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Some remarks on a class of weight functions

LOREDANA CASO, MARIA TRANSIRICO

Abstract. In this paper we obtain some results about a class of functions $\varrho : \Omega \rightarrow R_+$, where Ω is an open set of R^n , which are related to the distance function from a fixed subset $S_\varrho \subset \partial\Omega$. We deduce some imbedding theorems in weighted Sobolev spaces, where the weight function is a power of a function ϱ .

Keywords: weight functions, weighted Sobolev spaces

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Introduction

Let Ω be an open subset of R^n .

In [T₄] M. Troisi has studied the class $\mathcal{A}(\Omega)$ of functions $\varrho : \Omega \rightarrow R_+$ such that

$$(1) \quad \sup_{\substack{x,y \in \Omega \\ |x-y| < \varrho(y)}} \left| \log \frac{\varrho(x)}{\varrho(y)} \right| < +\infty.$$

Typical examples of functions $\varrho \in \mathcal{A}(\Omega)$ are the function

$$x \in R^n \rightarrow 1 + a|x|, \quad a \in]0, 1[,$$

and, if $\Omega \neq R^n$ and S is a nonempty subset of $\partial\Omega$, the function

$$x \in \Omega \rightarrow a \operatorname{dist}(x, S), \quad a \in]0, 1[.$$

For any $\varrho \in \mathcal{A}(\Omega)$ we put

$$(2) \quad S_\varrho = \{z \in \partial\Omega : \lim_{x \rightarrow z} \varrho(x) = 0\}.$$

We remark (see, e.g., [T₄], [CCD₁]) that if $\varrho \in \mathcal{A}(\Omega)$ and $S_\varrho \neq \emptyset$, then ϱ is related to the distance function from S_ϱ .

For examples and properties of functions $\varrho \in \mathcal{A}(\Omega)$ we refer to [T₄] and also to [CCD₁], [TT], [DT].

For a treatment of weight functions as the distance function from a nonempty subset of the boundary of a bounded open set of R^n or weight functions related to such distance function, and for related problems see, e.g., [K], [KJF].

In some papers (see, e.g., [F₁], [S₁], [MT₂], [T₁], [CCD₁]) some classes of weighted Sobolev spaces have been studied, where the weight function is a power of a function $\varrho \in \mathcal{A}(\Omega)$.

In various papers (see, e.g., [MT₁], [IMT], [IT], [T₂], [S₂], [T₃], [F₂], [Sg], [ST], [GTT], [DT], [CCD₂]) many applications of such spaces to the study of boundary value problems for elliptic and quasielliptic differential equations have been studied, also in unbounded open sets.

In particular in [CCD₁] the authors, for fixed $\varrho \in \mathcal{A}(\Omega)$, have studied the operator

$$(3) \quad u \rightarrow gu,$$

where g is singular near S_ϱ , as an operator defined in a weighted Sobolev space, denoted by $W_q^{r,p}(\Omega)$ (see n. 1 for such definition), and which takes values in $L^p(\Omega)$, where the weight function is a power of ϱ . They have given conditions on ϱ (e.g. S_ϱ closed), g and Ω , so that the operator defined by (3) is bounded and other conditions in order that it is compact.

As an application (see [CCD₂]) the authors have studied the Dirichlet problem in an open set, not necessarily bounded, for variational second order elliptic equations with coefficients singular near S_ϱ . They have obtained an existence and uniqueness theorem for the solution in the closure of $C_o^\infty(\Omega)$ in $W_q^{1,2}(\Omega)$.

In this paper our purpose is to give a contribution to the study of functions of $\mathcal{A}(\Omega)$.

We state some suitable characterizations of S_ϱ , from which, in particular, we deduce that S_ϱ is a closed subset of $\partial\Omega$ (see n. 1).

Because of these results, we can give (see n. 2) a contribute to the study of some functions which are singular near S_ϱ , as the function g in (3). Furthermore we obtain (see n. 3) a remarkable improvement of the imbedding results of [CCD₁].

