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REGULARITY CONDITIONS FOR LIFTINGS OF FUNCTIONS AND VECTOR FIELDS TO NATURAL BUNDLES

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INTRODUCTION

The concept of "natural bundle" was introduced by A. Nijenhuis [7] as a modern approach to the classical theory of geometric objects (see J. Aczel and S. Gołąb [1]). A natural bundle (over n-dimensional manifolds) is a covariant functor $\mathscr{F}$ from the category of n-dimensional manifolds and their embeddings into the category of differentiable locally trivial fibre bundles and their bundle mappings such that:

(1) For every n-dimensional manifold $M, \mathscr{F}M$ is a locally trivial fibre bundle over M.

(2) For every embedding $\varphi : M \to N$ of two n-dimensional manifolds, $\mathscr{F}\varphi :$ $: \mathscr{F}M \to \mathscr{F}N$ covers φ and for any point x of $M, \mathscr{F}\varphi$ maps diffeomorphically the fibre $\mathscr{F}_x M$ onto the fibre $\mathscr{F}\varphi(x)N$.

(3) $\mathscr{F}$ is a regular functor in the following sense: If $\varphi : U \times M \to N$ is a differentiable mapping (where U is an open subset of $\mathbf{R}^k$) such that for every point t of U, $\varphi_t : M \to N$, $\varphi_t(x) = \varphi(t, x)$ is an embedding, then

$$U \times \mathscr{F}M \ni (t, y) \to \mathscr{F}\varphi_t(y) \in \mathscr{F}N$$

is of class C^∞.

About 1960, S. Kobayashi, K. Yano and S. Ishihara began investigations of liftings of geometric objects from a manifold to the tangent bundle (see [10], [11]). Soon, S. Ishihara, K. Yano, K. P. Mok and A. Morimoto ([4], [5], [6]) began analogous research on other natural bundles such as p^r-velocities or tangent bundles of higher orders. All such works were begun by investigating "liftings" of functions and vector fields.

In [3], axiomatic descriptions of the concepts of liftings of functions and quasi-liftings of vector fields to natural bundles are given. We recall these definitions.

Definition 0.1. *Let $\mathscr{L} = \{\mathscr{L}_M\}$ be a family of mappings $\mathscr{L}_M : C^\infty(M) \to C^\infty(\mathscr{F}M)$, where M is an n-dimensional manifold. $\mathscr{L}$ is called a lifting of functions to the natural bundle $\mathscr{F}$ if $\mathscr{L}$ has the following properties:*

(1) *For any n-dimensional manifold M the mapping* $\mathscr{L}_M$ *is* **R**-*linear, i.e.* $\mathscr{L}_M(af + bg) = a\mathscr{L}_M f + b\mathscr{L}_M g$ *for all differentiable functions f, g and all real numbers a, b.*

(2) *For any embedding* $\varphi : M \to N$ *of two n-dimensional manifolds and for any function f on N we have* $\mathscr{L}_M(f \circ \varphi) = (\mathscr{L}_N f) \circ \mathscr{F}\varphi$.

(3) $\mathscr{L}$ *is regular in the following sense: If U is an open subset of* $\mathbf{R}^k$ *and f is a differentiable function on* $U \times M$, *then the function*

$$U \times \mathscr{F}M \ni (t, y) \to \mathscr{L}_M f_t(y) \in \mathbf{R}$$

is of class C^∞, *where* f_t *is a function on M given by* $f_t(x) = f(t, x)$.

