Abstracts of CSc. theses in mathematics

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ABSTRACTS OF THESES* IN MATHEMATICS

defended recently at Charles University, Prague

ON SIGMA-IDEALS OF EXCEPTIONAL SETS

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(January 14, 1997; supervisor L. Zajíček)

The doctoral thesis consists of the following four research papers: *The Banach-Mazur game and σ-porosity* (Fund. Math. **150** (1996), 197–210), *On singular boundary points of complex functions, Sets of extended uniqueness and σ-porosity* (Comment. Math. Univ. Carolinae **38** (1997), 337–341), *Calibrated thin coanalytic σ-ideals are G_δ* (accepted in Proc. Amer. Math. Soc.). These papers deal with σ-ideals of exceptional sets. Each paper studies σ-ideals from a different point of view.

In the paper *The Banach-Mazur game and σ-porosity* we give a characterization of certain σ-ideals concerning various kinds of porosity in a terminology of two-person games. Roughly speaking, we have proved results of this type: the second player in the classical Banach-Mazur game with a set M has a winning strategy satisfying some extra conditions if and only if M is a "small" set. Varying the demands on the winning strategy we obtain characterizations of different types of smallness. The aim of this paper is also to obtain a method to prove that some concrete sets are small.

The work *On singular boundary points of complex functions* studies in fact two σ-ideals of subsets of the real line, namely the σ-ideal of σ-porous subsets of $\mathbb{R}$ and the σ-ideal of subsets of $\mathbb{R}$ which can be covered by countably many sets in the form $p(F) = \{x \in F'; x \text{ is a point of porosity of } F\}$, where $F \subset \mathbb{R}$ is closed. The $\mathrm{G}_{\delta\sigma}$ sets, which belong to the second σ-ideal, are exactly so-called E_{VV}-sets, which appear in the study of boundary behaviour of complex functions. We show that there exists a G_δ porous subset of $\mathbb{R}$ which is not an E_{VV}-set. This solves negatively an open problem posed by Dolzhenko. Let us note that the notion of σ-porosity was introduced by Dolzhenko in order to study E_{VV}-sets.

The third paper (entitled *Sets of extended uniqueness and σ-porosity*) deals with the σ-ideal of sets of extended uniqueness. This σ-ideal appears in the theory of trigonometric series. It was shown by Debs – Saint-Raymond that Borel sets of extended uniqueness are meager. We show that this result cannot be improved by replacing meagerness by σ-porosity. We also obtain a new proof of Lyons' theorem concerning so-called large subclasses of the σ-ideal of closed sets of uniqueness.

In the last work (*Calibrated thin coanalytic sigma-ideals are G_δ*) we study σ-ideals of compact sets. The theory of σ-ideals of compact sets has been developed by Kechris, Louveau and Woodin. This theory has several interesting

*An equivalent to PhD.

applications in the theory of trigonometric series. We present one result solving an open problem posed by Kechris, Louveau and Woodin. A consequence of the obtained results for the theory of trigonometric series is also presented.

CONVERGENCE ANALYSIS OF SOME MULTILEVEL METHODS

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(February 7, 1997; supervisor I. Marek)

The thesis is concerned with multilevel methods for solving boundary value problems and their discretizations. In Chapter 2, the idea of a multilevel method for solving a positive definite elliptic boundary value problem is demonstrated. A methodological explanation of the basic concepts and theorems of the so called model problem is presented. It is shown in Chapter 3 how the theory of multigrid methods can be adapted to the situation where the investigated systems are neither symmetric, nor positive definite.

Our problem is to construct all stationary probability vectors (s.p.v.) of a given column stochastic matrix. The problem is to find all x such as

$$Ax = x, x \geq 0$$
$$a_{ij} \geq 0, \sum_{i=1}^{N} a_{ij} = 1, j = 1, \ldots, N.$$

For the solution, the multigrid-like method is analyzed and two problem classes are considered. The first class of problems concerns the global convergence of the main aggregation/disaggregation algorithm. A theorem states that the aggregation process is globally convergent as the number of relaxations t is taken sufficiently large enough.

The second class is concerned with the question how small the number of relaxations on the fine level should be. It is shown that $t = 1$ is sufficient for local convergence. This result implies that our aggregation algorithm is locally convergent with any number of relaxations on fine level.

LOCAL NUMERICAL ANALYSIS OF SYMMETRY-BREAKING BIFURCATION

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The main purpose of the thesis is to develop numerical methods for the exploration of bifurcation problems with symmetry of a group Γ, in particular, Hopf bifurcation. For this purpose, the generalized Liapunov-Schmidt reduction for equivariant mappings has been investigated.

