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REFERÁTY

THE THREE CONJUGATE THEOREM – QUASI-UNIVERSAL FLOWS

Referát o dvou přednáškách, které proslovil profesor R. D. ANDERSON ze Státní university v Louisianě (USA) dne 27. a 28. května 1963 na matematicko fysikální fakultě KU v Praze.

1. The Three Conjugate Theorem

The theorems below are stated for the n-dimensional sphere S^n but are valid in more general contexts as noted in the final remark. Let G_k be the set of all homeomorphisms of S^n onto itself each of which is the identity in some open cell. Let G_k^* be the group of all finite products of elements of G_k. A conjugate of a homeomorphism f is a homeomorphism $(\varphi^{-1}f\varphi)$, where φ is also a homeomorphism. Such a conjugate of f may be thought of as a "copy" of f but with respect to a new coordinate system, φ carrying the new coordinate system onto the old.

Theorem I. *Let f and g be elements of G_k^* neither being the identity. Then f is the product of three conjugates of g where the conjugations are by elements of G_k^*.*

Oddly, it is not known whether the identity is the product of three conjugates of g.

Theorem II. *There exist homeomorphisms h and γ of G_k such that h is not the product of two conjugates of γ.*

Thus the number "three" of Theorem I is the best possible number in that context. In the proof of Theorem II γ is an involution (period 2 homeomorphism). Theorem I follows from a more general lemma.

Lemma. *If α, β, γ, $\delta \in G_k^*$ with none the identity, then there exist $f_1, f_2 f_3 f_4 \in G_k^*$ such that*

$$(f_1^{-1}\alpha f_1)(f_2^{-1}\beta f_2) = (f_3^{-1}\gamma f_3)(f_4^{-1}\delta f_4).$$

As a corollary, α is the product of a conjugate of β^{-1}, a conjugate of δ, and a conjugate of γ. Here the identity is an exceptional case, for from Theorem II it follows that, in general, the identity is not the three-way product of conjugates of arbitrary non-identity elements β^{-1}, γ and δ of G_k^*.

If the "annulus" problem is true, then G_k^* is the group of all orientation-preserving homeomorphisms of S^n. If the "annulus" problem is not true, then some orientation-preserving homeomorphism of S^n is not expressible as any finite product of elements of G_k^*. For $n = 2, 3$ the annulus problem is known to be true.

The techniques and theorems are also applicable for many "invertible" spaces such as the Cantor set, the space of rationals (or irrationals), the universal curve, and even for the closed n-cell, $n > 1$, if it is assumed that the homeomorphisms concerned are not the identity on the boundary.

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2. Quasi-Universal Flows

A *flow* is a transformation group (G, X), where G is isomorphic to integers I (a *discrete* flow) or the reals (a *continuous* flow). For our purposes the space on which the group acts is assumed compact and metric.

A flow (G, Z) is said to be quasi-universal provided, for any compact metric space X, every flow (G, X) can be raised to a closed subflow of (G, Y) of (G, Z), i.e. the flow (G, Z) restricted to a closed invariant subset Y of Z.

Theorem I. *There exists a quasi-universal discrete flow on the Cantor Set C.*

This theorem follows from a theorem of Baayen and de Groot on the existence of a universal homeomorphism on C and from an earlier theorem of the author to the effect that every discrete flow can be raised to a discrete flow on C.

By a known construction and use of Theorem I on discrete flows, Theorems II and III can also be proved:

Theorem II. *There exists a quasi-universal continuous flow on a certain one-dimensional compactum.*

Theorem III. *There exists a quasi-universal continuous flow on the solid torus in* E^3.

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