Definition 0.2. *Let* $\mathscr{F}$ *be a natural bundle over n-dimensional manifolds and* $\Lambda = \{\Lambda_M\}$ *a family of mappings* $\Lambda_M : \mathfrak{X}(M) \to \mathfrak{X}(\mathscr{F}M)$ *where M is a manifold* $\dim M = n$. Λ *is called a quasi-lifting of vector fields to* $\mathscr{F}$, *if* Λ *satisfies the following conditions:*

(1) *For every manifold M,* Λ_M *is* **R**-*linear.*

(2) *If* $\varphi : M \to N$ *is an embeding of two n-dimensional manifolds, then for any vector field X on M,*

$$\Lambda_{\varphi(M)}(\varphi_* X) = (\mathscr{F}\varphi)_* (\Lambda_M X) .$$

(3) Λ_M *is regular in the following sense: If U is an open subset of* $\mathbf{R}^k$ *and* $X : U \times M \to TM$ *is a* C^∞-*mapping such that for every t in U,* $X_t : M \to TM$, $X_t(x) = X(t, x)$ *is a vector field on M, then the mapping*

$$U \times \mathscr{F}M \ni (t, y) \to (\Lambda_M X_t)(y) \in T(\mathscr{F}M)$$

is of class C^∞.

In [2] D. B. A. Epstein and W. P. Thurston showed that if $\mathscr{F}\mathbf{R}^n$ has a countable basis, then the "regularity condition" in the definition of a natural bundle is a consequence of conditions (1) and (2). From the above it follows that the question "Is the regularity condition in Definition 0.1 (or 0.2) a consequence of the other conditions in this definitions?" is natural.

The purpose of this paper is to give answer to the above question. In both cases answer is affirmative. In the case of liftings of functions the proof is not difficult. We will give it in section 1. The case of quasi-liftings of vector fields is more complicated. The main difficulty is the proof of the formula

$$\Lambda[X, Y] = [\Lambda X, \mathscr{F}Y] .$$

The above formula is a simple consequence of the regularity condition (see [3]) but we must show this formula without using the regularity condition. To prove this we shall use Lemma 2.3 which corresponds to Proposition 7.1 in [2].

I would like to thank Dr. J. Gancarzewicz for his suggestions and corrections.

1. THE REGULARITY CONDITION FOR LIFTINGS OF FUNCTIONS

In this section we show the following theorem:

Theorem 1.1. *If a family* $\mathscr{L} = \{\mathscr{L}_M\}$ *of mappings* $\mathscr{L}_M : C^\infty(M) \to C^\infty(\mathscr{F}M)$ *satisfies the conditions* (1) *and* (2) *of Definition* 0.1, *then it satisfies the condition* (3) *of this definition.*

Let us remark that if $\mathscr{L}$ is a family of mapping satisfying the conditions (1) and (2) of Definition 0.1, then $\mathscr{L}$ is local, that is, if U is an open subset of M and f_1, f_2 are C^∞-functions on M such that $f_1 \mid U = f_2 \mid U$, then $(\mathscr{L}_M f_1) \mid \mathscr{F}U = (\mathscr{L}_M f_2) \mid \mathscr{F}U$. Since every n-dimensional manifold is locally diffeomorphic to $\mathbf{R}^n$, it is sufficient to show the following proposition.

Proposition 1.2. *Let* $\mathscr{F}$ *be a natural bundle (over n-dimensional manifolds) and let* $\mathscr{L} : C^\infty(\mathbf{R}^n) \to C^\infty(\mathscr{F}\mathbf{R}^n)$ *be a mapping such that*:

(1) $\mathscr{L}$ *is* $\mathbf{R}$*-linear.*

(2) *If* $\varphi : \mathbf{R}^n \to \mathbf{R}^n$ *is an embedding, then* $\mathscr{L}(f \circ \varphi) = \mathscr{L}f \circ \mathscr{F}\varphi$ *for every f from* $C^\infty(\mathbf{R}^n)$.

Then $\mathscr{L}$ *satisfies the regularity condition: If N is a manifold and* $N \times \mathbf{R}^n \ni (t, x) \to f_t(x) \in \mathbf{R}$ *is of class* C^∞, *then the mapping*

$$N \times \mathscr{F}\mathbf{R}^n \ni (t, y) \to \mathscr{L} f_t(y) \in \mathbf{R}$$

is of class C^∞.

