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ON APPROXIMATE METHODS IN EIGENVALUE PROBLEMS[1])

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1. DEFINITIONS AND NOTATION

Let $\mathcal{X}_1$, $\mathcal{X}_2$ be complex Banach spaces, $x, y, \ldots; u, v, \ldots$ its elements and $\|x\|_1$, $\|u\|_2$ norms of $x \in \mathcal{X}_1$, $u \in \mathcal{X}_2$. Let H be an index set and $\mathcal{X}_{jh}$, $j = 1, 2$, $h \in H$, be Banach spaces with norms $\|\cdot\|_{j,h}$. We suppose, that there exist continuous linear transformations $P_{j,h}$, $j = 1, 2$, $h \in H$, of $\mathcal{X}_j$ onto $\mathcal{X}_{j,h}$. Further we suppose that there exist subspaces $\tilde{\mathcal{X}}_{j,h}$ of $\mathcal{X}_j$ isomorphic with $\mathcal{X}_{j,h}$. Let us denote by $S_{j,h}$ the corresponding isomorphisms. Hence

$$S_{j,h}\tilde{\mathcal{X}}_{j,h} = \mathcal{X}_{j,h} \quad \text{or} \quad \tilde{\mathcal{X}}_{j,h} = S_{j,h}^{-1}\mathcal{X}_{j,h}\,.$$

We shall assume that $H = (0, h_0\rangle$ and that for an arbitrary vector $x_j \in \mathcal{X}_j$ there holds the expression $x_j = \lim_{h\to 0} \tilde{x}_{j,h}$, where $\tilde{x}_{j,h} = S_{j,h}^{-1}P_{j,h}x_j \in \tilde{\mathcal{X}}_{j,h}$. Let us note, that in applications $\mathcal{X}_{j,h}$ are usually finite dimensional spaces.

Let $\mathscr{X}$ be a complex Banach space. Then $\mathscr{X}'$ denotes the adjoint space of continuous linear forms on $\mathscr{X}$. The space of all bounded linear operators from $\mathscr{X}$ into the Banach space $\mathscr{X}_1$, topologized by the norm $\|T\| = \sup_{\|x\|_{\mathscr{X}}=1} \|Tx\|_{\mathscr{X}_1}$, $x \in \mathscr{X}$, $Tx \in \mathscr{X}_1$, will be denoted by $[\mathscr{X}, \mathscr{X}_1]$. We put $[\mathscr{X}] = [\mathscr{X}, \mathscr{X}]$.

Let $T \in [\mathscr{X}, \mathscr{X}_1]$. Then T' denotes the adjoint operator, i.e. the operator T', for which $y' = T'x' \Leftrightarrow y'(x) = x'(Ty)$, where $y = Tx$ and $x \in \mathscr{X}$, $y \in \mathscr{X}_1$, $x' \in \mathscr{X}'_1$, $y' \in Z'$. Evidently $T' \in [\mathscr{X}'_1, \mathscr{X}']$.

If T is a linear (not necessarily) bounded transformation, then $\mathscr{D}(T)$, $\mathscr{D}(T) \subset \mathscr{X}$ denotes the domain and $\mathscr{R}(T)$, $\mathscr{R}(T) \subset \mathscr{X}_1$ the range of this transformation.

The following eigenvalue problem

$$Tx = \mu x \tag{1.1}$$

[1]) An announcement of the main results was made in the preliminary communication "*A principle of dehomogenization for eigenvalue problems*", Comment. Math. Univ. Carol. *6*, 2 (1965), 199–210.

shall be investigated together with the "approximate" eigenvalue problem

$$T_h x_h = \mu^{(h)} x_h , \tag{1.2}$$

where T, T_h are elements of $\mathscr{X}$, $\mathscr{X}_h$ and the symbols $\mathscr{X}$, $\mathscr{X}_h$ denote the corresponding pair of the spaces $\mathscr{X}_j$, $\mathscr{X}_{j,h}$, $j = 1, 2$. In agreement with this notation we also omit the index j, $j = 1, 2$, in $P_{j,h}$.

The problem (1.2) can be considered as an approximation of the problem (1.1) only if the closeness of the operators T and T_h is guaranteed. This closeness we shall define later. Then the eigenvalue problem (1.2) will be called the approximate problem of (1.1) and the eigenvector x_h of T_h an approximate eigenvector corresponding to the approximate eigenvalue $\mu^{(h)}$. The closeness of the operators mentioned will be defined by using the terms usual in the theory of approximate methods, in particular in the net method of approximate solution of differential equations.

Convention. The positive constants independent of $h \in H$ will be denoted by the unique symbol c without the distinguishing indices.

Definition 1. Suppose that T maps $\mathscr{D}(T) \subset \mathscr{X}_1$ into $\mathscr{X}_2$ and T_h maps $\mathscr{D}(T_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_{2,h}$. Let $\mathscr{M}_1 \subset \mathscr{X}_1$ and $\mathscr{M}_1 \cap \mathscr{D}(T) = \mathscr{M} \neq \emptyset$. Let $P_h \mathscr{M} = \mathscr{M}_h \subset \mathscr{D}(T_h)$. Finally let r be a positive integer. If the inequalities

$$\|P_{2h} T x - T_h P_{1h} x\|_2 \leqq c(x)\, h^r \tag{1.3}$$

where $c(x) \leqq c\|x\|_1$, hold for all vectors $x \in \mathscr{M}$, we shall say, that T_h have the approximation order r on the set $\mathscr{M} \subset \mathscr{X}$ with respect to the operator T.

Note, that the approximation order r depends on $\mathscr{M}$, $\mathscr{X}_2$, $\mathscr{X}_{1,h}$, $\mathscr{X}_{2h}$ and T.

Let $y \in \mathscr{X}_2$ and put $y_h = P_{2,h} y$; thus $y_h \in P_{2,h} \mathscr{X}_2$. Let u, $u^{(h)}$ be solutions of the equations

$$Tx = y , \quad T_h x^{(h)} = y_h , \tag{1.4}$$

where the operators T and T_h map $\mathscr{D}(T) \subset \mathscr{X}_1$ into $\mathscr{X}_2$ and $\mathscr{D}(T_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_{2,h}$ respectively.