1. Some properties of functions of $\mathcal{A}(\Omega)$

For all $x \in R^n$ and for all $r \in R_+$ we set

$$B(x, r) = \{y \in R^n : |y - x| < r\}.$$

If A is a Lebesgue measurable subset of R^n , $1 \leq p \leq +\infty$, and $f \in L^p(A)$ we put

$$\|f\|_{L^p(A)} = |f|_{p,A}.$$

Let Ω be an open subset of R^n , $n \geq 2$. We put

$$\Omega(x, r) = \Omega \cap B(x, r) \quad \forall x \in R^n, \quad \forall r \in R_+.$$

We denote by $\mathcal{A}(\Omega)$ the class of functions $\varrho : \Omega \rightarrow R_+$ verifying (1).

Obviously ϱ verifies (1) if and only if there exists $\gamma \in R_+$ such that

$$(1.1) \quad \gamma^{-1}\varrho(y) \leq \varrho(x) \leq \gamma\varrho(y) \quad \forall y \in \Omega \quad \text{and} \quad \forall x \in \Omega \cap B(y, \varrho(y)).$$

We remark that for any $\varrho \in \mathcal{A}(\Omega)$ there exist $a \in R_+$ and $b \in]0, 1]$ such that

$$(1.2) \quad \varrho(x) \leq a + b|x| \quad \forall x \in \Omega$$

(see, e.g., (19) and (20) of [TT]).

We denote by $\mathcal{A}_o(\Omega)$ the class of measurable functions $\varrho \in \mathcal{A}(\Omega)$.

From (1.1) and (1.2) follows that for all $\varrho \in \mathcal{A}_o(\Omega)$ we have

$$(1.3) \quad \varrho \in L_{loc}^\infty(\bar{\Omega}), \quad \varrho^{-1} \in L_{loc}^\infty(\Omega).$$

As we will see in (1.5) the second relation of (1.3) can be improved.

For all $\varrho \in \mathcal{A}(\Omega)$ we denote by S_ϱ the set defined by (2).

It is well-known (see, e.g., [T₄]) that, if $\varrho \in \mathcal{A}(\Omega)$ and $S_\varrho \neq \emptyset$, then

$$(1.4) \quad \varrho(x) \leq \text{dist}(x, S_\varrho) \quad \forall x \in \Omega.$$

We prove the following

Lemma 1.1. *For all $\varrho \in \mathcal{A}(\Omega)$ and for all $z \in \partial\Omega$, the following statements are equivalent:*

- (1) $z \in S_\varrho$,
- (2) $\varrho(x) \leq |x - z| \quad \forall x \in \Omega$,
- (3) $\inf_{\Omega(z,r)} \varrho = 0 \quad \forall r \in R_+$.

PROOF: (1) \Rightarrow (2) is a consequence of (1.4). (2) \Rightarrow (1) and (2) \Rightarrow (3) are evident. To prove (3) \Rightarrow (2), we observe that if there exists $x_1 \in \Omega$ such that $\varrho(x_1) > |x_1 - z|$ and if we put $\tau = \varrho(x_1) - |x_1 - z|$, we have

$$|x - x_1| < \varrho(x_1) \quad \forall x \in \Omega(z, \tau),$$

from which, by (1.1), follows

$$\gamma^{-1}\varrho(x_1) \leq \varrho(x) \leq \gamma\varrho(x_1) \quad \forall x \in \Omega(z, \tau),$$

and so we have $\inf_{\Omega(z,\tau)} \varrho > 0$. □

Theorem 1.1. *If $\varrho \in \mathcal{A}(\Omega)$, then S_ϱ is a closed subset in $\partial\Omega$.*

PROOF: Let $z \in \partial\Omega \setminus S_\varrho$. As a consequence of (3) of Lemma 1.1 there exists $\tau \in R_+$ such that $\inf_{\Omega(z,\tau)} \varrho > 0$. From this we have

$$\inf_{\Omega(y,\tau-|y-z|)} \varrho > 0 \quad \forall y \in B(z, \tau) \cap \partial\Omega,$$

and then, again from (3) of Lemma 1.1, $B(z, \tau) \cap \partial\Omega \subset \partial\Omega \setminus S_\varrho$. Thus we obtain our statement. □