In this section we shall use the following notations: $\mathscr{L}$ is such a mapping as in Proposition 1.2, $\pi : \mathscr{F}\mathbf{R}^n \to \mathbf{R}^n$ is the bundle projection and $q : \mathbf{R}^n \to \mathbf{R}^{n-1}$ as well as $p : \mathbf{R}^n \to \mathbf{R}$ are natural projections given by the formulas:

$$q(x_1, \ldots, x_n) = (x_2, \ldots, x_n)\,; \quad p(x_1, \ldots, x_n) = x_1\,.$$

First we prove the following lemma:

Lemma 1.3. *Let N be a manifold, let* $f : N \times \mathbf{R}^n \to \mathbf{R}$ *be of class* C^∞ *and let* (t_0, x_0) *be a point of* $N \times \mathbf{R}^n$. *If the Jacobi matrix of* (f_{t_0}, q) *at* x_0 *is non-singular, then there is a neighbourhood* $U_0 \times W_0$ *of* (t_0, x_0) *in* $N \times \mathbf{R}^n$ *such that the mapping*

$$U_0 \times \pi^{-1}(W_0) \ni (t, y) \to \mathscr{L}f_t(y) \in \mathbf{R}$$

is of class C^∞.

Proof. There exists a neighbourhood $U \times W$ of (t_0, x_0) in $N \times \mathbf{R}^n$ such that $(f_t, q)_{|W}$ is an embedding for every t from U. Let W_0, $\tilde{W}$ be open subsets of W and let $\psi : \tilde{W} \to \mathbf{R}^n$ be a diffeomorphism such that $x_0 \in W_0 \subset \tilde{W} \subset W$, $\psi_{|W_0} = id$. $\mathscr{L}$ is local, so

$$\mathscr{L} f_t(y) = \mathscr{L}(f_t \circ \psi^{-1})(y) = \mathscr{L}(p \circ (f_t, q) \circ \psi^{-1})(y) =$$
$$= \mathscr{L} p \circ \mathscr{F}((f_t, q) \circ \psi^{-1})(y)$$

for every t from U and every y from $\pi^{-1}(W_0)$. Lemma 1.3 is proved.

Now we will prove Proposition 1.2. For every (t_0, x_0) from $N \times \mathscr{F}\mathbf{R}^n$ there exists a real number u such that $f_t + up$ satisfies the assumptions of Lemma 1.3 for (t_0, x_0). So, there exists a neighbourhood $U_0 \times \pi^{-1}(W_0)$ of (t_0, x_0) such that the mapping

$$U_0 \times \pi^{-1}(W_0) \ni (t, y) \to \mathscr{L}(f_t + up)(y) \in \mathbf{R}$$

is of class C^∞. Because of the equality $\mathscr{L}(f_t + up) = \mathscr{L}f_t + u\mathscr{L}p$ the mapping

$$U_0 \times \pi^{-1}(W_0) \ni (t, y) \to \mathscr{L} f_t(y) \in \mathbf{R}$$

is also of class C^∞. This completes the proof of Proposition 1.2.

2. THE REGULARITY CONDITION FOR LIFTINGS OF VECTOR FIELDS

In this section we show the following theorem.

Theorem 2.1. *If a family $\Lambda = \{\Lambda_M\}$ of mappings $\Lambda_M : \mathfrak{X}(M) \to \mathfrak{X}(\mathscr{F}M)$ satisfies the conditions* (1) *and* (2) *of Definitions* 0.2, *then it satisfies the condition* (3) *of this definition.*

Using similar arguments as in section 1, we will prove

Proposition 2.2. *Let $\mathscr{F}$ be a natural bundle (over n-dimensional manifolds), let $\pi : \mathscr{F}\mathbf{R}^n \to \mathbf{R}^n$ be a bundle projection and $\Lambda : \mathfrak{X}(\mathbf{R}^n) \to \mathfrak{X}(\mathscr{F}\mathbf{R}^n)$ a mapping such that:*

(1) *Λ is $\mathbf{R}$-linear.*

(2) *If $\varphi : \mathbf{R}^n \to \mathbf{R}^n$ is a diffeomorphism and X a vector field on $\mathbf{R}^n$, then $\Lambda(\varphi_* X) = (\mathscr{F}\varphi)_* \Lambda X$.*