Definition 2. *Let the equations* (1.4) *have unique solutions* $u \in \mathscr{D}(T)$, $u^{(h)} \in \mathscr{D}(T_h)$ *for given* $y \in \mathscr{M}_2 \subset \mathscr{X}_2$. *Let* p *be a positive integer. We say, that* T_h *has the accuracy-order* p *on the set* $\mathscr{M}_2$ *with respect to the operator* T, *if there holds the inequality*

$$\|P_{1,h} u - u^{(h)}\| \leqq c(y)\, h^p , \tag{1.5}$$

where $c(y) \leqq c\|y\|_2$.

Evidently the accuracy-order depends on $\mathscr{X}_1$, $\mathscr{M}_2$, $\mathscr{X}_{1,h}$, $\mathscr{X}_{2,h}$ and T.

In definitions **1** and **2** there are given two significant characteristics of an approximate operator with respect to the investigated operator T. There are some relations between the approximation-order r and the accuracy-order p if the operator T has, say, some smoothness properties. For example, in the net method for the differential operator L, where

$$Lu \equiv -\frac{\mathrm{d}}{\mathrm{d}t}\left[k(t)\frac{\mathrm{d}u}{\mathrm{d}t}\right] + q(t)\,u(t)\,,\quad u(0) = \alpha\,,\quad u(1) = \beta\,,$$

in the space $\mathscr{C}^p(\langle 0, 1\rangle)$ of functions continuous with their derivatives of order p the accuracy-order p is equal to the approximation-order r, if $k \in \mathscr{C}^{p-1}(\langle 0, 1\rangle)$ and $q \in \mathscr{C}^{p-2}(\langle 0, 1\rangle)$ (see [5]). On the other hand, there is shown in [5] that a fixed operator L_h in $\mathscr{X}_h$ may be an approximate operator with respect to two different operators L_1, L_2 in $\mathscr{X}$, where the corresponding approximation-orders are identical in a class of sufficiently smooth coefficients. For example if the coefficients of L_1 are smooth and the coefficients of L_2 are discontinuous, then for the accuracy-orders the inequality $p_1 \geqq p_2$ is true, however $p_1 \neq p_2$ in general.

In definition **2** the equations were supposed to fulfil the unicity conditions. This definition is not convenient for the studying of eigenvalue problems.

Definition 3. Let μ_0 be an eigenvalue of the operator T from $\mathscr{D}(T) \subset \mathscr{X}$ into $\mathscr{X}$ and let $x_1, \ldots, x_s$, $1 \leqq s < +\infty$, be corresponding linear independent eigenvectors. We say, that an operator T_h from $\mathscr{D}(T_h) \subset \mathscr{X}_h$ into $\mathscr{X}_h$ has the accuracy-order p in $\mathscr{X}$ for the proper value μ_0 with respect to the operator T, if for every proper vector x_j there exist proper values $\mu_{j,t}^{(h)}$, $t = 1, \ldots, l_j$ of the operator T_h and corresponding proper vectors $x_{j,t}^{(h)}$ such, that the inequalities

$$(1.6)\qquad \left\|P_h x_j, - \sum_{t=1}^{l_j} \alpha_{jt} x_{jt}^{(h)}\right\| \leqq c(x_j)\,h^p\,,\quad c(x_j) \leqq c\|x_j\|\,,$$
$$\left|\mu_0 - \mu_{jt}^{(h)}\right| \leqq ch^p\,,\quad j = 1, \ldots, s\,,\quad t = 1, \ldots, l_j\,,$$

hold for appropriate complex numbers α_{jt} and positive integers l_j.

Convention. Let M and C be linear transformations from $\mathscr{D}(M) \subset \mathscr{X}_1$ and $\mathscr{D}(C) \subset \mathscr{X}_1$ into $\mathscr{X}_2$ and similarly M_h and C_h from $\mathscr{D}(M_h) \subset \mathscr{X}_{1h}$ and $\mathscr{D}(C_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_{2h}$. The pair $\{M_h, C_h\}$ will be called an approximate scheme for M and C, shortly a scheme.

Definition 4. Suppose that the equations

$$(1.7)\qquad Mx = Cv\,,\quad M_h x^{(h)} = C_h P_{1,h} v$$

have unique solutions $u \in \mathscr{D}(M)$ and $u^{(h)} \in \mathscr{D}(M_h)$. We say, that the scheme $\{M_h, C_h\}$ has the accuracy-order p in $\mathscr{X}_1$ with respect to the problem (1.7), if there hold the inequality

$$(1.8)\qquad \|P_{1h}u - u^{(h)}\| \leqq c(v)\,h^p\,,\quad c(v) \leqq c\|v\|\,.$$

Definition 5. Suppose that there exist the inverse operators M^{-1} and M_h^{-1} and that $\mathscr{R}(M^{-1}) \subset \mathscr{D}(C)$ and $\mathscr{R}(M_h^{-1}) \subset \mathscr{D}(C_h)$, where M, M_h, C, C_h are linear operators mapping $\mathscr{D}(M) \subset \mathscr{X}_1$ and $\mathscr{D}(C) \subset \mathscr{X}_1$ into $\mathscr{X}_2$ and $\mathscr{D}(M_h) \subset X_{1,h}$ and $\mathscr{D}(C_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_{2h}$ respectively. We say that the schema $\{M_2, C_h\}$ has the accuracy-order p for the characteristic value λ_0 of the problem

$$Mx = \lambda Cx\,, \tag{1.9}$$

if $M_h^{-1}C_h$ or $C_hM_h^{-1}$ has the accuracy-order p for the proper value $\mu_0 = \lambda_0^{-1}$ with respect to the corresponding operator $M^{-1}C$ in $\mathscr{X}_1$ or CM^{-1} in $\mathscr{X}_2$.

Convention. We write $\{M_h, C_h; \lambda_0\}$, if we want emphasize the fact, that the scheme $\{M_h, C_h\}$ is used for the construction of the characteristic value λ_0.

2. EIGENVALUE PROBLEMS

In this paragraph we introduce some assumptions and properties of operators M and C which work in our eigenvalue problem

$$Mx = \lambda Cx\,. \tag{2.1}$$

It is assumed that M and C are linear, generally unbounded, operators from $\mathscr{D}(M) \subset \subset \mathscr{X}_1$ and $\mathscr{D}(C) \subset \mathscr{X}_1$ into $\mathscr{X}_2$. Further it is assumed that $\mathscr{D}(M)$ and $\mathscr{D}(C)$ are dense in $\mathscr{X}_1$.

