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Czechoslovak Mathematical Journal, Vol. 16 (1966), No. 4, 624–626

Persistent URL: http://dml.cz/dmlcz/116965

СООБЩЕНИЯ — NEWS AND NOTICES

PROFESSOR VLASTIMIL PTÁK AWARDED THE 1966 STATE PRIZE

On the occasion of the First of May celebrations 1966 the President of the Republic has awarded the State Prize to Prof. Dr. VLASTIMIL PTÁK DrSc. for his contributions to Functional Analysis. These contributions have gained a wide recognition abroad and rank with the best achievements of Czechoslovak mathematics in the past years.

V. Pták's scientific activity is very extensive and includes results in algebra, combinatorial analysis, real functions, numerical analysis, matrix theory, analysis and topology. One of the characteristic features of his work is the fact that his results are frequently obtained by an ingenuous combination of methods from different branches of mathematics. As an example of this approach, his systematic application of methods of Functional Analysis in matrix theory should be mentioned. Another instance is an unexpected application of combinatorial methods to obtain deep results on weak compactness.

His first results appeared already during his University studies. In 1952, mainly under the influence of E. ČECH, his interest turned to Functional Analysis. He concentrated himself from the very beginning on a deep study of the fundamental principles of Analysis. It is exactly this part of his scientific activity which has been rewarded by the State Prize.

One of the most important problems in Functional Analysis is that of the continuity of the inverse operator. In his first paper on Functional Analysis V. Pták has attacked this question with great success and obtained results which received immediate recognition abroad. Let us describe in a few words the state of the problem in 1952, before the first paper of Pták's series of investigations concerning the open mapping theorem. In 1931 S. BANACH proved a highly important result which may be formulated as follows: Let φ be a continuous linear mapping of a Banach space E onto another Banach space F. Then the mapping φ is open. The rapid development of Functional Analysis after the appearance of Banach's "Théorie des opérations linéaires" has shown rather soon that the requirements of some applications in analysis make it necessary to consider a broader class of spaces. Moreover, the theory of normed spaces itself necessitates a widening of the classical framework. Indeed, the notion of weak convergence — introduced already by Banach — and later the theory of duality cannot be understood properly without the notion of a topological linear space. A Banach space, however, is topologically a metric space and the majority of methods used in the theory of Banach spaces is based essentially on the metric character of the space. The proof of the classical open mapping theorem of Banach consists in proving the convergence of a certain infinite series and becomes meaningless in spaces without the first axiom of countability. Since the proof of this theorem is so closely tied up with countability it becomes natural to ask whether the validity of the theorem itself is not restricted to metrizable spaces. The first step of V. Pták consisted in clearing up the substance of the theorem. One of the decisive moments for the further investigations is the full recognition of the fact that only one of the spaces considered in the open mapping theorem plays an essential role. An analysis of the classical open mapping theorem shows that the proof uses the completeness of the first space essentially while the completeness of the second space is not used to its full extent: indeed, it suffices to use the weaker Baire property. In fact, the theorem is based on the following alternative: Let E be a Banach space and let F be a normed space. If φ is a continuous linear mapping of E into F then the image of the unit cell of E is either nondense in F or a neighbourhood of zero in F. Hence if the image of E is of the second category in F the first case of the alternative becomes impossible and the mapping φ is open. One of the basic ideas of the further investigations of V. Pták consists in formulating this as a property of the mapping φ itself: he calls a mapping φ of a topological space P into a topological space Q almost open if, for each $x_0 \in P$ and each neighbourhood U of x_0 the set $\overline{\varphi(U)}$ is a neighbourhood of $\varphi(x_0)$ in $\varphi(P)$. It may be remarked in passing that some of the further investigations show that — from the point of view of duality theory — the notion of almost openness is more natural then that of openness. Hence the classical open mapping theorem may be formulated as follows: Every continuous linear mapping of a Banach space which is almost open is already open. This formulation of the open mapping theorem involving the properties of one space only and of the mapping itself is the starting point of the further investigations. The spaces which possess the property above form the object of a detailed study. Since it may be expected from the analogy with the classical case that this property will be closely connected with completeness, V. Pták suggested the name B-complete for these spaces. The next step consisted in characterizing these spaces in terms of properties of the dual space. The result is as follows. A locally convex space E is B-complete if and only if the adjoint space E' taken in the topology $\sigma(E', E)$ possesses the following property: *given a subspace $Q \subset E'$ such that the intersections $Q \cap U^0$ are closed for each neighbourhood of zero U in E, than Q itself is closed.* Now this condition is surprisingly similar to a result of Krein and Šmulyan which may be stated in the following manner. Given a subspace Q of the adjoint of a Banach space such that the intersection of Q with the closed unit sphere is weakly closed, then Q itself is weakly closed. The equivalence above shows that the open mapping theorem and the Krein-Šmulyan theorem actually are only two ways of stating one result, although, at first glance, they have hardly anything in common. It is surprising that this connection has not been noticed before, although 1952 was twenty years after the "Théorie des opérations linéaires". At the same time, the Krein-Šmulyan theorem shows that, for normed spaces, the notion of B-completeness coincides with that of completeness. It was thus necessary to clear up the connection between B-completeness and completeness in the general case. V. Pták has succeeded in obtaining an analogous dual characterization of completeness. The completion of a locally convex space E consits of those linear forms on E' the zero hyperplane of which has a closed intersection with every U^0. A comparison with the preceding characterization of B-completeness shows that the class of B-complete spaces is contained in that of complete spaces. The further results of the 1952 paper and of subsequent papers show that this class is in fact smaller. On the other hand, there are important classes of spaces which are B-complete. The class of Banach spaces, F-spaces and dual spaces to F-spaces taken in the compact-open topology.