(3) *If X_1 and X_2 are two vector fields on $\mathbf{R}^n$ such that $X_1 \mid U = X_2 \mid U$ for some open subset U of $\mathbf{R}^n$, then $\Lambda X_1 \mid \pi^{-1}(U) = \Lambda X_2 \mid \pi^{-1}(U)$.*

Then Λ has the following property: if N is a manifold and $N \times \mathbf{R}^n \ni (t, x) \to X_t(x) \in T\mathbf{R}^n$ is a C^∞-mapping such that for every t from N, X_t is a vector field on $\mathbf{R}^n$, then the mapping

$$N \times \mathscr{F}\mathbf{R}^n \ni (t, y) \to \Lambda X_t(y) \in T\mathscr{F}\mathbf{R}^n$$

is of class C^∞.

In this section we shall use the following notations: Λ is such a mapping as in Proposition 2.2, $\pi : \mathscr{F}\mathbf{R}^n \to \mathbf{R}^n$ is the bundle projection and $\tau_x : \mathbf{R}^n \to \mathbf{R}^n$ is the translation by x.

Lemma 2.3. *Let X and X_p be vector fields on $\mathbf{R}^n$ such that the ∞-jet of X_p at 0 converges to the ∞-jet of X at 0 if $p \to \infty$. Then $\Lambda X_p(y)$ converges to $\Lambda X(y)$ for every y from $\pi^{-1}(0)$.*

Proof. We fix a point y from $\pi^{-1}(0)$ and let

$$X_p = \sum_{i=1}^{n} \varphi_p^i \partial_i , \quad X = \sum_{i=1}^{n} \psi^i \partial_i , \quad \varphi_p = \{\varphi_p^i\}_{i=1}^n , \quad \psi = \{\psi^i\}_{i=1}^n .$$

Replacing φ_p by $\varphi_p - \psi + id - \varphi_p(0) + \psi(0)$ we see that there is no loss of generality in supposing that $\psi = id$ and $\varphi_p(0) = 0$. Now, the mapping φ_p is an embedding on $(-\delta_p, \delta_p)^n$ for p sufficiently large, so by applying the diffeomorphism $\psi_p : (\mathbf{R}^n, 0) \to ((-\delta_p, \delta_p)^n, 0)$ such that

$$\psi_p\big|_{[(\delta p)/2,(\delta p)/2]^n} = id$$

we may suppose that φ_p is an embedding.

We show that any subsequence X_{p_q} of X_p contains a subsequence $X_{p_{q_s}}$ such that

$$\Lambda X_{p_{q_s}}(y) \xrightarrow[s\to\infty]{} \Lambda X(y) .$$

Let X_{p_q} be a subsequence of X_p. By passing to a subsequence $X_{p_{q_m}}$ we can assume that $\|D^\alpha(\varphi_{p_{q_m}} - id)(0)\| < e^{-m}$ for every n-tuple α such that $|\alpha| \leqq m$. We choose $\varepsilon_m < e^{-m}$ such that $\|D^\alpha \varphi_{p_{q_m}}(x) - D^\alpha \varphi_{p_{q_m}}(\tilde{y})\| < \mathrm{e}^{-m}$ for $|\alpha| \leqq m$ and for every x, $\tilde{y}$ from $N_m = \{x \in \mathbf{R}^n : \|x\| < \varepsilon_m\}$. Let $x_m = (1/m, 0, \ldots, 0) \in \mathbf{R}^n$. By Whitney's extension theorem (see [9]), there are a C^∞-diffeomorphism $\varphi : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$ and m_0 such that

$$\varphi \mid \tau_{x_{2m+1}} N_{2m+1} = id \tag{2.1}$$

and

$$\varphi \mid \tau_{x_{2m}} N_{2m} = \tau_{x_{2m}} \circ \varphi_{p_{q_{2m}}} \circ \tau_{-x_{2m}} \tag{2.2}$$