Together with the problem (2.1) we shall investigate the problem

$$M_hx^{(h)} = \lambda^{(h)}C_hx^{(h)}\,, \tag{2.2}$$

where M_h and C_h fulfil the same conditions.

Our purpose is to determine the accuracy-order of the scheme $\{M_h, C_h; \lambda\}$ for the problem (2.1) under the assumption that the accuracy-order of the scheme $\{M_h, C_h\}$ in the sense of definition 4 is known.

In order to exploit this assumption the eigenvalue problems (2.1) and (2.2) need be transfered into the unhomogeneous problems of type

$$Mx = y\,, \quad M_hx^{(h)} = y_h\,. \tag{2.3}$$

Such procedure is suitable especially for that reason that using it, one can omit some assumptions which are usually laid on M and C and which are as a rule too restrictive. For example the symetry or positive definiteness. This method has also practical use. Using, e.g., Kellogg's iterations, the original eigenvalue problem is transfered into the system of countable unhomogeneous equations. As solution of this system of unhomogeneous equations the exact proper elements can be obtained.

If we restrict the infinite system to finite number of equations, we obtain some approximate proper elements. The method just described is often used for solution of timeindependent problems of reactor physics [1], [3].

Though on this place the iterative process mentioned only forms an auxiliary apparatus we shall demonstrate it here in such a form which it applies directly in practical problems.

At first, we introduce some properties of the investigated operators and using these properties we demonstrate some relations needed.

Suppose $T \in [\mathscr{X}]$, where $\mathscr{X}$ is some Banach space, has a dominant eigenvalue μ_0, i.e. $\mu_0 \in \sigma(T)$ and the inequality

$$|\lambda| < |\mu_0| \tag{2.4}$$

hold for every $\lambda \in \sigma(T)$, $\lambda \neq \mu_0$.

Let the symbol I denote the unity operator in $\mathscr{X}$ and $R(\lambda, T) = (\lambda I - T)^{-1}$ the resolvent of T. Let ϱ_0 be such that for $K \cap \sigma(T) = \{\mu_0\}$ holds for $K = \{\lambda|\, |\lambda - \mu_0| \leqq \varrho_0\}$. Let $C_0 = \{\lambda|\, |\lambda - \mu_0| = \varrho_0\}$ and

$$B_1 = \frac{1}{2\pi i} \int_{C_0} R(\lambda, T)\, d\lambda\,. \tag{2.5}$$

It is known ([4] p. 306) that the operators $B_1, B_{k+1} = (T - \mu_0 I) B_k$, $k = 1, 2, \ldots$ form the main part of Laurent expansion of the resolvent $R(\lambda, T)$ in a neighbourhood of the singularity μ_0. Thus

$$R(\lambda, T) = \sum_{k=0}^{\infty} A_k(\lambda - \mu_0)^k + \sum_{k=1}^{\infty} B_k(\lambda - \mu_0)^{-k}\,,$$

where $A_k \in [\mathscr{X}]$.

Suppose that $x', y', x'_n, y'_n, z'_n \in \mathscr{X}'$ and that for every vector $x \in \mathscr{X}$ there hold

$$\lim_{n\to\infty} x'_n(x) = x'(x)\,, \quad \lim_{n\to\infty} y'_n(x) = \lim_{n\to\infty} z'_n(x) = y'(x)\,. \tag{2.6}$$

Finally let there exists a positive integer s, $1 \leqq s < +\infty$ such that

$$x'(B_s x^{(0)}) \neq 0\,, \quad y'(B_s x^{(0)}) \neq 0\,, \quad B_{s+1} x^{(0)} = o\,, \tag{2.7}$$

where $x^{(0)} \in \mathscr{X}$ is a suitable element.

Theorem ([2]) *Let μ_0 be a dominant proper value of the operator $T \in [\mathscr{X}]$ and let the conditions* (2.6) *and* (2.7) *be fulfilled. Then* $\lim_{n\to\infty} \|x_{(n)} - x_0\| = 0$, $\lim_{n\to\infty} \lambda_{(n)} = \mu_0^{-1}$, *where* $x_0 = [x'(B_s x^{(0)})]^{-1} B_s x^{(0)} = \mu_0^{-1} T x_0$ *and where*

$$x_{(n)} = \frac{T^n x^{(0)}}{x'_n(T^n x^{(0)})}\,, \tag{2.8}$$

$$\lambda_{(n)} = \frac{y'_n(T^n x^{(0)})}{z'_n(T^{n+1} x^{(0)})}\,. \tag{2.9}$$

If, moreover, μ_0 is a pole of the resolvent of order q, then we have

$$\lim_{n\to\infty} \left\| n^{-q+1}\mu_0^{-n}T^n - \frac{\mu_0^{-q+1}}{(q-1)!} B_q \right\| = 0$$

and in particular for $q = 1$

$$\lim_{n\to\infty} \mu_0^{-n}T^n = B_1 .$$

In [2] it was shown how the iterative process (2.8) and (2.9) need be modified for problems with unbounded operators. This modification will be used for transfering the problems (2.1) and (2.2) into systems of unhomogeneous equations.

Suppose, that there exist the bounded inverses M^{-1} and M_h^{-1} that $M^{-1} \in [\mathscr{X}_2, \mathscr{X}_1]$ and $M_h^{-1} \in [\mathscr{X}_{2h}, \mathscr{X}_{1h}]$. Further let $\mathscr{D}(C) \supset \mathscr{R}(M^{-1})$ and $\mathscr{D}(C_h) \supset \mathscr{R}(M_h^{-1})$. Then the problems (2.1) and (2.2) are equivalent with the problems

$$x = \lambda M^{-1}Cx , \quad x^{(h)} = \lambda^{(h)}M_h^{-1}C_hx^{(h)} \tag{2.10}$$

and (putting $y = Cx$ and $y_h = C_hx^{(h)}$) also with the problems

$$y = \lambda CM^{-1}y , \quad y_h = \lambda^{(h)}C_hM_h^{-1}y_h . \tag{2.11}$$

If $C \in [\mathscr{X}_1, \mathscr{X}_2]$ and $C_h \in [\mathscr{X}_{1,h}, \mathscr{X}_{2,h}]$, then $M^{-1}C \in [\mathscr{X}_1]$ and $M_h^{-1}C_h \in [\mathscr{X}_{1,h}]$. Corresponding iterative processes are then defined as follows

$$Mu^{(n+1)} = Cu_{(n)} , \quad u_{(n+1)} = \lambda_{(n)}u^{(n+1)} , \quad u_{(0)} = x^{(0)} , \tag{2.12}$$

$$\lambda_{(n)} = \frac{y_n'(u_{(n)})}{z_n'(u^{(n+1)})} ;$$

$$M_hu_h^{(n+1)} = C_hu_{(n)}^{(h)} , \quad u_{(n+1)}^{(h)} = \lambda_{(n)}^{(h)}u_h^{(n+1)} , \quad u_{(0)}^{(h)} = P_{1,h}x^{(0)} , \tag{2.13}$$

$$\lambda_{(n)}^{(h)} = \frac{y_{n,h}'(u_{(n)}^{(h)})}{z_{n,h}'(u_h^{(n-1)})} .$$

