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Announcements of new results

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ANNOUNCEMENTS OF NEW RESULTS

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INTEGRATION WITH RESPECT TO A NONLINEAR HARMONIC MEASURE: A COUNTEREXAMPLE

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Let U be an open subset of a topological space, on which a potential theory is considered, and x be a point of U. In linear potential theories, the functional which associates with a function f on the boundary of U the value at x of the solution H_f^U of the Dirichlet problem, is the integral of f with respect to the harmonic measure μ_x^U at x (see [CC]). In nonlinear potential theories, we are led to a 'nonlinear integration'. It can be defined in the following way: Let X be a compact topological space and A be a real functional defined on $C(X)$ such that $A(f) \le A(g)$ whenever $f \le g$, $A(cf) = cA(f)$ and $A(f+c) = A(f) + c$. Here f and g denote arbitrary functions from $C(X)$ and c is an arbitrary constant. Then we define

$$A^o(u) = \sup\{A(f) : f \in C(X), f \le u\}$$

if u is l.s.c. and lower finite,

$$A^*(v) = \inf\{A^o(u) : u \ge v,\ u > -\infty,\ u \text{ is l.s.c.}\}\,,$$
$$A_*(v) = -A^*(-v)\,,$$

if v is arbitrary. We say that a function v is A-integrable, if $A^*(v) = A_*(v)$.

Now, let us describe the example. Let B_r be the disc in $\mathbf{R}^2$ with center at the origin and radius r, $S_r = \partial B_r$. Let σ_r be the uniformly distributed unit measure on S_r. If $f \in C(S_r)$, we define $A_r(f)$ to be the (unique) value $c \in \mathbf{R}$ for which both $\sigma_r\{f > c\} \le 1/2$ and $\sigma_r\{f < c\} \le 1/2$ (the so-called "median" of f). Now, in case $r = 1$, $A = A_1$, if we set

$$v(x) = \begin{cases} 1, & \text{if } x_1 \ge 0 \text{ and } x_2 > -1, \\ 0, & \text{if } x_1 \le 0 \text{ and } x_2 < 1, \end{cases}$$

then it is easily verified that $A^*(v) = 1$ and $A_*(v) = 0$, although v is a nice bounded Borel function.

We show that there is a "potential theory", for which A stands to be a "harmonic measure". We say that a function h is harmonic on an open set $G \subset \mathbf{R}^2$, if it is continuous, locally linear on the sets $G \cap \{tx : t \ge 0\}$ for all $x \in \mathbf{R}^2$, and $h(0) = A_r(h)$ for any $r > 0$ with $\overline{B}_r \subset G$. Such harmonic functions form a sheaf and bounded open strictly convex sets are regular. Furthermore, the comparison principle holds on every bounded open set. Nevertheless, it is still a pathological potential theory, as the Bauer convergence principle is not satisfied. We do not know, whether bounded Borel functions are "integrable with respect to the harmonic measure" in the framework of nonlinear harmonic spaces introduced by Laine [L].

References

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