Remark 1.1. If $\varrho \in \mathcal{A}(\Omega)$, for any compact set $\Omega_o \subset \bar{\Omega} \setminus S_\varrho$, from (1.1) and (3) of Lemma 1.1 we deduce easily that $\inf_{\Omega_o} \varrho > 0$. It follows that if $\varrho \in \mathcal{A}_o(\Omega)$ then

$$(1.5) \quad \varrho^{-1} \in L^\infty_{loc}(\bar{\Omega} \setminus S_\varrho).$$

□

If $r \in \mathbb{N}$, $1 \leq p \leq +\infty$, $q \in \mathbb{R}$ and $\varrho \in \mathcal{A}_o(\Omega)$, we denote by $W_q^{r,p}(\Omega)$ the space of distributions u on Ω such that $\varrho^{q+|\alpha|-r} \partial^\alpha u \in L^p(\Omega)$ for $|\alpha| \leq r$ with the norm

$$(1.6) \quad \|u\|_{W_q^{r,p}(\Omega)} = \sum_{|\alpha| \leq r} |\varrho^{q+|\alpha|-r} \partial^\alpha u|_{p,\Omega}.$$

We put

$$W_q^{0,p}(\Omega) = L^p_q(\Omega).$$

2. The spaces $K^p_q(\Omega)$

Let us fix $\varrho \in \mathcal{A}_o(\Omega)$.

We consider the spaces $K^p_q(\Omega)$, $\tilde{K}^p_q(\Omega)$, $\overset{\circ}{K}^p_q(\Omega)$, $1 \leq p < +\infty$, $q \in \mathbb{R}$, defined in [CCD₁] in correspondence with the family of open sets $\Omega(x, \varrho(x))$, $x \in \Omega$.

Let us recall that:

$K^p_q(\Omega)$ is the space of functions $g \in L^p_{loc}(\bar{\Omega} \setminus S_\varrho)$ such that

$$(2.1) \quad \|g\|_{K^p_q(\Omega)} = \sup_{x \in \Omega} \left(\varrho^{q-n/p}(x) |g|_{p,\Omega(x,\varrho(x))} \right) < +\infty,$$

with the norm defined by (2.1),

$\tilde{K}^p_q(\Omega)$ is the closure of $L^\infty(\Omega)$ in $K^p_q(\Omega)$,

$\overset{\circ}{K}^p_q(\Omega)$ is the closure of $C^\infty_o(\Omega)$ in $K^p_q(\Omega)$.

For some properties of the spaces $K^p_q(\Omega)$, $\tilde{K}^p_q(\Omega)$ and $\overset{\circ}{K}^p_q(\Omega)$ we refer to [CCD₁], [CCD₂].

In order to recall a result of [CCD₁] which we will use later, we introduce the following notations (see, e.g., n. 1 of [CCD₁]).

We denote by α a function of $C^{(0,1)}(\bar{\Omega}) \cap C^\infty(\Omega)$ such that $\alpha(x) \sim \text{dist}(x, \partial\Omega)$ and we put

$$\Omega_k = \{x \in \Omega : |x| < k, \alpha(x) > 1/k\}, \quad \forall k \in \mathbb{N}.$$

We denote, furthermore, by $(\psi_k)_{k \in \mathbb{N}}$ a sequence of functions in $C^\infty_o(\Omega)$ such that

$$0 \leq \psi_k \leq 1, \quad \psi_k|_{\Omega_k} = 1, \quad \text{supp } \psi_k \subset \Omega_{2k}.$$

The following result holds (see Lemma 2 of [CCD₁]): a function $g \in \overset{\circ}{K}^p_q(\Omega)$ if and only if $g \in K^p_q(\Omega)$ and

$$(2.2) \quad \lim_{k \rightarrow \infty} \|(1 - \psi_k)g\|_{K^p_q(\Omega)} = 0.$$

Because of this result, of Theorem 1.1 and of Remark 1.1, we can prove the following condition so that a function in $K^p_q(\Omega)$ is in $\overset{\circ}{K}^p_q(\Omega)$.

Lemma 2.1. *If $g \in K_q^p(\Omega)$, $1 \leq p < +\infty$, $q \in R$, and if moreover*

$$(2.3) \quad \lim_{|x| \rightarrow +\infty} \varrho^q(x) g(x) = 0,$$

$$(2.4) \quad \lim_{x \rightarrow x_o} \varrho^q(x) g(x) = 0 \quad \forall x_o \in S_\varrho,$$

then $g \in \overset{\circ}{K}_q^p(\Omega)$.

