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Miroslav Sova

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RELATION BETWEEN REAL AND COMPLEX PROPERTIES OF THE LAPLACE TRANSFORM

MIROSLAV SOVA, Praha

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It is well-known that Widder's theory [1] of representability by Laplace transform of numerical functions gives necessary and sufficient conditions for the existence of originals of certain classes. These conditions are especially simple for the class of images of exponentially bounded measurable functions and we shall deal in the sequel only with this type.

Widder's theory was generalized to reflexive Banach spaces by MIYADERA [2] and further results and generalizations to non-reflexive Banach spaces were obtained by the author in [3] and [4].

All above mentioned results are of Widder's type, i.e. they are based on the behavior of the derivatives of the Laplace images on the real halfaxis. But there are also other sufficient conditions based on the behavior of the images on lines parallel to the imaginary axis. In the sequel we shall show a simple way how to get also conditions of complex character from Widder's type theories.

For the sake of simplicity we restrict ourselves to reflexive spaces only because Miyadera's theorem [2] will be our basic tool. But it is easy to obtain in this way also the corresponding results for the situations examined in [3] and [4].

1. We shall use the following notation: (1) $\mathbb{R}$ – the real number field, (2) $\mathbb{R}^+$ – the set of all positive real numbers, (3) (ω, ∞) – the set of all real numbers greater than ω if $\omega \in \mathbb{R}$, (4) $\mathbb{C}$ – the complex number field, (5) $(\operatorname{Re} z > \omega)$ – the set of all complex numbers whose real part is greater than ω if $\omega \in \mathbb{R}$, (6) $M_1 \to M_2$ – the set of all mappings of the whole set M_1 into the set M_2.

2. In the whole paper, E will denote a Banach space over $\mathbb{C}$ with the norm $\|\cdot\|$.

3. Functional analysis (including the theory of vector-valued functions) is used to the extent of the first three chapters of [7], certain special subjects (e.g. II.4, III.3) being omitted. The reader interested only in the numerical case needs nothing more than the basic facts from the modern differential and integral calculus.

4. Lemma. *Let $\alpha \in \mathbb{R}$, $J \in \{z : \operatorname{Re} z \geqq \alpha\} \to E$ and $k \in \{0, 1, \dots\}$. If*

(α) *the function J is continuous on the set $\{z : \operatorname{Re} z \geqq \alpha\}$,*

(β) *the function J is analytic on the set $\{z : \operatorname{Re} z > \alpha\}$,*

(γ) *there is a $K \geqq 0$ so that $\|J(z)\| \leqq K(1 + |z|)^k$ for every $z \in \mathbb{C}$, $\operatorname{Re} z \geqq \alpha$,*

then

$$J^{(p)}(\lambda) = (-1)^p \frac{p!}{2\pi} \int_{-\infty}^{\infty} \frac{J(\alpha + i\beta)}{(\lambda - \alpha - i\beta)^{p+1}} \, d\beta$$

for every $\lambda > \alpha$ and $p \in \{k + 1, k + 2, \dots\}$.

Proof. Let us fix a $\lambda > \alpha$.

Further, let $K \geqq 0$ be chosen so that (γ) holds.

By virtue of Cauchy's integral theorem we obtain from (α) and (β) that (a sketch will be helpful)

$$(1) \qquad \frac{2\pi}{p!} J^{(p)}(\lambda) = - \int_{-N}^{N} \frac{J(\alpha + i\beta)}{(\alpha + i\beta - \lambda)^{p+1}} \, d\beta +$$

$$+ \int_{-N}^{N} \frac{J(\alpha + 2N + i\beta)}{(\alpha + 2N + i\beta - \lambda)^{p+1}} \, d\beta + i \int_{0}^{2N} \frac{J(\alpha + \eta + iN)}{(\alpha + \eta + iN - \lambda)^{p+1}} \, d\eta -$$

$$- i \int_{0}^{2N} \frac{J(\alpha + \eta - iN)}{\alpha + \eta - iN - \lambda)^{p+1}} \, d\eta$$

for every $p \in \{0, 1, \dots\}$ and $N > \frac{1}{2}\lambda$.