The definition of the contact equivalence follows the standard frame in works by M. Golubitsky, I. Stewart and D. Schaeffer. The main result consists in the proof of the equivariant contact equivalence for the reduced mappings that are obtained from the symmetry adapted reduction. It verifies the adaptability of the generalized reduction on the equivariant problems. However, in the case of equivariant Hopf bifurcation, several questions had to be answered. The equivariant reduction deals with irreducible subspaces of the group representation while the analytical results obtained for the equivariant Hopf bifurcation uses the notion of Γ-simplicity. We succeeded in a reformulation of the Equivariant Hopf Theorem and suggested an equivariant generalized reduction in a complex space of the half (complex) dimension in which the irreducibility is naturally preserved.

Due to the local one-to-one correspondence of the roots of the reduced mapping with either steady-states, or periodic solutions of the original dynamical system, the reduction makes it possible to analyze the various local properties of the emanating branches in the reduced space that leads to various numerical methods for the localization of equivariant Hopf points on a path of steady-states and their numerical analysis including the prediction of emanating branches of periodic orbits with the symmetry of an isotropy subgroup of $\Gamma \times \mathbf{S}^1$ (standing waves, rotating waves).

SOME ASPECTS OF DIFFERENTIABILITY IN GEOMETRY ON BANACH SPACES

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This work consists of two parts.

In the first part, the smallness of continuous linear images of the sets of k-dimensional nondifferentiability of a convex function, or k-dimensional multiplicity of a monotone operator on a real Banach space, is characterized in the following way:

1) Smallness of these sets in the sense of category and also in the sense of Aronszajn is proved.

2) There is also a counterexample: there exists a convex function f, for which the projection of the set of k-dimensional nondifferentiability can not be covered by a countable union of Lipschitz surfaces.

The second part deals with the properties of Asplund generated spaces and their subspaces. A new property of Banach spaces, "countable dentability," is introduced, and the behaviour of convex functions and monotone operators on spaces with this property is studied.

It is proved that the dual of a countable dentable Banach space is weak* fragmentable. Also an example is found, showing that these two categories of Banach spaces do not coincide.

Using the principle of countable dentability, Zajíček's theorem about the points of singlevaluedness of monotone operators is extended. In particular, it is proved that every monotone operator on an Asplund generated space is singlevalued on the whole space except a σ-cone supported set. Since a σ-cone supported set is of first category, this also strengthens a theorem by Christensen and Kenderov.

Finally, compact spaces are dealt with, especially those with properties of countably dentable compacta and adequate compacta. An equivalent characterization is proved for countably dentable compact subsets of 2^Γ in the flavour of Rosenthal. Properties equivalent to Eberleinity for adequate compacta are also proved. As an application, it is shown that there exists a non-countably dentable Banach space whose dual is weak* fragmentable.

CONTINUITY-LIKE PROPERTIES OF MAPPINGS

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(September 23, 1997; supervisor P. Holický)

This doctoral thesis consists of four research papers. The first three papers deal with functions of the generalized first Borel class while the last two concern Stegall class of compact spaces.

In the first paper (*Remark on the Point of Continuity Property II*, joint paper with P. Holický, Bull. Acad. Polon. Sci., **50** (1995), 105–111) it is proved that if X is a hereditarily Baire space which has the tightness less than the least weakly inaccessible cardinal and each (closed) subspace of X is *ccc*, then every extended Borel class one map of X into a metric space M has the point of continuity property. In the case of countable tightness the *ccc*-assumption is not needed, which yields a result of R.W. Hansell (1991).

In the second paper (*Note on Connections of Point of Continuity Property and Kuratowski Problem on Function Having the Baire Property*, Acta Univ. Carolinae, Math. et Phys., **38** (1), (1997), 3–12), it is shown that the question whether every extended Borel class one map of any hereditarily Baire space into a metric space has the point of continuity property is equivalent to the Kuratowski question whether the function with the Baire property of any topological space into a metric space is continuous apart from a meager set. The method of the proof is used to get, under the assumption that it is consistent to suppose that there is a measurable cardinal, examples of ordinary Borel class one maps (i.e. $\mathcal{F}_\sigma$-measurable) of a hereditarily Baire space into a metric space which do not have the point of continuity property. These examples complete and strengthen an example of G. Koumoullis (1993).