PROOF: Let us fix $\epsilon > 0$.

From (2.3) it follows that there exists $r_\epsilon > 0$ such that

$$(2.5) \quad |\varrho^q(y) g(y)| < \epsilon \quad \forall y \in \Omega, \quad |y| > r_\epsilon.$$

If we put

$$A_\epsilon = \{x \in \Omega : \text{dist}(x, B_{r_\epsilon} \cap \Omega) < \varrho(x)\},$$

from Theorem 1.3 of [T₄] it follows that A_ϵ is bounded.

Let $r_\epsilon^* > r_\epsilon$ such that $A_\epsilon \subset B_{r_\epsilon^*} \cap \Omega$.

We remark that if $x \in \Omega$, $|x| \geq r_\epsilon^*$ and $y \in \Omega(x, \varrho(x))$, then $|y| > r_\epsilon$.

Thus, because of (2.5), for any $k \in N$ we have

$$(2.6) \quad \begin{aligned} & \sup_{\substack{x \in \Omega \\ |x| \geq r_\epsilon^*}} \varrho^{qp-n}(x) \int_{\Omega(x, \varrho(x))} |1 - \psi_k|^p |g|^p dy \\ & \leq c_1 \sup_{\substack{x \in \Omega \\ |x| \geq r_\epsilon^*}} \varrho^{-n}(x) \int_{\Omega(x, \varrho(x))} |1 - \psi_k|^p |\varrho^q g|^p dy \leq c_2 \epsilon^p, \end{aligned}$$

where the constants $c_1, c_2 \in R_+$ are independent of x and k .

Clearly, if $x \in \Omega$, $|x| < r_\epsilon^*$ and $y \in \Omega(x, \varrho(x))$, then $|y| < r_\epsilon^* + \sup_{\Omega \cap B_{r_\epsilon^*}} \varrho = \tilde{r}_\epsilon$.

From (2.4) and from Theorem 1.1 it follows that $S_\varrho \cap \overline{B_{\tilde{r}_\epsilon}}$ can be covering by a finite number of open balls, with center on S_ϱ , $I_{\epsilon, i}$, $i = 1, \dots, m$, such that, letting $K_\epsilon = \cup_{i=1}^m I_{\epsilon, i}$, we have

$$(2.7) \quad |\varrho^q(y) g(y)| < \epsilon \quad \forall y \in \Omega \cap K_\epsilon.$$

From (2.7), for any $k \in N$ we have

$$(2.8) \quad \begin{aligned} & \sup_{\substack{x \in \Omega \\ |x| < r_\epsilon^*}} \varrho^{qp-n}(x) \int_{\Omega(x, \varrho(x)) \cap K_\epsilon} |1 - \psi_k|^p |g|^p dy \\ & \leq c_3 \sup_{\substack{x \in \Omega \\ |x| < r_\epsilon^*}} \varrho^{-n}(x) \int_{\Omega(x, \varrho(x)) \cap K_\epsilon} |1 - \psi_k|^p |\varrho^q g|^p dy \leq c_4 \epsilon^p, \end{aligned}$$

where the constants $c_3, c_4 \in R_+$ are independent of x and k .

Moreover, from (1.5), we get

$$\begin{aligned}
 & \sup_{\substack{x \in \Omega \\ |x| < r_\epsilon^*}} \varrho^{qp-n}(x) \int_{\Omega(x, \varrho(x)) \setminus K_\epsilon} |1 - \psi_k|^p |g|^p dy \\
 (2.9) \quad & \leq c_5 \sup_{\substack{x \in \Omega \\ |x| < r_\epsilon^*}} \int_{\Omega(x, \varrho(x)) \setminus K_\epsilon} \varrho^{qp-n}(y) |1 - \psi_k|^p |g|^p dy \\
 & \leq c_6 \int_{(\Omega \cap B_{\tilde{r}_\epsilon}) \setminus K_\epsilon} |1 - \psi_k|^p |g|^p dy,
 \end{aligned}$$

where the constants $c_5, c_6 \in R_+$ are independent of x and k .