Using (γ), we obtain

$$(2) \qquad \left\| \frac{J(\alpha + i\beta)}{(\alpha + i\beta - \lambda)^{p+1}} \right\| \leqq \frac{K(1 + (\alpha^2 + \beta^2)^{1/2})^k}{((\lambda - \alpha)^2 + \beta^2)^{(p+1)/2}}$$

for every $\beta \in \mathbb{R}$ and $p \in \{0, 1, \dots\}$.

$$(3) \quad \left\| \int_{-N}^{N} \frac{J(\alpha + 2N + i\beta)}{(\alpha + 2N + i\beta - \lambda)^{p+1}} \, d\beta \right\| \leqq K \int_{-N}^{N} \frac{[1 + ((\alpha + 2N)^2 + \beta^2)^{1/2}]^k}{[(\lambda - \alpha + 2N)^2 + \beta^2]^{(p+1)/2}} \, d\beta \leqq$$

$$\leqq K \int_{-N}^{N} \frac{[1 + ((\alpha + 2N)^2 + N^2)^{1/2}]^k}{(\lambda - \alpha + 2N)^{p+1}} \, d\beta = \frac{2NK[1 + ((\alpha + 2N)^2 + N^2)^{1/2}]^k}{(\lambda - \alpha + 2N)^{p+1}},$$

$$\left\| \int_{0}^{2N} \frac{J(\alpha + \eta + iN)}{(\alpha + \eta + iN - \lambda)^{p+1}} \, d\eta \right\| \leqq K \int_{0}^{2N} \frac{[1 + ((\alpha + \eta)^2 + N^2)^{1/2}]^k}{[(\lambda + \eta - \alpha)^2 + N^2]^{(p+1)/2}} \, d\eta \leqq$$

$$\leqq K \int_{0}^{2N} \frac{[1 + ((\alpha + 2N)^2 + N^2)^{1/2}]^k}{N^{p+1}} \, d\eta \leqq \frac{2K[1 + ((\alpha + 2N)^2 + N^2)^{1/2}]^k}{N^p},$$

$$\left\| \int_0^{2N} \frac{J(\alpha + \eta - iN)}{(\alpha + \eta + iN - \lambda)^{p+1}} d\eta \right\| \leqq \frac{2K[1 + ((\alpha + 2N)^2 + N^2)^{1/2}]^k}{N^p}$$

for every $p \in \{0, 1, \ldots\}$ and $N > \frac{1}{2}\lambda$.
It follows from (2) that

(4) $$\int_{-\infty}^{\infty} \frac{J(\alpha + i\beta)}{(\alpha + i\beta - \lambda)^{p+1}} d\beta \quad \text{exists for every} \quad p \in \{k + 1, k + 2, \ldots\},$$

(5) $$\frac{J(\alpha + i\beta)}{(\alpha + i\beta - \lambda)^{p+1}} d\beta \to_{N-\infty} \int_{-\infty}^{\infty} \frac{J(\alpha + i\beta)}{(\alpha + i\beta - \lambda)^{p+1}} d\beta$$

for every $p \in \{k + 1, k + 2, \ldots\}$.

Further, by (3) we obtain

(6) $$\int_{-N}^{N} \frac{J(\alpha + 2N + i\beta)}{(\alpha + 2N + i\beta - \lambda)^{p+1}} d\beta \to_{N\to\infty} 0,$$

$$\int_0^{2N} \frac{J(\alpha + \eta + iN)}{(\alpha + \eta + iN - \lambda)^{p+1}} d\eta \to_{N\to\infty} 0,$$

$$\int_0^{2N} \frac{J(\alpha + \eta - iN)}{(\alpha + \eta - iN - \lambda)^{p+1}} d\eta \to_{N\to\infty} 0$$

for every $p \in \{k + 1, k + 2, \ldots\}$.

The desired result follows from (1), (4), (5) and (6).