In the third paper (*New Examples of Hereditarily t-Baire Spaces*, to appear in Bull. Acad. Polon. Sci.) a new subclass of hereditarily t-Baire spaces (defined by G. Koumoullis (1993)) is introduced. These spaces need not have the restricted Baire property in a compactification, and are used (together with a modified

construction of D. Fremlin (1987)) to get, (under the assumption that there is a measurable cardinal) an example of a first class function of a hereditarily t-Baire space into a metric space which has no point of continuity, which answers a question of G. Koumoullis (1993).

In the fourth paper (*Stegall Compact Spaces Which Are Not Fragmentable*, to appear in Topology Appl.), consistent examples of nonfragmentable compact Hausdorff spaces which belong to Stegall's class $\mathcal{S}$ are constructed. The construction is based on modifying the well-known "double-arrow" space.

The last paper (*Few remarks on structure on certain spaces of measures*) deals with the spaces of measures on compacta constructed in the previous one.

AN APPROXIMATED SOLUTIONS OF GENERALIZED DIFFERENTIAL EQUATIONS

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The thesis deals with generalizations of initial value problems for linear differential systems of the form

$$x(t) = x_0 + \int_a^t \mathrm{d}[A(s)]\, x(s), \quad t \in [a,b], \tag{1}$$

where $A \in \mathbf{BV}^{n\times n}$, $x \in \mathbf{BV}^n$, $x_0 \in \mathbb{R}^n$ and the integral is the Perron-Stieltjes one.

We define the solution of (1) as the limit of a sequence $\{x_k\}_{k=1}^{\infty}$ of the solution of the ordinary differential equations

$$x_k(t) = x_0 + \int_a^t \mathrm{d}[A_k(s)]\, x_k(s), \quad t \in [a,b], \quad k \in \mathbb{N},$$

where the sequence $\{A_k\}_{k=1}^{\infty} \subset \mathbf{AC}^{n\times n}$ approximate in a proper way the given A.

The existence and uniqueness results are obtained. Relations between the classical integral solution and the *approximated solution* are also presented.

ALGEBRAS DETERMINED BY THEIR ENDOMORPHISM MONOID

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For a category K the endomorphism monoid of an K-object A will be denoted by $\mathrm{End}_K(A)$. Various properties of $\mathrm{End}_K(A)$ in the given category K have been studied by numerous papers. Many authors have dealt with problems of

universality and their results show that in the algebraic universal (and therefore in universal) categories, the relation between algebraic properties of endomorphism monoid of the given object and the given object is very free. It means that in algebraic universal (or universal) categories there is impossible to draw any conclusions about the structure of an object from its endomorphism monoid.

The aim of this paper is to solve the opposite problem. To formalize the fact that algebraic properties of the endomorphism monoid of an object determine this object, we use the following definition.

Let α be a cardinal. A category K is called α-*determined* if for every monoid M, every set X of K-objects such that $\mathrm{End}_K(A)$ is isomorphic to M for every $A \in X$ has cardinality smaller than α.

The aim of the present paper is to describe general methods for the decision whether a given concrete category is α-determined for any cardinal α. These methods have been found by generalization of known results.

Important results for given categories mentioned in this paper are proven in articles of the following authors: L.M. Gluskin, B.M. Schein, C.J. Maxson, K.D. Magill, P. Ribenboim, R. McKenzie, C. Tsinakis, M.E. Adams, J. Sichler and V. Koubek. The goal of this paper has not been to show further examples of α-determined categories but to find general features of known results and to build up a homogeneous frame and to show how to reach the known results within this frame.

The general method which is used for investigation of α-determinacy has been splitted into two main parts. In the first part, the relation between an isomorphism of endomorphism monoids and the underlying sets of objects is found. In the second part, relations (resp. a topology) forming the given object are built up on the basis of results of the first part.

All the paper is splitted into nine sections. In the first three sections, the main aims of the present paper are mentioned and the results being reached in past. The main facts concerning the Priestley duality are given in the fourth section. The main results of the paper are concentrated in the fifth and sixth sections. The fifth section is dealing with an investigation of sufficient conditions for finding the carrier of an isomorphism between endomorphism monoids. Since the construction of a carrier for the whole object is often impossible the method is modified so that the carrier is constructed step by step. The exact correlation among determinacy, homogeneity and concrete determinacy is explained in the sixth section. The seventh section is devoted to the investigation of the categories of distributive lattices, distributive p-algebras and Heyting algebras. The eight section deals with the categories of Abelian groups and categories of distributive dp-algebras. The ninth section is devoted to the investigation of necessary conditions for a category to be concretely β-determined for a cardinal β and there is also the brief review of the reached results there.