for $m \geqq m_0$. From (2.1) we have

$$(\Lambda(\sum_{i=1}^{n} \varphi^i \partial_i)) \mid \bigcup_{m \geqq m_0} \pi^{-1}(\tau_{x_{2m+1}} N_{2m+1}) = (\Lambda X) \tag{2.3}$$

and from (2.2) we have

$$((\mathscr{F}\tau_{-x_{2m}})_* \Lambda(\sum_{i=1}^{n} \varphi^i \partial_i)) \mid \pi^{-1}(N_{2m}) = \frac{1}{2m}(\Lambda\partial_1) + + (\Lambda(\sum_{i=1}^{n} \varphi^i_{p_{q_{2m}}} \partial_i)) \tag{2.4}$$

for $m \geqq m_0$ (because $(\tau_{-x_m})_* (\varphi_m^i \partial_i) = (\varphi_m^i \circ \tau_{x_m}) \partial_i)$. The inclusion

$$\pi^{-1}(0) \subset \overline{\bigcup_{m \geqq m_0} \pi^{-1}(\tau_{x_{2m+1}} N_{2m+1})}$$

together with (2.3) implies

$$\Lambda(\sum_{i=1}^{n}\varphi^i\,\partial_i)(y) = \Lambda\,X(y)\,.$$

Now, from (2.4) we get

$$\Lambda(\sum_{i=1}^{n}\varphi^i_{p_{q_{2m}}}\,\partial_i)(y)\xrightarrow[m\to\infty]{}\Lambda(\sum_{i=1}^{n}\varphi^i\,\partial_i(y) = \Lambda\,X(y)\,.$$

Lemma 2.3 is proved.

Lemma 2.4. *Let U be a neighbourhood of* 0 *in* $\mathbf{R}$. *If* $U\times\mathbf{R}^n\ni(t,x)\to X_t(x)\in$ $\in T\mathbf{R}^n$ is a *C^∞-mapping such that for every t from U, X_t is a vector field on $\mathbf{R}^n$, then*

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Lambda\,X_t(y))_{t=0} = \Lambda\left(\frac{\mathrm{d}}{\mathrm{d}t}X_t\mid t=0\right)(y)$$

for all y from $\pi^{-1}(0)$.

Proof. Let t_p be any sequence of points in $\mathbf{R}$ converging to 0. From

$$j_0^\infty\left(\frac{X_{t_p}-X_0}{t_p}\right)\xrightarrow[p\to\infty]{} j_0^\infty\left(\frac{\mathrm{d}}{\mathrm{d}t}X_t\mid t=0\right)$$

and from Lemma 2.5 we get that

$$\lim_{p\to\infty}\frac{\Lambda\,X_{t_p}(y)-\Lambda\,X_0(y)}{t_p} = \lim_{p\to\infty}\Lambda\left(\frac{X_{t_p}-X_0}{t_p}\right)(y) = \Lambda\left(\frac{\mathrm{d}}{\mathrm{d}t}X_t\mid t=0\right)(y)$$

for every y from $\pi^{-1}(0)$.

Now, we prove a very important lemma.

Lemma 2.5. *If X, Y are vector fields on $\mathbf{R}^n$, then* $\Lambda[X,Y] = [\Lambda X,\mathscr{F}Y]$.

Remark. $\mathscr{F}Y$ is the complete lift of Y from $\mathscr{R}^n$ to $\mathscr{F}\mathbf{R}^n$, i.e. the vector field on $\mathscr{F}\mathbf{R}^n$ such that, if φ_t is a local 1-parameter group of transformations of Y defined on some open subset U of $\mathbf{R}^n$, then $\mathscr{F}\varphi_t$ is a local 1-parameter group of transformations defining $\mathscr{F}Y$ on $\mathscr{F}\mathbf{R}^n\mid U$ (see [3]); $\mathscr{F}(\cdot)$ is the mapping defined by $\mathscr{F}(Y) = \mathscr{F}Y$.

