Abstracts of CSc. theses in mathematics

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ABSTRACTS OF CSc. THESES IN MATHEMATICS

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STUDENTIZED M-ESTIMATORS AND THEIR k-STEP VERSIONS

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(December 17, 1991; supervisor J. Jurečková)

Robust statistical methods, namely estimators, became well-known part of mathematical statistics. M-estimators (maximum likelihood type estimators), introduced by Huber in 1964, became one of the most important classes of robust estimators. Estimators of this type are not scale-equivariant. Thus, they should be studentized unless we know the scale parameter of underlying distribution. The present work deals just with the studentized M-estimators of location parameter and with their one- and k-step versions.

The M-estimators are defined implicitly as solutions of minimization problems and eventually as those of the corresponding equations. As such, they should be calculated in iterations and then there is a question of consistency of the obtained solution. The basic tool to attack this problem is the second order asymptotic representation of the estimator — an approximation of estimator by a sum of independent random variables supplemented by the order of the remainder term. These representations enable to show the precision of one- and k-step versions as approximations of M-estimators. The quality of the approximations as well as the technique of the proof heavily depends on the shape of the function ψ generating the estimator.

The objectives of Malý's work can be expressed in next four points.

1. To study asymptotic representation of studentized M-estimators of location parameter with exact order of remainder for the following three types of generating ψ-functions:
   (i) ψ is a step-function having finitely many jumps of finite magnitudes;
   (ii) ψ is absolutely continuous with a derivative ψ' which is a step-function;
   (iii) ψ is absolutely continuous with an absolutely continuous derivative ψ'.
2. Using this representation, to prove the asymptotic equivalence of M-estimators and their one-step version and determine the accuracy of approximation.
3. To introduce k-step version of studentized M-estimators, to generalize the results mentioned above, to compare the accuracy of approximation of studentized M-estimator by one- and k-step version and to discuss the usefulness of k-step versions for various types of ψ-functions.
4. To illustrate some of the theoretical results (especially for Huber's function ψ) by numerical simulations.

The results are proved with the aid of the uniform second order asymptotic linearity of studentized M-estimators and their k-step versions. These results, mostly new in the present form, provide a useful generalization of the corresponding ones in the non-studentized case. The dissertation deals mainly with the three-parametric

stochastic processes connected with M-estimators, gives some justifications for studentization and studies the asymptotic properties of k-step estimators.

SOME ADAPTIVE ESTIMATORS IN REGRESSION MODELS

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(December 17, 1991; supervisor M. Hušková)

The thesis concerns sequentially adaptive estimators based on ranks in simple linear models. First these estimators are constructed and then their asymptotic properties are studied. As the main tools of the proof certain asymptotic linearity results for rank statistics are used.

The presented results are certain extensions of the assertions proved by Beran (1974) and Hušková and Sen (1985 and 1986).

AN EXTENDED DEPENDENCY BASED SPECIFICATION OF UNDERLYING REPRESENTATIONS OF SENTENCES

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The thesis formally specifies one of the main components of the Functional Generative Description of Language (FGD), its generative component, which is responsible for the forming of the representations of meaning of sentences of a natural language within FGD. The generative component is defined as an ordered pair (automaton, lexicon). In the thesis, automation is a pushdown store automaton which has one output tape and no input tape (i.e. it is a pushdown store generator) and it generates the representations of meaning of sentences of a natural language on output using the lexicon which contains individual lexical meanings of a given language. The generated representations have the shape of so-called complex dependency structures (CDS) which formalize the components constituting the meaning of sentences, such as the hierarchy of the syntactic dependency, coordination/apposition, topic-focus articulation, the scope of the sentence negation, coreference and other. These components of meaning are not described as isolated phenomena but in their mutual relations which are often very complex. The set of representations generated by the automaton forms a language belonging to the type 2 of the Chomskyan hierarchy of grammars. The thesis shows that even structurally complex relations, such as coreference, can be represented by a relatively weak (from the viewpoint of the generative power) mathematical means. In addition to the definition of the generator as such, the study also contains the thorough linguistic justification of individual formal solutions which substantiate the choice of the pushdown store generator rather than a context-free grammar. The thesis can also be a solid basis for the computer implementation of the generative component within the FGD and its results can be exploited in the construction of the synthesis of language within the machine translation.

