Abstracts of CSc. theses in mathematics

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ABSTRACTS OF CSc. THESES IN MATHEMATICS

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BIFURCATION OF PERIODIC SOLUTIONS OF DIFFERENTIAL VARIATIONAL INEQUALITIES AND OF STATIONARY SOLUTIONS OF QUASI VARIATIONAL INEQUALITIES

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(January 12, 1994; supervisor B. Pondělíček)

The dissertation is divided in two parts:

In the part A we study the bifurcation of periodic solutions of differential inequalities of the type

$$\begin{aligned} & U() \in K, \quad t \in [0,T), \\ & (\dot{U}(t) - F(\lambda, , U(t)), v - U(t)) \geq 0 \qquad \forall\, v \in K, \\ & \qquad\qquad\qquad\qquad\qquad\qquad \text{a.e. } t \in [0,T), \end{aligned} \tag{0.6}$$

where K is a closed convex cone in $\mathbb{R}^n$, and $F : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear mapping such that $F(\lambda, 0) \equiv 0$.

In Section 1, the basic properties of the solutions of differential inequalities are introduced.

In Section 2, the main bifurcation theorem is formulated and proved. An essential step in its proof is provided by a topological lemma which is also proved. The use of the theorem is illustrated in several examples. As a particular case, the well-known Hopf bifurcation theorem for ordinary differential equations is obtained.

In Section 3, an elementary method is worked out to deal with the case of inequalities in $\mathbb{R}^3$. Owing to the simple geometrical structure of three dimensional cones slightly sharper results are obtained for this particular type of inequalities.

The part B is devoted to the study of bifurcation and of eigenvalues of a certain type of quasi variational inequalities. The problem is approached with the method of a jump in the degree and our bifurcation theorems are extensions to the quasi variational case of M. Kučera and P. Quittner.

In Section 1, a general theory of bifurcation for quasi variational inequalities is given which is illustrated in two examples.

In Section 2, a system of quasi variational inequalities of reaction-diffusion type is studied. A bifurcation theorem is proved and its assumptions are verified in an example concerning a system of partial differential equations (of reaction-diffusion type) with unilateral boundary value conditions depending on the solution of the inequality. Also, a certain destabilizing effect is proved to take place under such unilateral condition.

MIX AND MAX OBJECTIVE FUNCTIONS IN LOCATION PROBLEMS

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(May 31, 1994; supervisor K. Zimmermann)

A general graph location problem consists in finding the optimum location of one or several objects in a system which can be modelled by a graph. Any location problem is well defined by the optimality criterion (objective function), the specification of the demand set (set of already existing points) and specification of the choices available for the set of facilities (new points). The appropriate objective function may vary by application. The common features of the location problems considered in the thesis are graph representation, objective function consisting of min and max operators, involving distances points in the graph. It is assumed the located objects have two or finite number of possibilities for location, or location of each object is confined to an edge of the underlying graph. Everywhere the demand set is a finite set of points of the graph and it is expected a finite number of objects is to be located. For the location problems defined, the complexity and polynomial algorithms are presented.

THE FINITE ELEMENT METHOD FOR NONLINEAR ELLIPTIC PROBLEMS

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(May 31, 1994; supervisor M. Feistauer, K. Najzar)

The thesis considers the finite element solution of second-order nonlinear elliptic equations with discontinuous coefficients in a general nonpolygonal domain Ω with nonhomogeneous mixed Dirichlet-Neumann boundary conditions. In the discretization of the problem it is proceeded in the usual way: the domain Ω is approximated by a polygonal one, conforming piecewise linear triangular elements are used and the integrals are evaluated by numerical quadratures. The solvability of the discrete problem is proved and the convergence of the method is studied.

In case of problems with Lipschitz-continuous operator the strong convergence of the approximate solutions to the weak solution u of variational problem is proved on the only assumption that the exact solution $u \in H^1(\Omega)$. Provided u is piecewise of class H^2 and the problem is strongly monotone, the error estimate $O(h)$ can be obtained.

There are studied problems with locally Lipschitz-continuous operator too. On the assumption the coefficient of the equation are quazilinear and the weak solution is of the class $H^1(\Omega)$ the weak convergence of the method is proved. Provided the exact solution u is of class $W^{2,r}(\Omega)$, $r > 1$, the convergence is strong.

HAUSDORFF MEASURE AND REAL ANALYSIS

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(September 13, 1994; supervisor B. Novák)

The thesis considers Hausdorff measure and dimension and some of their applications in classical real analysis.

In the first part, there are studied metric rectifiable spaces and derived basic informations about their local geometric structure. This is used to conclude the almost everywhere regularity of the Hausdorff measure on such spaces and to prove an area formula for Lipschitz mappings from Euclidean into metric spaces. The key result in this part is the metric version of the well-known Rademacher theorem.

The second part deals with sets of small but stable Hausdorff dimension and a new method how to construct them. A typical result is the following:

For any $0 < T \leq n$ there is a set $M \subset \mathbf{R}^n$ with $\mathcal{H}^T(M) = 0$ but such that the Hausdorff dimension of any set of the kind $\bigcap_i f_i(M)$, where the $f_i : \mathbf{R}^n \to \mathbf{R}^n$ are C^1 and surjective, is again at least T. From this the construction of certain pathological σ-ideals follows. This result can also be extended to the case that the f_i are only bihoelderian and it can be proved that our dimension estimate for this case is quite sharp.

The last part of the work investigates level sets of typical functions and their Hausdorff dimension. It is shown, among other things, that typical continuous functions and typical Darboux functions in the first class of Baire have all level sets of Hausdorff dimension zero. On the contrary, in the intermediate space of all approximately continuous functions a typical function has all level sets of Hausdorff dimension one. At the end precise estimates of the Hausdorff dimension of typical continuous functions between Euclidean spaces are given. For example, if $n \geq m \geq 1$ then a typical continuous function $[0,1]^n \to \mathbf{R}^m$ has all level set $f + -1(\mathrm{y})$, where y is an interior point of the image of f, of Hausdorff dimension $n - m$ but with non-σ-finite measure.

STRONGLY RECTIFABLE AND HOMOGENEOUS MODUL OF TYPE S

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Strongly rectifiable and homogeneous moduls of type S (where $S = R/I$, R is a ring with identity, I is maximal left, two-sided ideal of the ring R) are defined in the thesis. Further it is proved that if conditions (X) and (Y) are fulfilled then every I-prime modul is strongly rectifiable and homogeneous of the type S. The condition (X) requires every I-coprime ideal to be two-sided and the condition (Y) requires I to be two-sided ideal and I/I^2 cyclical either trivial or simple as both right and left modul.

Further the thesis studies strongly rectifiable and homogeneous moduls of the type S. It is proved that every countable strongly rectifiable homogeneous modul of the type S is a direct sum of uniserial submoduls if and only if has no non-zero elements of infinite height; which is an analogy of 2. Prüfer theorem.

In the thesis pure and refined submoduls and Hill moduls are studied and used to build Ulm's theory in the field of the strongly rectifiable and homogeneous moduls of type S. Ulm submoduls, Ulm sequences and Ulm-Kaplansky invariants are defined and the possibility of raising homomorphisms is proved.

In the conclusion of the thesis Ulm's theorem is proved, i.e. strongly rectifiable homogeneous moduls of the type S (if they are direct sum of countably generated moduls) are isomorphic if and only if their Ulm sequences are isomorphic it means that their Ulm-Kaplansky invariants are equal.