If $CM^{-1} \in [\mathscr{X}_2]$ and $C_hM_h^{-1} \in [\mathscr{X}_{2,h}]$, the iterative processes are defined by formulae

$$Mv^{(n)} = v_{(n)} , \quad v_{(n+1)} = \nu_{(n)}Cv^{(n)} , \quad v_{(0)} = Cx^{(0)} , \tag{2.14}$$

$$\nu_{(n)} = \frac{y_n'(v_{(n)})}{z_n'(Cv^{(n)})} ;$$

$$M_hv_h^{(n)} = v_{(n)}^{(h)} , \quad v_{(n+1)}^{(h)} = \nu_{(n)}^{(h)}C_hv_h^{(n)} , \quad v_{(0)}^{(h)} = C_hP_{1h}x^{(0)} , \tag{2.15}$$

$$\nu_{(n)}^{(h)} = \frac{y_{n,h}'(v_{(n)}^{(h)})}{z_{n,h}'(C_hv_h^{(n)})} .$$

It is easy to see that using iterations (2.12) and (2.13) or (2.14) and (2.15) respectively, the problems (2.1) and (2.2) are transfered into the systems of unhomogeneous equations of type

$$Mu_{(n)} = v_n\,, \quad M_h u_{(n)}^{(h)} = v_n^{(h)}\,.$$

The procedure just described forms the base of the method of dehomogenization. In the theoretical considerations an appropriate choice of the initial element of iterations enables us to derive some estimates needed.

3. AUXILIARY ASSERTIONS

Suppose $T \in [\mathscr{X}]$ and $T \in [\mathscr{X}_h]$. Define the operators

$$S = \mu_0^{-1} T\,, \quad S_h = \mu_{0,h}^{-1} T_h\,, \tag{3.1}$$

where μ_0 and $\mu_{0,h}$ are dominant proper values of the operators T and T_h. Moreover, assume that μ_0 and $\mu_{0,h}$ be simple poles of the resolvents $R(\lambda, T)$ and $R(\lambda, T_h)$. Evidently $S \in [\mathscr{X}]$ and $S_h \in [\mathscr{X}_h]$ and for the spectral radii $r(S)$ and $r(S_h)$ we have

$$r(S) = 1\,, \quad r(S_h) = 1\,. \tag{3.2}$$

We shall consider the operators

$$Q_n = \sum_{k=0}^{n} S_h^k\,, \quad n = 1, 2, \ldots \tag{3.3}$$

From the assumptions about the dominantness of poles μ_0, $\mu_{0,h}$ there hold the relations $\tilde{C}_0 \cap \sigma(S) = \{1\}$, $\tilde{C}_{0,h} \cap \sigma(S_h) = \{1\}$, where $\tilde{C}_0 = \mu_0^{-1} C_0$ and $\tilde{C}_{0,h} = \mu_{0,h}^{-1} C_{0,h}$. Let $C_{1,h} = \{\lambda \mid |\lambda| = \varrho_1\}$, $K_{1,h} = \{\lambda \mid |\lambda| \leq \varrho_1\}$ and $K_{1,h} \cap \sigma(S_h) = \sigma(S_h) - \{1\}$. Hence $\varrho_1 < 1$.

Let us put

$$V_n = \frac{1}{2\pi i} \sum_{k=0}^{n} \int_{\tilde{C}_{0,h}} \lambda^k R(\lambda, S_h)\, d\lambda\,, \tag{3.4}$$

$$W_n = \frac{1}{2\pi i} \sum_{k=0}^{n} \int_{\tilde{C}_{1,h}} \lambda^k R(\lambda, S_h)\, d\lambda\,. \tag{3.5}$$

Thus one can write $Q_n = V_n + W_n$.

Lemma 3.1. *There exists a constant c independent of n such that*

$$\|W_n\| \leq c\,. \tag{3.6}$$

Proof. Easily one can see, that

$$W_n = \frac{1}{2\pi i} \int_{C_{1,h}} \frac{1 - \lambda^n}{1 - \lambda} R(\lambda, S_h)\, d\lambda\,.$$

According to the inequality $\varrho_1 < 1$ there follows the estimate

$$\|W_n\| \leqq \frac{2\varrho_1}{1-\varrho_1} \sup_{\lambda \in C_{1,h}} \|R(\lambda, S_h)\| = c .$$

Lemma 3.2. *Suppose $\mu_{0,h}$ is a simple pole of the resolvent $R(\lambda, T_h)$. Then we have*

(3.7) $$V_n = (n+1)\, B_{1,h} ,$$

where

$$B_{1,h} = \frac{1}{2\pi i} \int_{C_{0,h}} R(\lambda, T_h)\, d\lambda .$$

Corollary 1. *If $u_0 = \mu_0^{-1} T u_0$, then we have*

(3.8) $$v_n^{(h)} = V_n(P_h S u_0 - S_h P_h u_0) = o .$$

Evidently this relation is non trivial especially, if

(3.9) $$B_{1,h} P_h u_0 \neq o .$$

Proof. The relation (3.7) follows from the Cauchy theorem as a consequence the simplicity of the pole $\mu_{0,h}$.

Further we shall prove the validity of (3.8). Since $u_0 = S u_0$, we have

$$v_n^{(h)} = (n+1)\, B_{1,h}(P_h u_0 - S_h P_h u_0) .$$

On the other hand $B_{1,h} S_h = S_h B_{1,h}$. In the case $B_{1,h} P_h u_0 = o$ the equation (3.8) is trivial. In the case $B_1 P_h u_0 \neq o$ the vector $u_0^{(h)} = B_{1,h} P_h u_0$ is a proper vector of S_h corresponding to the proper value 1 and thus

$$B_{1,h} S_h P_h u_0 = S_h B_{1,h} P_h u_0 = B_{1,h} P_h u_0 .$$

From those relations (3.8) follows immediately.