NUMERICAL ANALYSIS OF TAKENS-BOGDANOV POINTS

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Takens-Bogdanov points (TB-points) are singular points of a smooth mapping $F : \mathbb{R}^N \times \mathbb{R}^{1+k} \to \mathbb{R}^N$ with additional spectral degeneracy of the Jacobian matrix F_u. The simplest TB-points are defined by algebraic multiplicity 2 of the zero eigenvalue with the Jordan cell $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. In the unfolding of the TB-points both of Hopf and steady-state bifurcations occur. Following the well-tried concept of imperfect bifurcations given by Golubitsky and Schaeffer we can investigate TB-points as organizing centres for both of those bifurcation points.

The aim of the thesis is to contribute to the constructive numerical analysis of these phenomena. The numerical technique is based upon a special kind of reduction — TB-reduction, which has been developed and in detail studied. The involved reduction process gives a straight connection between the quite pure theory of imperfect bifurcation, classification of matrices depending on parameters and standard numerical techniques. Computation of TB-points as isolated organizing centers together with an *à posteriori* computer aided asymptotic analysis of a detected organizing center (being subjected to a given perturbation) are discussed. The pathfollowing methods are used for tracking branches of TB-points.

The asymptotic formulae are applied as analytical predictors for numerical detection of (e.g.) Hopf bifurcation points on a neighborhood of an already computed organizing center. Another application is a local stability analysis of imperfect bifurcation diagrams. The special attention is directed towards local qualitative analysis and its dynamical consequences.

BASES, ESSENTIAL BASES AND WIENER'S CRITERION IN BALAYAGE SPACES

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(March 26, 1992; supervisor I. Netuka)

In the thesis it is proved that the Wiener test for regularity provides a test for bases, essential bases and the Choquet boundary of an open set of a balayage space. The result which is obtained in the context of balayage spaces can be applied to a wide class of second order partial differential operators of elliptic or parabolic type and some classes of pseudodifferential operators.

The first chapter deals with the notion of a balayage space. It is proved that some pseudodifferential operators generate balayage spaces. The second chapter is concerned with the notion of a **K**-capacity to be a Choquet capacity. These results on **K**-capacities are generalizations of Brelot's results from the year 1967. The third and fourth chapters deal with the notion of a base, an essential base and that of a semipolar set. In the fifth chapter an α-capacity is introduced and its fundamental properties are established. The relations between the α-capacity and

the so-called continuous capacity are investigated. Some open problems from the book of G. Anger "Funktionalanalytische Betrachtungen bei Differentialgleichungen unter Verwendug von Methoden der Potentialtheorie", Akademie-Verlag, Berlin, 1967, are solved.

The theoretical results of the thesis are applied to the Laplace, the Kolgomorov and the heat operators, some classes of elliptic and parabolic operators and to the heat operator of order α. Some new results for these operators are obtained.

10-VERTICES COLOURED 3-CONFIGURATIONS AND SOME QUASIGROUP IDENTITIES

KOSTROVÁ Jela, Department of Mathematics, Faculty of Machinery and Electrotechnics, University of Transport and Communications, Žilina, Slovak Republic
(April 8, 1992; supervisor V. Havel)

The algorithmic procedure for a construction of all non-isomorphic configurations in 3-nets with finite number of the vertices is set. For some of those 3-configurations the closure conditions and the corresponding algebraic identities are found.

ON THE REGULARITY OF WEAK SOLUTIONS TO ELLIPTIC PROBLEMS

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(June 5, 1992; supervisor B. Novák)

The dissertation is concerned with the regularity problem for weak solutions of elliptic systems with nonsmooth coefficients. It is shown that a weak solution is regular (i.e. Hölder continuous) if the dispersion of the eigenvalues of the coefficient matrix is small. The condition of such type was firstly established by A. Koshelev. New results are presented in the fourth chapter which contains eight paragraphs.

The first paragraph contains various preliminaries and basic ideas of the method.

The second paragraph is devoted to the study of the behaviour of the solutions of systems with the right hand side from the Morrey spaces.

In the third paragraph it is shown that the perturbation by lower order terms does not changes the regularity result.

The fourth paragraph is concerned with regularity results for some type of variational inequalities.

In the fifth paragraph the sharp theorem is proved. The modification of the method due to J. Nečas gives new improvements of his result.

The sixth paragraph is concerned with elliptic systems which are "not far" from the systems with constant coefficients and results obtained in this paragraph are used in the seventh paragraph for the study of $W^{2,2}$-regularity of weak solutions. The main tool of this paragraph is the difference quotient technique in Nikolski spaces.

The eighth paragraph brings a new method for obtaining the regularity up to the boundary of a convex domain.