5. Proposition. *Let M, ω be two nonnegative constants and $F \in (\operatorname{Re} z > \omega) \to E$. If the function F is analytic in the domain $(\operatorname{Re} z > \omega)$, then the following two statements (A), (B) are equivalent:*

(A) (I) *for every $\alpha > \omega$, there exist a $k \in \{0, 1, \ldots\}$ and a $K \geqq 0$ so that $\|F(z)\| \leqq K(1 + |z|)^k$ for every $z \in C$, $\operatorname{Re} z > \alpha$,*

(II) *for every $\alpha > \omega$, there exists an $l \in \{0, 1, \ldots\}$ so that*

$$\frac{1}{2\pi} \left\| \int_{-\infty}^{\infty} \frac{F(\alpha + i\beta)}{(1 - is\beta)^r} d\beta \right\| \leqq M$$

for every $s > 0$ and $r \in \{l + 2, l + 3, \ldots\}$,

(III) $F(\lambda) \to 0$ $(\lambda \to \infty, \lambda > \omega)$;

(B) $$\|F^{(p)}(\lambda)\| \leqq \frac{Mp!}{(\lambda - \omega)^{p+1}}$$

for every $\lambda > \omega$ and $p \in \{0, 1, \ldots\}$.

Proof. (A) $\Rightarrow$ (B). We first fix an arbitrary $\alpha > \omega$.

Now we choose $k, l \in \{0, 1, \dots\}$ so that the assumptions A (I), (II) hold.

Denoting $J(z) = F(z)$ for $z \in \mathbb{C}$, $\operatorname{Re} z > \alpha$, we observe that according to (A) (I), all assumptions of Lemma 4 are fulfilled and consequently we obtain

$$F^{(p)}(\lambda) = (-1)^p \frac{p!}{2\pi} \int_{-\infty}^{\infty} \frac{F(\alpha + \mathrm{i}\beta)}{(\lambda - \alpha - \mathrm{i}\beta)^{p+1}} \, \mathrm{d}\beta \tag{1}$$

for every $\lambda > \alpha$ and $p \in \{k + 1, k + 2, \dots\}$.

Further, we write

(2) $q = \max(k, l)$.

It follows from (A) (II) and from (1) and (2) that

$$\|F^{(p)}(\lambda)\| = \frac{p!}{2\pi} \left\| \int_{-\infty}^{\infty} \frac{F(\alpha + \mathrm{i}\beta)}{(\lambda - \alpha - \mathrm{i}\beta)^{p+1}} \, \mathrm{d}\beta \right\| = \tag{3}$$

$$= \frac{p!}{(\lambda - \alpha)^{p+1}} \frac{1}{2\pi} \left\| \int_{-\infty}^{\infty} \frac{F(\alpha + \mathrm{i}\beta)}{\left(1 - \mathrm{i}\dfrac{1}{\lambda - \alpha}\beta\right)^{p+1}} \, \mathrm{d}\beta \right\| \leqq \frac{Mp!}{(\lambda - \alpha)^{p+1}}$$

for every $\lambda > \alpha$ and $p \in \{q + 1, q + 2, \dots\}$.

Let us now define

$$F_0(\lambda) = \frac{(-1)^q}{q!} \int_{\lambda}^{\infty} (\mu - \lambda)^q \, F^{(q+1)}(\mu) \, \mathrm{d}\mu \quad \text{for} \quad \lambda > \alpha \tag{4}$$

which is admissible according to (3).

Further, we obtain easily from (3) and (4) that

(5) $F_0^{(p)}(\lambda) = \dfrac{(-1)^{q-p}}{(q-p)!} \displaystyle\int_{\lambda}^{\infty} (\mu - \lambda)^{q-p} \, F^{(q+1)}(\mu) \, \mathrm{d}\mu$ for every $\lambda > \alpha$ and $p \in \{0, 1, \dots, q\}$,

(6) $F_0^{(q+1)}(\lambda) = F^{q+1}(\lambda)$ for every $\lambda > \alpha$.