Proof. It is known that Λ, [;], $\mathscr{F}(\cdot)$ are local. By Whitney's extension theorem ([9]) there is a C^∞-mapping $\lambda:\mathbf{R}^n\to\mathbf{R}$ such that $\lambda\mid\{x\in\mathbf{R}^n:\|x\|\leqq 1\}\equiv 1$ and $\lambda\mid\{x\in\mathbf{R}^n:\|x\|\geqq 2\}\equiv 0$. Let $\tilde{Y}=\lambda Y$ and let φ_t be a global 1 $-$ parameter group of transformations of $\tilde{Y}$. By Lemma 2.4 we have

$$\Lambda[X,Y](y) = \Lambda[X,\tilde{Y}](y) = \Lambda\left(\frac{\mathrm{d}}{\mathrm{d}t}((\varphi_t)_*X)\mid t=0\right)(y) =$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}(\Lambda((\varphi_t)_*X)(y)\mid t=0 = \frac{\mathrm{d}}{\mathrm{d}t}(((\mathscr{F}\varphi_t)_*\Lambda X)(y))\mid t=0 =$$

$$= [\Lambda X,\mathscr{F}\tilde{Y}](y) = [\Lambda X,\mathscr{F}Y](y)$$

for all y from $\pi^{-1}(0)$. For all v from $\mathscr{F}\mathbf{R}^n$, $\mathscr{F}\tau_{-\pi(v)}(v)$ is a point of $\pi^{-1}(0)$, so we have

$$\begin{aligned}
\Lambda[X, Y](v) &= ((\mathscr{F}\tau_{\pi(v)})_* \Lambda((\tau_{-\pi(v)})_* [X, Y]))(v) = \\
&= ((\mathscr{F}\tau_{\pi(v)})_* \Lambda[(\tau_{-\pi(v)})_* X, (\tau_{-\pi(v)})_* Y])(v) = \\
&= \mathrm{d}\mathscr{F}\tau_{\pi(v)} \circ \Lambda[(\tau_{-\pi(v)})_* X, (\tau_{-\pi(v)})_* Y](\mathscr{F}\tau_{-\pi(v)}(v)) = \\
&= \mathrm{d}\mathscr{F}\tau_{\pi(v)} \circ [\Lambda((\tau_{-\pi(v)})_* X), \mathscr{F}((\tau_{-\pi(v)})_* Y)](\mathscr{F}\tau_{-\pi(v)}(v)) = \\
&= ((\mathscr{F}\tau_{\pi(v)})_* [(\mathscr{F}\tau_{-\pi(v)})_* \Lambda X, (\mathscr{F}\tau_{-\pi(v)})_* \mathscr{F}Y])(v) = \\
&= [\Lambda X, \mathscr{F}Y](v).
\end{aligned}$$

Lemma 2.5 is proved.

Lemma 2.6. *Let $\tilde{N}$ be a manifold. If $\tilde{N} \times \mathbf{R}^n \ni (t, x) \to \tilde{X}_t(x) \in T\mathbf{R}^n$ is a C^∞-mapping such that for every t from $\tilde{N}$, $\tilde{X}_t$ is a vector field on $\mathbf{R}^n$, then the mapping*

$$\tilde{N} \times \pi^{-1}(0) \ni (t, y) \to \Lambda\tilde{X}_t(y) \in T\mathscr{F}\mathbf{R}^n$$

is of class C^∞.

Proof. In [8], R. S. Palais and C. R. Terng showed that every natural bundle has a finite order. (A natural bundle $\mathscr{F}$ is of order r if any two embeddings $\varphi, \psi : M \to N$ such that $j^r_x\varphi = j^r_x\psi$ satisfy $\mathscr{F}\varphi(y) = \mathscr{F}\psi(y)$ for any point y of the fibre $\mathscr{F}_xM$ over x and r is the smallest number which has the above property.) Let $\mathscr{F}$ be of order r. By Lemma 2.5 and by Theorem 5.13 in [3] (the above theorem is a consequence only of the equality $\Lambda[X, Y] = [\Lambda X, \mathscr{F}Y]$) we have that if $j^r_0X = j^r_0Y$, then $\Lambda X(y) = \Lambda Y(y)$ for all y of $\pi^{-1}(0)$. Let $\tilde{X}_t(x) = \sum_{i=1}^n \alpha^i_t(x)\,\partial_i(x)$. Then