Open problems are listed in the fifth chapter.

CONTRIBUTION TO THE SPECTRAL THEORY OF VARIATIONAL INEQUALITIES

NEUMANN Jan, AQUATEST Construction Geology, a.s., Gorkého nám. 7, Praha 1, Czech Republic
(June 5, 1992; supervisor B. Novák)

In the thesis higher bifurcation points of certain variational inequalities are investigated. The main result consists in the generalization of the well known potential bifurcation theorems concerning variational equations given by M.I. Krasnoselskii and I.V. Skrypnik.

The Krasnoselskii's methods have been applied to the spectral theory of variational inequalities by E. Miersemann and later by P. Quittner. These authors confine themselves almost exclusively to the problems with cones as determining sets.

The approach presented in the thesis is based on the use of the Skrypnik's proof technique which is rather more constructive and enables us to prove the validity of the extension of the Miersemann's assertions to the inequalities with general convex determining sets.

Auxiliary results on the evolution differential inequalities which are used in the proofs of the bifurcation theorems represent an interesting contribution to the elementary theory of the time-dependent variational inequalities.

ESTIMATION OF LIFETIME CHARACTERISTICS FROM RANDOMLY CENSORED SAMPLES PROPORTION HAZARDS MODEL

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(June 22, 1992; supervisor J. Hurt)

The Koziol-Green model of random censorship is considered (Koziol, Green (1976)). The maximum likelihood estimate of the survival function is studied and its weak convergence in the Skorokhod space is proved. The estimate is employed to construct estimates of other lifetime characteristics. Estimates of lifetime moments and of residual lifetime moments are proposed and their asymptotic properties are studied. They are proved to be asymptotically normally distributed and consistent in strong sense. Moreover, joint asymptotic normality of the suggested estimates is obtained. Quantile estimates are also proposed and their asymptotic normality is derived including a generalization to multidimensional case. Finally, some tests of fit with the Koziol-Green model of random censorship are proposed.

THE PROJECTIVE SPECIAL LINEAR GROUP PSL$(6, 2^a)$

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(July 3, 1992; supervisor K. Drbohlav)

In the thesis the 6-dimensional projective special linear group over a field of characteristic two, PSL$(6, 2^a)$ is dealt with. The types of collineations of this group, the structure of centralizer corresponding to each type and some other properties of

these collineations are determined. The main outcome of this investigation is a list of maximal subgroups.

EULER EQUATIONS AND THEIR NUMERICAL SOLUTION BY THE FINITE VOLUME METHOD

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(October 13, 1992; supervisors M. Feistauer, I. Netuka)

The contribution is devoted to the study of Euler equations as a particular example of general hyperbolic system of partial differential equations both from the theoretical and numerical point of view. In the first part the general hyperbolic systems of partial differential equations are concerned. The thesis studies the existence of a weak entropy solution to the hyperbolic system of partial differential equations and uses the Kružkov existence result for the case of one equation. Then it studies general questions on the existence of entropy showing that the one exists if and only if the system in question is symmetrizable. To finish the theoretical part up, it applies the Young measure technique to show the existence result for one equation in one space dimension. Namely, it uses a version of the Murat-Tartar identity for nonconvex entropies. After introducing the concept of measure-valued solution, it reminds DiPerna's uniqueness theorem.

In the numerical part the thesis deals with the general finite volume method for solving hyperbolic conservation laws. We prove the L^p-stability result in the case of scalar monotone scheme. Then it generalizes Lax-Wendroff theorem to the case of unstructured grids. Next, it studies the general questions of convergence and proves the convergence theorem for scalar monotone scheme.

Finally, it chooses one particular scheme (the one of Vijayasundaram) as an example to the previous general considerations and calculates two numerical experiments, namely the examples of both stationary and unstationary flow.

ASYMPTOTIC METHODS IN THE THEORY OF STOCHASTIC CONTROLLED SYSTEMS

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(October 15, 1992; supervisor P. Mandl)

The dissertation consists of three parts. Asymptotic inequalities for the cost functional are derived in the first part containing two chapters.

Chapter 1 presents the approach using martingale methods together with the Skorokhod's representation of random variables by the stopping of the Wiener process in the case of discrete time linear systems. Asymptotic inequalities comparing the probability distributions of monotonous functionals of the cost under a general control and under the optimal stationary one are presented.