From (2.6), (2.8) and (2.9) it follows that

$$\begin{aligned}
 (2.10) \quad & \sup_{x \in \Omega} \varrho^{qp-n}(x) \int_{\Omega(x, \varrho(x))} |1 - \psi_k|^p |g|^p dy \\
 & \leq c_7 (\epsilon^p + \int_{(\Omega \cap B_{\tilde{r}_\epsilon}) \setminus K_\epsilon} |1 - \psi_k|^p |g|^p dy),
 \end{aligned}$$

where the constant $c_7 \in R_+$ is independent of x and k .

From (2.10) we obtain (2.2) and thus our statement. □

3. Imbedding results

For any $x \in R^n$ and for any $\theta \in]0, \frac{\pi}{2}[$ we denote by $C_\theta(x)$ an open indefinite cone with vertex in x and opening θ .

For a fixed $C_\theta(x)$, we put

$$C_\theta(x, r) = C_\theta(x) \cap B(x, r) \quad \forall r \in R_+.$$

We denote by $\Gamma(\Omega, \theta, r)$ the family of open cones C of opening θ , height r and such that $\overline{C} \subset \Omega$.

We suppose that the following condition holds:

(h_0) there exist $b \in]0, 1]$ and $\theta \in]0, \frac{\pi}{2}[$ such that

$$(3.1) \quad \forall x \in \Omega \quad \exists C_\theta(x) : \quad \overline{C_\theta(x, b\varrho(x))} \subset \Omega.$$

Remark 3.1. We remark that if, for example, $\varrho \in \mathcal{A}(\Omega) \cap L^\infty(\Omega)$ and Ω verifies the condition

(h_1) there exists an open subset Ω^* of R^n with the cone property such that

$$\Omega \subset \Omega^*, \quad \partial\Omega \setminus S_\varrho \subset \partial\Omega^*,$$

then the condition (h_0) holds.

In fact, we consider $\theta \in]0, \frac{\pi}{2}[$ and $r \in R_+$ such that for all $x \in \Omega$ there exists $C_\theta(x)$ such that $\overline{C_\theta(x, r)} \subset \Omega^*$.

Let us fix $b \in]0, 1]$ such that $b \operatorname{ess\,sup}_\Omega \varrho < r$. Then (see n.2 of [CCD₁]) we have that for any $x \in \Omega$ it results $\overline{C_\theta(x, b\varrho(x))} \subset \Omega$. □

Let us fix $\gamma \in R_+$ such that (1.1) is verified.

For all $x \in \Omega$ and for all $\lambda \in]0, 1]$ we denote by $G_{\lambda,b}(x)$ the subset of R^n union of the family of open cones $C \in \Gamma(\Omega, \theta, \lambda\gamma^{-1}b\varrho(x))$ such that $x \in C$.

We fix $\Omega_b(x)$, $x \in \Omega$, with the condition that there exists $\lambda \in]0, 1]$ such that

$$(3.2) \quad G_{\lambda,b}(x) \subset \Omega_b(x) \subset \Omega(x, b\varrho(x)) \quad \forall x \in \Omega.$$

We put, in the case $b = 1$,

$$G_\lambda(x) = G_{\lambda,1}(x), \quad \Omega(x) = \Omega_1(x) \quad \forall x \in \Omega.$$

Remark 3.2. In n.5 of [CCD₁], fixed $\varrho \in \mathcal{A}_o(\Omega)$, the authors assumed that the following hypotheses are satisfied:

- (i₁) S_ϱ is closed, $\varrho^{-1} \in L^\infty_{loc}(\overline{\Omega} \setminus S_\varrho)$,
- (i₂) $\exists \theta \in]0, \frac{\pi}{2}[$ such that

$$\forall x \in \Omega \quad \exists C_\theta(x) : \quad \overline{C_\theta(x, \varrho(x))} \subset \Omega,$$

- (i₃) $\Omega(x)$ has the cone property with a cone $C \in \Gamma(\Omega, \theta_o, \lambda_o \varrho(x))$, where θ_o and λ_o are constants independent of x ,
- (i₄) r, q, p, s are numbers such that

$$(3.3) \quad r \in N, \quad q \in R, \quad 1 \leq p \leq s < +\infty, \quad s \geq \frac{n}{r}, \quad s > \frac{n}{r} \quad \text{if} \quad \frac{n}{r} = p > 1.$$

Condition (i₃) is not really contained in n.5 of [CCD₁], but in the note ⁽¹⁾ of page 115 of [CCD₂] it is explained that hypothesis (i₃) must be added in order to prove the results of n.5 of [CCD₁].