Note that one can write (3.8) explicitely as follows

(3.10) $$B_{1h}(\mu_0^{-1} P_h T u_0 - \mu_{0,h}^{-1} T_h P_h u_0) = o .$$

Corrolary 2. *Assume*

(3.11) $$\|\mu_0^{-1} P_h T u_0 - \mu_{0,h}^{-1} T_h P_h u_0\| \leqq c(u_0)\, h^p ,$$

where $u_0 = \mu_0^{-1} T u_0$, $u_0 \neq o$, and $c(u_0)$ does not depend on h. Then the estimate

(3.12) $$\lim_{n\to\infty} \|\mu_0^{-n} P_h T^n u_0 - \mu_{0,h}^{-n} T_h^n P_h u_0\| \leqq c h^p$$

is true with a constant c independent of h.

Proof. It is easy to see that

$$\mu_0^{-(n+1)} P_h T^{n+1} u_0 - \mu_{0,h}^{-(n+1)} T_h^{n+1} P_h u_0 = P_h u_0 - S_h^{n+1} P_h u_0 =$$

$$= \sum_{k=0}^{n} [S_h^k - S_h^{k+1}] P_h u_0 = \sum_{k=0}^{n} S_h^k [P_h S - S_h P_h] u_0 .$$

Lemmae (3.1) and 3.2 imply the inequalities

$$\left\| \sum_{k=0}^{n} S_h^k (P_h S - S_h P_h) u_0 \right\| =$$

$$= \|(V_n + W_n)(P_h S - S_h P_h) u_0\| \leqq c \|P_h S u_0 - S_h P_h u_0\| \leqq c h^p .$$

From this we easily obtain the estimate wanted.

Theorem 3.1. *Assumptions*:

(a) *Operators $T \in [\mathscr{X}]$ and $T_h \in [\mathscr{X}_h]$ have dominant proper values μ_0 and $\mu_{0,h}$ and these values are simple poles of resolvents $R(\lambda, T)$ and $R(\lambda, T_h)$. Let $u_0 \in \mathscr{X}$ be a proper vector of the operator T corresponding to μ_0. Then we put $u_0^{(h)} = B_{1,h} P_h u_0$.*

(b) *There exists such a constant c that*

$$\|T_h\| \leqq c .$$

(c) *There exists such a constant c that the estimate*

$$\|P_h T u_0 - T_h P_h u_0\| \leqq c h^p \tag{3.13}$$

holds for $h \in H$.

(d) *There exist linear forms $\mathfrak{x}_h' \in \mathscr{X}_h'$, $h \in H$, with the following properties*:

(di) *There exists a constant c such that*

$$\|\mathfrak{x}_h'\| \geqq c . \tag{3.14}$$

(dii) *The relation*

$$\mathfrak{x}_h'(T_h x_h) = \mu_{0\,h} \mathfrak{x}_h'(x_h) \tag{3.15}$$

holds for arbitrary vector $x_h \in \mathscr{X}_h$.

(diii) *There exists a constant c such that*

$$|\mathfrak{x}_h'(P_h u_0)| \geqq c > 0 . \tag{3.16}$$

Then the following assertions are valid:

There exists a constant c such that

$$|\mu_0 - \mu_{0,h}| \leqq c h^p , \tag{3.17}$$

$$\|P_h u_0 - u_0^{(h)}\| \leqq c h^p . \tag{3.18}$$

Proof. At first we shall prove the inequality (3.17). For this purpose we shall derive an equation for the quantity $\Delta\mu = \mu_0^{-1} - \mu_{0,h}^{-1}$. We introduce the vector $z_0^{(h)} = u_0^{(h)} - P_h u_0$ and substitute $P_h u_0 + z_0^{(h)}$ for $u_0^{(h)}$ in the equation $u_0^{(h)} = \mu_{0,h}^{-1} T_h u_0^{(h)}$. We obtain

$$z_0^{(h)} + P_h u_0 = \mu_{0,h}^{-1} T_h P_h u_0 + \mu_{0,h}^{-1} T_h z_0^{(h)}$$

or equivalently

$$z_0^{(h)} - \mu_{0,h}^{-1} T_h z_0^{(h)} = w_h\,, \tag{3.20}$$

where

$$w_h = -P_h u_0 + \mu_{0,h}^{-1} T_h P_h u_0 = \mu_0^{-1}(T_h P_h - P_h T u_0) - (\mu_0^{-1} - \mu_{0,h}^{-1})\, T_h P_h u_0\,.$$

By assumption $\mu_{0,h}$ is a simple pole of $R(\lambda, T_h)$ and hence the equation (3.20) has a solution iff

$$u_h'(w_h) = 0 \tag{3.21}$$

holds for every form $u_h' \in \mathscr{X}_h'$ for which

$$u_h'(x_h) = \mu_{0,h}^{-1} u_h'(T_h x_h) \tag{3.22}$$

for $x_h \in \mathscr{X}_h$. In particular the relation (3.22) must be fulfilled for the form $\hat{x}_k'$ having properties (di) to (diii). From the equation (3.21) we get the expression for $\Delta\mu$ as follows

$$\Delta\mu = \frac{\hat{x}_h'(T_h P_h u_0 - P_h T u_0)}{\mu_0 \hat{x}_h'(T_h P_h u_0)} = \frac{\hat{x}_h'(T_h P_h u_0 - P_h T u_0)}{\mu_0 \mu_{0,h} \hat{x}_h'(P_h u_0)}\,.$$

Thus there exists a constant c such that the estimate (3.17) is true.

Using (3.17) and (3.13) the inequality (3.11) can be easily proved and this inequality implies (3.12) as a consequence of corollary 2 of lemma 3.2. The estimate (3.12) can then be written as $\|P_h u_0 - B_{1,h} P_h u_0\| \leqq c h^p$ and this is the inequality (3.18) which was to be proved.

4. ACCURACY-ORDER AND APPROXIMATE METHODS IN EIGENVALUE PROBLEMS

In this paragraph we shall investigate the accuracy-order of an approximate scheme $\{M_h, C_h; \lambda\}$ under the assumption that the accuracy-order of the scheme $\{M_h, C_h\}$ with respect to the unhomogeneous problem $Mu = Cv$ is known.

