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ON DILATIONS AND CONTRACTIONS IN RIESZ GROUPS

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Summary. In the paper the notion of an (m, n)-transposition in a partially ordered group is introduced (m and n are positive integers). If $m < n$ $(m > n)$, then an (m, n)-transposition in an isolated partially ordered group is called a dilation (contraction). The main result establishes the relations between the (m, n)-transpositions in an isolated abelian Riesz group G and the direct decompositions of G. Further, it is shown that (m, n)-transpositions in G preserve certain convex subsets of G.

Keywords: (m, n)-transposition, dilation, contraction, isometry, Riesz group.

AMS classification: 06F.

In [8] K. L. N. Swamy introduced the notion of an intrinsic metric in an abelian lattice ordered group H by putting $d(x, y) = |x - y|$ for any x, y in H. In [9], [10] K. L. N. Swamy studied isometries in an abelian lattice ordered group H, i.e. bijections $f: H \to H$ preserving the intrinsic metric of H. Isometries in non-abelian lattice ordered groups have been studied by J. Jakubík [3], [4]. J. Jakubík proved that for every isometry f in a lattice ordered group H such that $f(0) = 0$ there exists a uniquely determined direct decomposition $H = A \times B$ of H such that $f(x) = x(A) - x(B)$ is valid for each $x \in H$ ($x(A)$ and $x(B)$ are the components of x in the direct factors A and B, respectively). W. Ch. Holland [2] showed that the only intrinsic metrics in lattice ordered groups are the multiples $n|x - y|$ of the metric $|x - y|$. Isometries in Riesz spaces and f-rings have been studied by J. T. Pairó [11], [13]. In [5] J. Jakubík and M. Kolibiar extended the results on the relations between isometries and direct decompositions to abelian distributive multilattice groups. J. Rachůnek [7] generalized the notion of an intrinsic metric and an isometry to any partially ordered group and showed that every 2-isolated abelian Riesz group G is metrized by $d(a, b) = |a - b|$ for each $a, b \in G$ (where $|x| = U(x, -x)$ for any x in G). Analogously (using the relation $n|a| = |na|$) it can be proved that in an isolated abelian Riesz group G the multiples $n|x - y|$ of the metric $|x - y|$ are intrinsic metrics in G, too. In an f-ring A with a central superunity u (central subunity s) J. T. Pairó [12] studied the mappings $F: A \to A$ satisfying $|F(x) - F(y)| = u|x - y|$ ($|F(x) - F(y)| = s|x - y|$) for each $x, y \in A$ and called them u-dilations (s-contractions) because $|F(x) - F(y)| \geqq |x - y|$ ($|F(x) - F(y)| \leqq |x - y|$) holds for each $x, y \in A$.

First we recall some notions and notation used in the paper. The set of all positive integers will be denoted by N. Let H be a partially ordered group (notation po-group). The group operation will be written additively. We denote $H^+ = \{x \in H;\ x \geqq 0\}$. If $A \subseteq H$, we denote by $U(A)$ and $L(A)$ the set of all upper bounds and the set of all lower bounds of A in H, respectively. For $A = \{a_1, \ldots, a_n\}$ we shall write $U(a_1, \ldots, a_n)$ $(L(a_1, \ldots, a_n))$ instead of $U(\{a_1, \ldots, a_n\})$ $(L(\{a_1, \ldots, a_n\}))$. For each $a \in G$, $|a| = U(a, -a)$. If $A_1, \ldots, A_n \subseteq H$, then $A_1 + \ldots + A_n = \{a_1 + \ldots + a_n;\ a_1 \in A_1, \ldots, a_n \in A_n\}$. If $A_1 = \ldots = A_n = A$, then we set $nA = A_1 + \ldots + A_n$.

If $m, n \in N$, then a bijection $f\colon H \to H$ is called an (m, n)-transposition in H if $m|f(x) - f(y)| = n|x - y|$ for each $x, y \in H$. $(1, 1)$-transposition is an isometry in H. A mapping $f\colon H \to H$ is said to be a dilation (contraction) in H if $|f(x) - f(y)| \subseteq |x - y|$ $(|f(x) - f(y)| \supseteq |x - y|)$ for each $x, y \in H$. If $a \in H$, then the mapping $f_a\colon H \to H$ defined by $f_a(x) = x + a$ for each $x \in H$ is called a right translation in H. Every right translation in H is an isometry. A mapping $f\colon H \to H$ is called homogeneous if $f(0) = 0$.