$$j^r_0\tilde{X}_t = j^r_0\left(\sum_{|p|\leq r}\sum_{i=1}^n \frac{D^p\alpha^i_t(0)}{p!}\,x^p\,\partial_i\right)$$

yields

$$\Lambda\tilde{X}_t(y) = \sum_{|p|\leq r}\sum_{i=1}^n \frac{D^p\alpha^i_t(0)}{p!}\,\Lambda(x^p\,\partial_i)(y).$$

The above equality immediately implies Lemma 2.6.

Proof of Proposition 2.2. Let y be a point from $\mathscr{F}\mathbf{R}^n$ and t a point of the manifold N. We have the equality

$$\Lambda X_t(y) = \mathrm{d}\mathscr{F}\tau_{\pi(y)} \circ \Lambda(\mathrm{d}\tau_{-\pi(y)} \circ X_t \circ \tau_{\pi(y)}) \circ \mathscr{F}\tau_{-\pi(y)}(y).$$

The mapping

$$(N \times \mathscr{F}\mathbf{R}^n) \times \mathbf{R}^n \ni ((t, y), x) \to \mathrm{d}\tau_{-\pi(y)} \circ X_t \circ \tau_{\pi(y)}(x) \in T\mathbf{R}^n$$

is of class C^∞ and $\mathrm{d}\tau_{-\pi(y)} \circ X_t \circ \tau_{\pi(y)}$ is a vector field on $\mathbf{R}^n$ for every (t, y) from the manifold $N \times \mathscr{F}\mathbf{R}^n$. We also have that $\mathscr{F}\tau_{-\pi(y)}(y)$ lies in $\pi^{-1}(0)$ for all y from $\mathscr{F}\mathbf{R}^n$.

So, by Lemma 2.6, the mapping

$$N \times \mathscr{F}\mathbf{R}^n \ni (t, y) \to \Lambda X_t(y) \in T\mathscr{F}\mathbf{R}^n)$$

is of class C^∞. Proposition 2.2 is proved.

References

[1] *J. Aczel, S. Gołąb:* Funktionalgleichungen der Theorie der Geometrischen Objekte. PWN Warszawa 1960.

[2] *D. B. A. Epstein, W. P. Thurston:* Transformation groups and natural bundles, Proc. London Math. Soc XXXVIII. 1979, 219—237.

[3] *J. Gancarzewicz:* Liftings of functions and vector fields to natural bundles, Institute of Math. Polish Academy of Sciences Preprint No 207, Warsaw March 1980, Demonst Math. to appear.

[4] *K. P. Mok:* Complete lifts of tensor fields and connections from a manifold to the linear frame bundle. Proc. London Math. Soc. *38* (1979), 72—88.

[5] *A. Morimoto:* Liftings of some types of tensor fields and connections to tangent bundles of p^r-velocities. Nagoya Math. J. *40* (1970), 13—31.

[6] *A. Morimoto:* Liftings of tensor fields and connections to tangential fibre bundles of higher order. Nagoya Math. J. *40* (1970), 85—97.

[7] *A. Nijenhuis:* Natural bundles and their general properties. Diff. Geom. in honor of K. Yano Kinokuniya Tokio 1972, 317—334.

[8] *P. S. Palais, C. L. Terng:* Natural bundles have a finite order, Topology *16* (1978), 271—277.

[9] *J. C. Tougeron:* Ideaux des fonctions differentiables Springer, Berlin 1972.

[10] *K. Yano, S. Ishihara:* Differential geometry in tangent bundles, Kodai Math. Sem. Rep. *18* (1966), 271—292.

[11] *K. Yano, S. Kobayashi:* Prolongations of tensor fields an connections to tangent bundle. J. Math. Soc. Japan *18* (1966), 194—210.

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