Theorem 4.1. *Assumptions*:

1. $C \in [\mathscr{X}_1, \mathscr{X}_2]$, $C_h \in [\mathscr{X}_{1,h}, \mathscr{X}_{2,h}]$.

2. *The operators M and M_h from $\mathscr{D}(M) \subset \mathscr{X}_1$ and $\mathscr{D}(M_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_2$ and $\mathscr{X}_{2,h}$ respectively have inverses M^{-1} and M_h^{-1} such that $M^{-1}C \in [\mathscr{X}_1]$ and $M_h^{-1}C_h \in \in [\mathscr{X}_{1,h}]$.*

3. *The operators $T = M^{-1}C$ and $T_h = M_h^{-1}C_h$ have dominant proper values μ_0 and $\mu_{0,h}$ and these values are simple poles of $R(\lambda, T)$ and $R(\lambda, T_h)$.*

We put $u_0^{(h)} = B_{1,h}P_{1,h}u_0$, if u_0 is a proper vector of T corresponding to μ_0.

4. *The operators T_h, $h \in H$, are uniformly bounded*:

$$\|T_h\| \leqq c .$$

5. *For every $h \in H$ there is a form $\hat{x}_h' \in \mathscr{X}_{1,h}'$ such that*

(i) $$\|\hat{x}_h'\| \leqq c ,$$

(ii) $$\hat{x}_h'(T_h x_h) = \mu_{0,h}\hat{x}_h'(x_h) \quad for \quad x_h \in \mathscr{X}_h ,$$

(iii) *there exists a constant c such that*

$$|\hat{x}_h'(P_h u_0)| \geqq c > 0 .$$

6. *The scheme $\{M_h, C_h\}$ has accuracy-order p with respect to problem (1.7) in $\mathscr{X}_1$.*

Assertion. *The scheme $\{M_h, C_h; \lambda_0\}$ has the accuracy-order p for the characteristic value λ_0 with minimal modulus of the problem $Mu = \lambda Cu$.*

Proof. Assumptions 1 to 5 guarantee the fulfilment of assumptions (a), (b) and (d) of theorem **3.1.** Thus it is sufficient to legalize only assumption (c) of theorem **3.1.** Let us put $v = u_0$, where $Mu_0 = \lambda_0 Cu_0$, $\lambda = \mu_0^{-1}$, $u_0 \neq o$. Then the equations

$$Mu = Cv , \quad M_h u^{(h)} = C_h P_h v$$

have unique solutions Tu_0 and $T_h P_{1,h} u_0$. From assumption 6 it follows that

$$\|P_h T u_0 - T_h P_h u_0\| \leqq ch^p .$$

But this is an inequality required in (c) of theorem **3.1**. The assertion of theorem **4.1** is then a direct consequence of theorem **3.1.**

If C is an unbounded operator the situation is more complicated.

Theorem 4.2. *Assumptions*:

1. *The operators M and M_h from $\mathscr{D}(M) \subset \mathscr{X}_1$ and $\mathscr{D}(M_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_2$ and $\mathscr{X}_{2,h}$ have inverses $M^{-1} \in [\mathscr{X}_2, \mathscr{X}_1]$ and $M_h^{-1} \in [\mathscr{X}_{2,h}, \mathscr{X}_{1,h}]$.*

2. *$\mathscr{R}(M^{-1}) \subset \mathscr{D}(C)$ and $\mathscr{R}(M_h^{-1}) \subset \mathscr{D}(C_h)$.*

3. *There exist a constant c such that*

(4.1) $$\|Cy\|_2 \leqq c\|y\|_1$$

for every $y \in \mathscr{R}(M^{-1})$ *and*

(4.2) $$\|C_h y_h\|_2 \leqq c\|y_h\|_1$$

for every $y_h \in \mathscr{R}(M_h^{-1})$.

4. *For every* $h \in H$

(4.3) $$P_{1,h}M^{-1}\mathscr{X}_2 \subset M_h^{-1}\mathscr{X}_{2,h}.$$

5. *The operators* $CM^{-1} \in [\mathscr{X}_2]$ *and* $C_hM_h^{-1} \in [\mathscr{X}_{2,h}]$ *have dominant proper values* μ_0 *and* $\mu_{0,h}$ *and these values are simple poles of resolvents* $R(\lambda, CM^{-1})$ *and* $R(\lambda, C_hM_h^{-1})$

If v_0 is a proper vector of CM^{-1} corresponding to μ_0, we put $v_0^{(h)} = B_{1,h}^{(2)}P_{2,h}v_0$, where

$$B_{1,h}^{(2)} = \frac{1}{2\pi i}\int_{C_{h,0}} R(\lambda, C_hM_h^{-1})\,d\lambda$$

and

$$C_{h,0} = \{\lambda |\, |\lambda - \mu_{0,h}| = \varrho_{h,0}\}\,, \quad K_{h,0} = \{\lambda |\, |\lambda - \mu_0| \leqq \varrho_{h,0}\}\,,$$
$$K_{h,0} \cap \sigma(C_hM_h^{-1}) = \{\mu_{0,h}\}\,.$$

6. *For every* $u \in \mathscr{R}(M^{-1})$ *there exists a vector* $y_h \in \mathscr{X}_{2,h}$ *such that*

$$C_hP_{1,h}u = P_{2,h}Cu + y_h\,,$$

where $\|y_h\| \leqq ch^p$. *In other words, the approximation-order of* C_h *is equal p whith respect to C in* $\mathscr{R}(M^{-1})$.

7. *The operators* $C_hM_h^{-1}$, $h \in H$, *are uniformly bounded*: $\|C_hM_h^{-1}\| \leqq c$.

8. *There are forms* $\mathfrak{x}_h' \in \mathscr{X}_{2,h}'$, $h \in H$, *for which relations* (i) *to* (iii) *of assumptions of theorem* **4.1** *are valid, where* $T_h = C_hM_h^{-1}$.

9. *The scheme* $\{M_h, I_h\}$ *has the accuracy order p in* $\mathscr{X}_2$ *with respect to the problem* $Mu = v$, $v \in \mathscr{X}_2$.

Assertion. *The scheme* $\{M_h, C_h\}$ *has the accuracy-order p for the characteristic value* λ_0 *with minimal modulus of the problem* $Mu = \lambda Cu$.

