOF: By Theorem 0.1 (see also Remark 0.1) there exists an injectively different family $\{L_a : a \in \{0, 1\}^\infty\}$ of h.s.i.d. Cantor manifolds. Put $X = L_{(0,0,\dots)}$ and $M_{(b_1, b_2, \dots)} = L_{(1, b_1, b_2, \dots)}$ for every $(b_1, b_2, \dots) \in \{0, 1\}^\infty$. Note that for every $b \in \{0, 1\}^\infty$, M_b does not embed into X . Let L be an everywhere dense countable subset of X . By Propositions 2.1 (e), (f) and 2.2 the family $\{B(X, M_b, L) : b \in \{0, 1\}^\infty\}$ is locally r -incomparable and it consists of h.s.i.d. Cantor manifolds. \square

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