Further it was necessary to decide the question whether the class of B-complete spaces is the correct or natural generalization. The only possible way of verifying this consists in showing that all the classical results connected with the open mapping theorem have their natural generalizations in this broader class. This is indeed the case. To mention at least one result, V. Pták has succeeded in proving the following "closed relation theorem" which generalises both the open mapping theorem and the closed graph theorem.

Let E and F be two locally convex spaces and let R be a closed subspace of $E \times F$. Suppose that, for each neighbourhood of zero U in E, the set $\overline{RU}$ is a neighbourhood of zero in F. If E is B-complete then $RE = F$ and RU itself is a neighbourhood of zero in F for each U.

Another important problem of Analysis to which V. Pták has devoted much attention is that of inverting the order of two limit processes.

After a detailed study of this problem V. Pták has recognized that they can always be formulated as questions about weak compactness of appropriate sets and that these questions, in their turn, may be formulated as questions whether a certain operation which is continuous on a certain set will stay continuous if the set is enlarged by adjoining to it some "ideal" points. To this end he has formulated and proved an extension theorem for separately continuous functions which contains the majority of the classical results on weak compactness and forms a powerful tool of general analysis. To formulate the extension theorem let us recall that every completely regular topological space T may be imbedded in a natural manner in $C(T)'$, where $C(T)'$ is taken in the weak star topology corresponding to $C(T)$, the Banach space of all bounded continuous functions on T. Now we may formulate the main problem.

Let f be a bounded and separately continuous function on $S \times T$. Under what conditions does there exist a separately continuous bilinear form on $C(S)' \times C(T)'$ which extends f?

The extension theorem consists in showing that such an extension exists if and only if the function f satisfies the double limit condition: if $s_i \in S$ and $t_j \in T$ are two sequences such that both $\lim_i \lim_j f(s_i, t_j)$ and $\lim_j \lim_i f(s_i, t_j)$ exist then they have to be equal to each other.

The main (and only) tool is a combinatorial lemma published in 1959 which contains everything essential: all the results on weak compactness follow from it just by pure logic. Especially it is to be noted that this lemma eliminates all integration theory from the proof of Krein's theorem.

All these results have already been included in many monographs and textbooks abroad. They have also appeared in translation in the USSR. Some authors even use the name Pták-space for B-complete spaces. Also, the theorem on weak compactness as sometimes referred to as Pták's theorem.

All papers of V. Pták are characterized by an unusual clarity of style and by a penetrating analysis which enables him to gain a deep insight and grasp the substance of the problem.

V. Pták is head of the Section of Topology and Functional Analysis in the Institute of Mathematics of the Czechoslovak Academy of Sciences. His activity is by no means limited to his scientific work at the Institute. For many years he has been teaching graduate courses at Charles University and other scientific institutions, including other Institutes of the Academy. His assistance in the preparation of beginning mathematicians is also highly appreciated. Among his other activities his work as member of the Board of Editors of the Czechoslovak Mathematical Journal should be mentioned. Very frequently he is invited by Universities and scientific institutions abroad to give lectures on his work.

In the name of the Czechoslovak mathematicians we congratulate V. Pták on the tribute paid to his work by the bestowal of the highest scientific award and wish him the best of health and every success in his further work.

The Editors