We say that a po-group H is isolated if $a \in H$ and $na \geqq 0$ for some $n \in N$ imply $a \geqq 0$. A po-group H is called directed if $U(x, y) = \emptyset$ and $L(x, y) \neq \emptyset$ for each $x, y \in H$. A Riesz group is any po-group H which is directed and has the Riesz interpolation property, i.e. for each $a_i, b_j \in H$ $(i, j = 1, 2)$ such that $a_i \leqq b_j$ $(i, j = 1, 2)$ there exists $c \in H$ such that $a_i \leqq c \leqq b_j$ $(i, j = 1, 2)$. See [1].

1. Lemma. *Let G be an isolated po-group, $a, b \in G$, $m, n \in N$. Let $m|a| = n|b|$, $m > n$. Then $|b| \subseteq |a|$.*

Proof. Let $x \in |b|$. Then $nx \in n|b| = m|a|$. Thus $nx = y_1 + \ldots + y_m$, where $y_1, \ldots, y_m \in |a|$. Since G is isolated, $|a| \subseteq U(0)$. Then $y_i \geqq 0$ for $i = 1, \ldots, m$. From the relations $y_1 \geqq a$, $y_1 \geqq -a, \ldots, y_n \geqq a$, $y_n \geqq -a$, $y_{n+1} \geqq 0, \ldots, y_m \geqq 0$ for the element $nx = y_1 + \ldots + y_m$ we obtain $nx \geqq na$, $nx \geqq -na$. Since G is isolated, we have $x \in |a|$.

2. Corollary. *Let G be an isolated po-group and let f be an (m, n)-transposition in G.*

(i) *If $m > n$, then f is a contraction.*

(ii) *If $m < n$, then f is a dilation.*

If $m > n$ $(m < n)$, then an (m, n)-transposition in an isolated po-group is called an (m, n)-contraction ((m, n)-dilation).

3. Theorem. *Let f be an (m, n)-transposition in a po-group H. Then there exists a uniquely determined homogeneous (m, n)-transposition h in H such that $f(x) = h(x) + f(0)$ for each $x \in H$.*

Proof. If we put $h(x) = f(x) - f(0)$ for each $x \in H$, then h is clearly the required homogeneous (m, n)-transposition.

So every (m, n)-transposition can be uniquely represented as a composition of a homogeneous (m, n)-transposition and a right translation.

4. Theorem. *The set of all transpositions in a po-group H is a group with respect to the composition of mappings.*

Proof. It is easy to verify that the composition of an (m_1, n_1)-transposition and an (m_2, n_2)-transposition is an $(m_1 m_2, n_1 n_2)$-transposition. The inverse of an (m, n)-transposition is an (n, m)-transposition.

5. Lemma. *Let H be a po-group, $A, B_1, \ldots, B_n \subseteq H$ and let $A = B_1 + \ldots + B_n$. An element $u \in H$ is the least element of A if and only if $u = u_1 + \ldots + u_n$, where u_i is the least element of B_i for $i = 1, \ldots, n$.*

Proof. a) Let u be the least element of A and let $A = B_1 + \ldots + B_n$. Then $u = u_1 + \ldots + u_n$, where $u_i \in B_i$ for $i = 1, \ldots, n$. Assume that u_i is not the least element of B_i for some $i \in \{1, \ldots, n\}$. Then there exists $u'_i \in B_i$ such that either $u'_i \leqq u_i$ or $u'_i \parallel u_i$.

If $u'_i \leqq u_i$, then $u_1 + \ldots + u_{i-1} + u'_i + u_{i+1} + \ldots + u_n \leqq u_1 + \ldots + u_n = u$, which contradicts the assumption that u is the least element of A.

If $u'_i \parallel u_i$, then $u_1 + \ldots + u_{i-1} + u'_i + u_{i+1} + \ldots + u_n \parallel u$, a contradiction.