Now we are able to prove that

$$\|F_0^{(p)}(\lambda)\| \leqq \frac{Mp!}{(\lambda - \alpha)^{p+1}} \quad \text{for every} \quad \lambda > \alpha \quad \text{and} \quad p \in \{0, 1, \dots, q + 1\}. \tag{7}$$

Indeed we see from (5) that

$$(8) \qquad F_0^{(p)}(\lambda) = \int_\lambda^\infty F_0^{(p+1)}(\mu)\,\mathrm{d}\mu \quad \text{for every} \quad \lambda > \alpha \quad \text{and} \quad p \in \{0, 1, \ldots, q\}.$$

By (3) and (6)

$$(9) \qquad \|F_0^{(q+1)}(\lambda)\| \leqq \frac{M(q+1)!}{(\lambda - \alpha)^{q+2}} \quad \text{for every} \quad \lambda > \alpha\,.$$

Now (7) follows from (8) and (9) by a simple finite induction.

On the other hand, we see from (6) that $F_0 - F$ is a polynomial. Further, by assumption (A) (III) and by (7), $F_0(\lambda) - F(\lambda) \to_{\lambda\to\infty} 0$. Both these facts imply that

$$(10) \qquad F_0(\lambda) = F(\lambda) \quad \text{for every} \quad \lambda > \alpha\,.$$

Since $\alpha > \omega$ was chosen arbitrary, we see from (3), (7) and (10) that

$$(11) \quad \|F^{(p)}(\lambda)\| \leqq \frac{Mp!}{(\lambda - \alpha)^{p+1}} \quad \text{for every} \quad \alpha > \omega\,, \quad \lambda > \alpha \quad \text{and} \quad p \in \{0, 1, \ldots\}.$$

Now letting $\alpha \to \omega_+$ in (11) we obtain at once the desired property (B).

The proof of (A) $\Rightarrow$ (B) is complete.

(B) $\Rightarrow$ (A). We need the following relation

$$(1) \qquad \lambda - |\lambda - z| \to_{\substack{\lambda\to\infty \\ \lambda > \mathrm{Re}\, z}} \mathrm{Re}\, z \quad \text{for every} \quad z \in C\,, \quad \mathrm{Re}\, z > 0\,.$$

Indeed, we can write

$$\lambda - |\lambda - z| = \lambda - [(\lambda - \mathrm{Re}\, z)^2 + (\mathrm{Im}\, z)^2]^{1/2} =$$

$$= \lambda - (\lambda - \mathrm{Re}\, z)\left[1 + \left(\frac{\mathrm{Im}\, z}{\lambda - \mathrm{Re}\, z}\right)^2\right]^{1/2} =$$

$$= \lambda\left(1 - \left[1 + \left(\frac{\mathrm{Im}\, z}{\lambda - \mathrm{Re}\, z}\right)^2\right]^{1/2}\right) + \mathrm{Re}\, z\left[1 + \left(\frac{\mathrm{Im}\, z}{\lambda - \mathrm{Re}\, z}\right)^2\right]^{1/2} =$$

$$= -\frac{\lambda}{2}\int_0^{(\mathrm{Im} z/(\lambda - \mathrm{Re} z))^2} \frac{1}{(1+\alpha)^{1/2}}\,\mathrm{d}\alpha + \mathrm{Re}\, z\left[1 + \left(\frac{\mathrm{Im}\, z}{\lambda - \mathrm{Re}\, z}\right)^2\right]^{1/2}.$$

Clearly the second member in the last term tends to $\mathrm{Re}\, z$ as $\lambda \to \infty$. The first tends to zero because

$$\lambda\int_0^{(\mathrm{Im}/(\lambda - \mathrm{Re} z))^2} \frac{1}{(1+\alpha)^{1/2}}\,\mathrm{d}\alpha \leqq \lambda\left(\frac{\mathrm{Im}\, z}{\lambda - \mathrm{Re}\, z}\right)^2.$$

Hence (1) holds.