Proof. Similarly as in the proof of theorem **4.1** it is sufficient to legalize the fulfilment of the assumption (c) of theorem **3.1**, since the other assumptions of this theorem are fulfilled as a consequence of assumptions of theorem **4.2.**

We shall consider vectors $P_hCM^{-1}v_0 - C_hM_h^{-1}P_{2,h}v_0$, $h \in H$, where $v_0 = \lambda_0CM^{-1}v_0$, $\lambda_0 = \mu_0^{-1}$.

The assumption 9 implies the existence of a constant c such that $\|w_h\| \leqq ch^p$, where $w_h = P_{1h}M^{-1}v_0 - M_h^{-1}P_{2,h}v_0$. Using (4.3) we see that $w_h \in \mathscr{R}(M_h^{-1})$. From assumption 3 we deduce the inequalities

$$\|C_h w_h\|_2 \leqq c\|w_h\|_1 \leqq ch^p . \tag{4.4}$$

Assumption 6 guarantees the validity of relations

$$P_{2h}CM^{-1}v_0 - C_hM_h^{-1}P_{2h}v_0 =$$
$$= C_h(M_h^{-1}P_{2h}v_0 + w_h) - (C_hM_h^{-1}P_{2h}v_0 + y_h) = C_hw_h - y_h ,$$

from where

$$\|P_{2h}CM^{-1}v_0 - C_hM_h^{-1}P_{2h}v_0\| \leqq c\|w_h\|_1 + \|y_h\|_2 \leqq ch^p$$

and that was to be legalized. The assertion of theorem **4.2** then directly follows from theorem **3.1**.

The accuracy-order of the scheme $\{M_h, C_h\}$ for the eigenvalue problem $Mu = \lambda Cu$ can be investigated without the dominantness of the proper values μ_0 and $\mu_{0,h}$. A result in this direction is contained in theorem **4.3**.

An important class of problems for which theorem **4.3.** can be applied is formed by positive irreducible operators CM^{-1}.

Suppose that the spectrum of the operator CM^{-1} contains a finite number of simple poles $\mu_1, \ldots, \mu_s$ of the resolvent $R(\lambda, CM^{-1})$, where $|\mu_j| = r(CM^{-1})$ and $r(CM^{-1})$ denotes the spectral radius of CM^{-1}. Then to every μ_j there exists a complex number ν_j such that $\mu_j + \nu_j$ is a dominant point of the spectrum $\sigma(T_j)$, where

$$T_j = CM^{-1} + \nu_j I . \tag{4.5}$$

For fixed j we put

$$D_j = \nu_j M + C , \quad L = M . \tag{4.6}$$

Then evidently $D_jL^{-1} = T_j$ and T_j has a dominant proper value $\varrho_j = \mu_j + \nu_j$. For solution of the problem

$$Lu_j = \sigma_j D_j u_j , \quad \sigma_j = \varrho_j^{-1} \tag{4.7}$$

theorem 4.2 can be used. It is easy to see that

$$Mu_j = \mu_j^{-1}Cu_j , \tag{4.8}$$

where u_j is a proper vector of the problem (4.7).

Theorem 4.3. *Assumptions.*

1. *The operators M and M_h which map $\mathscr{D}(M) \subset \mathscr{X}_1$ and $\mathscr{D}(M_h) \subset \mathscr{X}_{1,h}$ into $\mathscr{X}_2$ and $\mathscr{X}_{2,h}$ have inverses $M^{-1} \in [\mathscr{X}_2, \mathscr{X}_1]$ and $M_h^{-1} \in [\mathscr{X}_{2,h}, \mathscr{X}_{1,h}]$.*
2. *$\mathscr{R}(M^{-1}) \subset \mathscr{D}(C)$ and $\mathscr{R}(M_h^{-1}) \subset \mathscr{D}(C_h)$.*

3. *There exists a constant c such that*

$$\|Cy\|_2 \leqq c\|y\|_1 \tag{4.9}$$

for $y \in \mathscr{R}(M^{-1})$ *and*

$$\|C_h y_h\|_2 \leqq c\|y_h\|_1 \tag{4.10}$$

for $y_h \in \mathscr{R}(M_h^{-1})$.

4. *For every* $h \in H$

$$P_{1h} M^{-1} \mathscr{X}_2 \subset M_h^{-1} \mathscr{X}_{2,h} \,. \tag{4.11}$$

5. *The resolvents* $R(\lambda, CM^{-1})$ *and* $R(\lambda, C_h M_h^{-1})$ *have s and* $s_h = s(h)$ *simple poles* $\mu_1, \ldots, \mu_s$ *and* $\mu_1^{(h)}, \ldots, \mu_{s(h)}^{(h)}$ *on the circles* $|\lambda| = r(CM^{-1})$ *and* $|\lambda| = r(C_h M_h^{-1})$ *respectively.*

If v_j' is a proper vector of CM^{-1} corresponding to μ_j, then we put $v_{jk}^{(h)} = B_{1,k}^{(h)} P_{2h} v_j$, where $B_{1,k}^{(h)}$ is the unique coefficient of the main part of Laurent expansion of $R(\lambda, C_h M_h^{-1})$ in a neighbourhood of $\mu_k^{(h)}$.

6. *For every vector* $u \in \mathscr{R}(M^{-1})$ *there exists vector* $y_h \in \mathscr{X}_{2,h}$ *such that*

$$C_h P_{1h} u = P_{2h} C u + y_h$$

and

$$\|y_h\| \leqq ch^p \,,$$

7. *The operators* $C_h M_h^{-1}$, $h \in H$, *are uniformly bounded*: $\|C_h M_h^{-1}\| \leqq c$.

8. *There are forms* $\mathfrak{x}_{h,j,k}' \in \mathscr{X}_{2,h}'$, $h \in H$, such that

$$\text{(i)} \qquad \|\mathfrak{x}_{h,j,k}'\| \leqq c \,,$$

$$\text{(ii)} \qquad \mathfrak{x}_{h,j,k}'(C_h M_h^{-1} x_h) = \mu_k^{(h)} \mathfrak{x}_{h,j,k}'(x_h) \,, \quad x_h \in \mathscr{X}_{2,h} \,,$$

$$\text{(iii)} \qquad |\mathfrak{x}_{h,j,k}'(P_{2h} v_j)| \geqq c > 0 \,, \quad j = 1, \ldots, s \,; \quad k = 1, \ldots, s_h \,.$$

9. *The scheme* $\{M_h, I_h\}$ *has the accuracy-order p in* $\mathscr{X}_2$ *with respect to the problem* $Mu = v$.