Finite controlled Markov chains with discounted cost criterion are considered in Chapter 2. The method of Skorokhod's representation is applied, too. It is proved that the average cost optimal control yields the stochastically smallest distribution of the discounted cost asymptotically as the discount rate tends to zero.

The following two chapters of Part II contain contributions to the theory of continuous linear systems. Chapter 3 deals with self-tuning controls constructed by inserting the estimates for the unknown parameters. The model of a linear controlled system containing a constant drift is considered. The unknown parameters are estimated by the least squares method. Recursive formula for the estimate is introduced and a sufficient condition for its consistency is presented. Assuming the consistency the asymptotic distributions of the estimate and of the quadratic functionals are investigated. From the asymptotic distributions the quality of the self-tuning can be assessed.

In Chapter 4 slow unmodelled oscillations of a system are regarded as a parameter and estimated by the discounted least square method. The estimate is used to eliminate the oscillations. Properties of the procedure are presented for vanishing discount factor. Asymptotic expansion of the criterion is derived.

The last chapter of Part III deals with analysis of the discrete observation of the input and output of the system presented in Chapter 4. Numerical values of the criterion for discrete time systems are compared with the asymptotic expansion of the criterion.

VARIATIONAL ANALYSIS ON FIBERED MANIFOLDS OVER ONE-DIMENSIONAL BASIS

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(November 11, 1992; supervisor M. Ráb)

The thesis is intended as a contribution to develop a systematic and self-contained variational theory within the range of the calculus of variations on fibered manifolds, which would cover classical results as a special case. It contains both a survey of the author's results and original research in higher-order mechanics (i.e. the calculus of variations on higher-order jet prolongations of fibered manifolds over one-dimensional basis). It develops the theory for the so-called locally variational forms. This corresponds to the situation when "globally defined variational equations" are given — i.e. globally defined solutions can be searched for, but, in general, the lagrangians are only locally defined (there even may exist no globally defined

A generalization of the Hamilton theory and the integration theories (Liouville theorem, Hamilton-Jacobi theory, Van Hove theorem) to arbitrary higher-order variational equations (i.e. to any, not only regular, lagrangians) is given, and geometrical structures related with extremals of locally variational forms are obtained and studied. Nonvariational equations are also considered.

ESTIMATIONS OF THE PARAMETERS IN THE LINEAR PROCESSES

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(December 15, 1992; supervisor J. Anděl)

At the beginning, a review of fundamental methods for the estimation of the parameters in the linear processes is given.

Three classes of estimators of the autoregressive parameters in the case of non-negative AR process of the general order are investigated: minimum quotients estimators (MQE), conditional maximum likelihood estimators (CMLE) and modified CMLE (MCMLE). It is proved that all these estimators are strongly consistent. In a simulation study some AR(3) models with exponential white noise were simulated. The results show that CMLE and MCMLE behave better than the least square estimators.

In the last part of the thesis a strongly consistent estimator of the parameter in the fractionally differenced white noise (FDWN) model (i.e. in the class of the so called long memory models) is constructed. This estimator is asymptotically equivalent to maximum likelihood one.

NON-NEWTONIAN INCOMPRESSIBLE FLUIDS WITHOUT MEMORY

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(December 16, 1992; supervisor J. Nečas)

The Navier-Stokes system can be included as a special case into the class of Non-newtonian incompressible fluids with the nonlinear stress tensor $\tau = \tau(\mathbf{e})$, the components of which satisfy the p-growth condition. This contribution extends some former existence and uniqueness results for nonstationary flow even for that case when the tensor function τ satisfies so-called bad (subquadratic) growth condition.

Measure-valued solutions exist already for $p > \frac{2n}{n+2}$. For the space periodic problem, the existence of the weak solution is then obtained for $p > \frac{3n}{n+2}$. For $p \geq 1 + \frac{2n}{n+2}$, these solutions are regular and unique provided that initial values are smoother (for threedimensional flow).

It does not allow to prove the existence of the global attractor by standard way. Therefore, the spaces of snakes (short trajectories) are introduced, which provides not only a topology convenient to prove the existence of a global attractor but also a direct method for estimate the fractal dimension of the attractor.

Some existence and uniqueness questions are discussed also for stationary case.

The modification of the Navier-Stokes solver for both steady and unsteady flow is presented. The method, using discretely divergence-free finite elements to eliminate the pressure, a fixed point-defect correction method to solve the nonlinear equations and an adapted multigrid method for linear subproblems, is applied on the alternate model of the boundary layer theory. The first numerical tests for pipe flow, driven cavity, backward step and venturi pipe flows are described.