According to (1), for every $z \in C$, $\operatorname{Re} z > \omega$, there exists $\lambda(z) > \omega$ so that

(2) $$\lambda - \omega > |z - \lambda| \quad \text{for every} \quad \lambda > \lambda(z).$$

Because the function F is assumed analytic in the domain ($\operatorname{Re} z > \omega$) we obtain from (B) and (2) that

(3) $$\|F(z)\| = \left\| \sum_{k=0}^{\infty} \frac{F^k(\lambda)}{k!} (z - \lambda)^k \right\| \leqq \sum_{k=0}^{\infty} \frac{\|F^{(k)}(\lambda)\|}{k!} |z - \lambda|^k \leqq$$

$$\leqq M \sum_{k=0}^{\infty} \frac{1}{(\lambda - \omega)^{k+1}} |z - \lambda|^k = \frac{M}{\lambda - \omega} \sum_{k=0}^{\infty} \left(\frac{|z - \lambda|}{\lambda - \omega} \right)^k =$$

$$= \frac{M}{\lambda - \omega} \frac{1}{1 - \frac{|z - \lambda|}{\lambda - \omega}} = \frac{M}{\lambda - \omega - |z - \lambda|} \quad \text{for every} \quad z \in C,$$

$\operatorname{Re} z > \omega$ and $\lambda > \lambda(z)$.

Letting $\lambda \to \infty$, we get from (1) and (3) that

(4) $$\|F(z)\| \leqq \frac{M}{\operatorname{Re} z - \omega} \quad \text{for every} \quad z \in C, \quad \operatorname{Re} z > \omega.$$

It is clear from (4) that the conditions (A) (I), (III) are fulfilled and it remains to prove (A) (II).

Given a fixed $\alpha > \omega$, let us denote $J(z) = F(z)$ for $z \in C$, $\operatorname{Re} z \geqq \alpha$, we see from (4) that all assumptions of Lemma 4 are fulfilled with $k = 0$ and consequently we obtain

(5) $$F^{(p)}(\lambda) = (-1)^p \frac{p!}{2\pi} \int_{-\infty}^{\infty} \frac{F(\alpha + i\beta)}{(\lambda - \alpha - i\beta)^{p+1}} d\beta$$

for every $\alpha > \omega$, $\lambda > \alpha$ and $p \in \{1, 2, \ldots\}$.

Writing $\lambda - \alpha = s$ and $p + 1 = r$ in (5) we obtain that

(6) $$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{F(\alpha + i\beta)}{(1 - is\beta)^r} d\beta = (-1)^p \frac{s^r}{(r - 1)!} F^{(r-1)}(s + \alpha)$$

for every $\alpha > \omega$, $s > 0$ and $r \in \{2, 3, \ldots\}$.

It is now immediate that (B) and (6) give (A) (II) with $l = 0$.

The proof of (B) $\Rightarrow$ (A) is complete.

6. Auxiliary theorem (Miyadera [2], Widder [1] in the numerical case). *Let M, ω be two nonnegative constants and $F \in (\omega, \infty) \to E$. If the space E is reflexive, then* *the following two statements* (A), (B) *are equivalent*:

(A) (I) *the function F is infinitely differentiable on* (ω, ∞),

(II) $\left\|\frac{d^p}{d\lambda^p} F(\lambda)\right\| \leqq \frac{Mp!}{(\lambda-\omega)^{p+1}}$ *for every* $\lambda > \omega$ *and*

$p \in \{0, 1, \ldots\}$;

(B) *there exists a function* $f \in \mathbb{R}^+ \to E$ *such that*

(I) *f is measurable on* $\mathbb{R}^+$,

(II) $\|f(t)\| \leqq M e^{\omega t}$ *for almost all* $t \in \mathbb{R}^+$,

(III) $F(\lambda) = \int_0^\infty e^{-\lambda\tau} f(\tau)\, d\tau$ *for every* $\lambda > \omega$.