In n.5 of [CCD₁], under the hypotheses (i₁), (i₂), (i₃) and (i₄) the authors have proved (see Theorem 1) that for all $g \in L^s_{loc}(\overline{\Omega} \setminus S_\varrho)$ such that $\sup_{x \in \Omega} (\varrho^{-q+r-\frac{n}{s}}(x)|g|_{s,\Omega(x)}) < +\infty$ and for all $u \in W_q^{r,p}(\Omega)$, it results $gu \in L^p(\Omega)$ and

$$(3.4) \quad |gu|_{p,\Omega} \leq c \sup_{x \in \Omega} \left(\varrho^{-q+r-\frac{n}{s}}(x)|g|_{s,\Omega(x)} \right) \|u\|_{W_q^{r,p}(\Omega)},$$

where the constant $c \in R_+$ is independent of g and u .

Moreover, they have proved some consequences of the above Theorem 1.

We remark (see Theorem 1.1 and Remark 1.1) that the hypothesis (i₁) can be dropped.

Moreover we remark that the hypothesis (i₃) is not necessary in order to obtain Theorem 1 of [CCD₁]. In fact, this theorem holds with $\Omega(x) = G_\lambda(x)$, because $G_\lambda(x)$ verifies the (i₃). Then from this we deduce that the above theorem holds also without (i₃), because for any $q \in R$ and $p \in [1, +\infty[$ we have

$$\sup_{x \in \Omega} \varrho^{q-\frac{n}{p}}(x) |g|_{p, G_\lambda(x)} \leq \sup_{x \in \Omega} \varrho^{q-\frac{n}{p}}(x) |g|_{p, \Omega(x)}.$$

□

From Remark 3.2 it follows that, the hypotheses (h₀) and (3.3) are enough to prove Theorem 1 of [CCD₁] in the case $\Omega_b(x)$ and thus also in the case $\Omega(x, \varrho(x))$. So we have

Theorem 3.1. *If the conditions (h₀) and (3.3) hold, then for all $g \in K^s_{-q+r}(\Omega)$ and for all $u \in W^{r,p}_q(\Omega)$ we have $gu \in L^p(\Omega)$ and*

$$(3.5) \quad |gu|_{p, \Omega} \leq c \|g\|_{K^s_{-q+r}(\Omega)} \|u\|_{W^{r,p}_q(\Omega)},$$

where the constant $c \in R_+$ is independent of g and u . □

Using arguments similar to those in n.5 of [CCD₁], from Theorem 3.1 we deduce that:

- (a) if the hypotheses of Theorem 3.1 hold and therefore $g \in \tilde{K}^s_{-q+r}(\Omega)$, then for any $\epsilon \in R_+$ there exists $c(\epsilon) \in R_+$ such that

$$(3.6) \quad |gu|_{p, \Omega} \leq \epsilon \|u\|_{W^{r,p}_q(\Omega)} + c(\epsilon) \|u\|_{L^p_{q-r}(\Omega)} \quad \forall u \in W^{r,p}_q(\Omega);$$

- (b) if, moreover, $g \in \overset{\circ}{K}^s_{-q+r}(\Omega)$, then for any $\epsilon \in R_+$ there exist $c(\epsilon) \in R_+$ and a bounded open set Ω_ϵ , with the cone property and with $\overline{\Omega}_\epsilon \subset \Omega$, such that

$$(3.7) \quad |gu|_{p, \Omega} \leq \epsilon \|u\|_{W^{r,p}_q(\Omega)} + c(\epsilon) |u|_{p, \Omega_\epsilon} \quad \forall u \in W^{r,p}_q(\Omega)$$

and we have that the operator

$$(3.8) \quad u \in W^{r,p}_q(\Omega) \rightarrow gu \in L^p(\Omega)$$

is compact.

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