10. *For every* j, $j = 1, \ldots, s$, *there is a complex* v_j *such that the operators* $T_j = CM^{-1} + v_j I$, $T_{h,j} = C_h M_h^{-1} + v_j I_h$ *have dominant proper values* $\mu_j + v_j$ *and* $\mu_k^{(h)} + v_j$ *for suitable k.*

Assertion. *The scheme* $\{M_h, C_h; \lambda_1, \ldots, \lambda_s\}$, *where* $\lambda_j = \mu_j^{-1}$, $j = 1, \ldots, s$, *has the accuracy-order p for the characteristic values* $\lambda_1, \ldots, \lambda_s$ *of the problem* $Mu = \lambda Cu$.

Proof. Let us choose j fixed and let us investigate operators T_j, $T_{h,j}$ together with problems

$$Lu = \sigma D_j u \,, \quad L_h u_h = \sigma_h D_{h,j} u_h \,, \tag{4.12}$$

where L and D_j are defined by (4.6) and

$$L_h = M_h\,, \quad D_{h,j} = \nu_j M_h + C_h\,. \tag{4.13}$$

We shall prove that for constructing $\sigma_j = \varrho_i^{-1}$, where $\varrho_j = \mu_j + \nu_j$, and corresponding proper vectors, theorem **4.2** can be applied. From this the validity of theorem **4.3** will follow.

Similarly as in proofs of theorems **4.1** and **4.2** it is sufficient to legalize only the fulfilment of the assumption (c) of theorem **3.1** for the operator T_j.

Let us consider vectors $P_{2h}T_j v_j - T_{h,j}P_{2h}v_j$, where $v_j = \mu_j^{-1}CM^{-1}v_j$. Assumption 10 guarantees the dominantness of $\mu_j + \nu_j$ and $\mu_k^{(h)} + \nu_j$. Assumption 9 gives a constant c such that $\|w_h\| \leqq ch^p$, where $w_h = P_{1h}M^{-1}v_j - M_h^{-1}P_{2h}v_j$. According to (4.11) $w_h \in \mathscr{R}(L_h^{-1})$. Assumption 3 implies the inequalities

$$\|C_h w_h\|_2 \leqq c\|w_h\|_1 \leqq ch^p\,. \tag{4.14}$$

From definition of T_j and $T_{h,j}$ using assumption 6 we obtain the relations

$$\begin{aligned} &P_{2h}T_j v_j - D_{h,j}L_h^{-1}P_{2h}v_j = \\ &= P_{2h}(C + \nu_j M)\,M^{-1}v_j - (C_h + \nu_j M_h)\,M_h^{-1}P_{2h}v_j = \\ &= C_h P_{1h}M^{-1}v_j - y_h - C_h M_h^{-1}P_{2h}v_j\,. \end{aligned}$$

All these considerations yield to the relations $\|P_{2h}T_j - D_{h,j}L_h^{-1}P_{2h}v_j\| \leqq \|C_h w_h\| + \|y_h\|$ and consequently to the required inequality $\|P_{2h}T_j v_j - T_{h,j}P_{2h}v_j\| \leqq ch^p$. In other words the requirement (c) of theorem **3.1** is fulfilled. From this theorem we conclude that

$$|\mu_j - \mu_k^{(h)}| \leqq |(\mu_j + \nu_j) - (\mu_k^{(h)} + \nu_j)| \leqq ch^p\,, \quad \|P_{2h}v_j - B_{1,k}^{(h)}P_{2h}v_j\| \leqq ch^p$$

and these estimates show that the scheme $\{M_h, C_h; \mu_j^{-1}\}$ has the accuracy-order p in $\mathscr{X}_2$.

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Výtah

K TEORII PŘIBLIŽNÝCH METOD V PROBLÉMECH VLASTNÍCH HODNOT

Ivo MAREK, Praha

Vyšetřují se problémy vlastních hodnot pro rovnice typu $Mu = \lambda Cu$, $M_h u_h = \lambda^{(h)} C_h u_h$, kde $u \in \mathscr{X}$, Mu, $Cu \in \mathscr{Y}$; $u_h \in \mathscr{X}_h$, $L_h u_h$, $C_h u_h \in \mathscr{Y}_h$ při čemž $\mathscr{X}$, $\mathscr{Y}$ resp. $\mathscr{X}_h$, $\mathscr{Y}_h$ jsou Banachovy prostory a M, C resp. M_h, C_h lineární zobrazení zobrazující husté podmnožiny $\mathscr{X}$ resp. $\mathscr{X}_h$ do $\mathscr{Y}$ resp. $\mathscr{Y}_h$. Jmenovitě se vyšetřuje řád přesnosti „přibližných" vlastních prvků u_h ,$\lambda^{(h)}$ vzhledem k vlastním prvkům určeným pomocí operátorů L a C. Jsou uvedeny postačující podmínky, jež zaručují, že řád přesnosti pro úlohu na vlastní hodnoty je roven řádu přesnosti odpovídající úlohy nehomogenní (princip dehomogenisace).

РЕЗЮМЕ

О ПРИБЛИЖЕННЫХ МЕТОДАХ В ЗАДАЧАХ О СОБСТВЕННЫХ ЗНАЧЕНИЯХ

ИВО МАРЕК (Ivo Marek), Прага

В статье рассматриваются проблемы о собственных значениях для уравнений типа $Mu = \lambda Cu$, где $u \in \mathscr{X}$, Mu, $Cu \in \mathscr{Y}$ и $\mathscr{X}$, $\mathscr{Y}$ банаховы пространства и M, C – линейные отображения плотных областей определения из $\mathscr{X}$ в $\mathscr{Y}$. Вместе с этой задачей рассматривается „приближенная" проблема $M_h u_h = \lambda^{(h)} C_h u_h$, где линейные операторы M_h, C_h отображают плотные подмножества из банахова пространства $\mathscr{X}_h$ в банахово пространство $\mathscr{Y}_h$. Особенно исследован порядок точности „приближенных" собственных элементов u_h, $\lambda^{(h)}$ задачи $M_h u_h = \lambda^{(h)} C_h u_h$ относительно точных собственных элементов уравнения $Mu = \lambda Cu$. Приводятся достаточные условия обеспечивающие равенство порядков точности задачи о собственных значениях и соответствующей неоднородной задачи. Эти исследования являются базой так называемого принципа дегомогенизации.