7. **Theorem.** *Let M, ω be two nonnegative constants and* $F \in (\operatorname{Re} z > \omega) \to E$. *If the space E is reflexive, then the following two statements* (A), (B) *are equivalent*:

(A) (I) *the function F is analytic in the domain* $(\operatorname{Re} z > \omega)$,

(II) *for every $\alpha > \omega$, there exist a $k \in \{0, 1, \ldots\}$ and a $K \geqq 0$ so that* $\|F(z)\| \leqq K(1 + |z|)^k$ *for every* $z \in C$, $\operatorname{Re} z > \alpha$,

(III) *for every $\alpha > \omega$, there exists an $l \in \{0, 1, \ldots\}$ so that*

$$\frac{1}{2\pi}\left\|\int_{-\infty}^{\infty} \frac{F(\alpha + i\beta)}{(1 - is\beta)^r}\, d\beta\right\| \leqq M$$

for every $s > 0$ and $r \in \{l + 2, l + 3, \ldots\}$,

(IV) $F(\lambda) \to 0$ $(\lambda \to \infty,\ \lambda > \omega)$;

(B) *there exists a function* $f \in \mathbb{R}^+ \to E$ *such that*

(I) *f is measurable on* $\mathbb{R}^+$,

(II) $\|f(t)\| \leqq M e^{\omega t}$ *for almost every* $t \in \mathbb{R}^+$,

(III) $\int_0^\infty e^{-z\tau} f(\tau)\, d\tau = F(z)$ *for every* $z \in C$, $\operatorname{Re} z > \omega$.

Proof. Immediate consequence of Theorem 6 and Proposition 5.

8. **Remark.** In an analogous way as above, it is possible to get complex characterizations of Laplace transforms of exponentially Lipschitzian and exponentially weakly compactly bounded functions – cf. [3], [4] – and also of analogous types of integrable functions.

In the case of exponentially Lipschitzian functions the reader obtains easily the corresponding result from Theorem 4 of [3] by means of Proposition 5 which plays the fundamental role in the relation between "real" and "complex" characteristic properties of Laplace transform.

In the case of exponentially weakly compactly bounded functions, we apply Theorem 13 of [4] but before applying Proposition 5, this must be somewhat modified. Namely, we first choose a convex circled closed subset C of E and replace the inequality in (A) (III) by

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{F(\alpha+i\beta)}{(1-is\beta)^r}\,d\beta\in C \quad \textit{for every} \quad \alpha>\omega\,,\quad s>0 \quad \textit{and} \quad r\in\{l+2,\ l+3,\ \ldots\}\,,$$

and further (B) by

$$F^{(p)}(\lambda)\in\frac{p!}{(\lambda-\omega)^{p+1}}C \quad \textit{for every} \quad \lambda>\omega \quad \textit{and} \quad p\in\{0,1,\ldots\}\,.$$

The proof of such a modified Proposition 5 proceeds without essential changes and may be left to the reader.

9. Remark. The condition (A) (III) of Theorem 7 represents a weakening of classically known sufficient conditions of the type of absolute integrability of F over lines parallel to imaginary axis, i.e. of the type

$$\int_{-\infty}^{\infty}\|F(\alpha+i\beta)\|\,d\beta<\infty\,.$$

See, for example, [5, Chap. VII] or [6].

10. Remark. It is clear that the inequality in (A) (III) of Theorem 7 cannot be replaced by

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\|F(\alpha+i\beta)\|}{|(1-is\beta)^r|}\,d\beta\leqq M$$

for every $s>0$ and $r\in\{l+2,\ l+3,\ \ldots\}$,

since this implies, by Fatou's lemma,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\|F(\alpha+i\beta)\|\,d\beta\leqq M$$

and this inequality is essentially less general than that of (A) (III) as may be seen from the function $F(z)=1/z$.

11. Remark. The conditions (A) (I) and (II) of Theorem 7 may be understood as necessary and sufficient conditions for the function F to be the Laplace transform of an exponentially bounded distribution with nonnegative support (cf. [8]). Thus the conditions (A) (III) and (IV) of the same theorem specify the class of functions whose distribution originals are functions.

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Author's address: 115 67 Praha 1, Žitná 25 (Matematický ústav ČSAV).