Thus u_i is the least element of B_i for $i = 1, \ldots, n$.

b) Let u_i be the least element of B_i for $i = 1, \ldots, n$. Let v be an arbitrary element of A. Then $v = v_1 + \ldots + v_n$, where $v_i \in B_i$ for $i = 1, \ldots, n$. Since $v_i \geqq u_i$ for $i = 1, \ldots, n$, we have $v = v_1 + \ldots + v_n \geqq u_1 + \ldots + u_n$. Thus $u = u_1 + \ldots + u_n$ is the least element of A.

6. Theorem. *Let F be an isolated po-group, $m, n \in N$ and let $f: F \to F$ be a mapping such that $m|f(x) - f(y)| = n|x - y|$ for each $x, y \in F$. Then f is an injection.*

Proof. Let $x, y \in F$ and let $f(x) = f(y)$. Then $n|x - y| = m|f(x) - f(y)| = m|0| = m\,U(0) = U(0)$. By 5, $0 = nb$, where b is the least element of $|x - y|$. Since F is isolated, we have $b = 0$. Then the relations $0 \geqq x - y$, $0 \geqq y - x$ yield $x = y$.

7. Lemma. *Let f be a homogeneous (m, n)-transposition in an isolated abelian directed group F. Then*

(i) *for each $c \in F$ there exists only one element $d \in F$ such that $mc = nd$,*

(ii) *for each $c' \in F$ there exists only one element $d' \in F$ such that $nc' = md'$.*

Proof. (i) Let $b \in F^+$ and let $a = f^{-1}(b)$. Then $n|a| = m|f(a)| = m|b| = m\,U(b) = U(mb)$. Since mb is the least element of $U(mb)$, 5 implies that $mb = na_1$, where a_1 is the least element of $|a|$.

Let $c \in F$. Since F is a directed group, $c = c_1 - c_2$ for some $c_1, c_2 \in F^+$ (cf. [1], Chap. II, Proposition 1). Then $mc = mc_1 - mc_2$. Further, there exist elements $c'_1, c'_2 \in F$ such that $mc_1 = nc'_1$, $mc_2 = nc'_2$. Thus $mc = n(c'_1 - c'_2)$.

Let $mc = nd_1$ and $mc = nd_2$ for some $d_1, d_2 \in F$. Then $n(d_1 - d_2) = 0$. Since F is isolated, we have $d_1 = d_2$. (ii) Since the mapping f^{-1} is an (n, m)-transposition, the assertion (ii) follows from (i).

Let G be a po-group, $a \in G$. For $m, n \in N$ let there exist only one element $b \in G$ such that $ma = nb$. Then b will be denoted by ma/n.

If G is an isolated Riesz group, then the relation $n|a| = |na|$ is valid for each $a \in G$, $n \in N$ (cf. [1], p. 114).

The following example shows that in a non-isolated abelian Riesz group G the following relations can be valid:

(i) $m|a| \neq |ma|$ for some $m \in N$, $a \in G$,

(ii) $n|b| = n|c|$ and $|b| \neq |c|$ for some $n \in N$, $b, c \in G$.

Example. Let G_1 be the additive group of all real numbers with the natural order and let G_2 be the additive group of residue classes modulo 4 with the trivial order. Let $G = G_1 \,.\, G_2$ be the lexicografic product of the po-groups G_1, G_2. Then G is a non-isolated abelian Riesz group.

Let $a = (0, \bar{1})$. Then $-a = (0, \bar{3})$, $|a| = \{(x, y) \in G;\ x > 0\}$, $2|a| = |a|$, $2a = -2a = (0, \bar{2})$, $|2a| = U((0, \bar{2}))$. Since $2a = (0, \bar{2}) \in |2a|$, $2a \notin 2|a|$, we have $2|a| \neq |2a|$.

Let $b = (0, \bar{0})$, $c = (0, \bar{2})$. Then $|b| = U((0, \bar{0}))$, $|c| = U((0, \bar{2}))$, $2|b| = U((0, \bar{0}))$, $2|c| = U((0, \bar{0}))$. Thus $2|b| = 2|c|$, but $|b| \neq |c|$.

Throughout the rest of this paper let G be an isolated abelian Riesz group.

8. Lemma. *Let $a, b \in G$, $n \in N$. If $n|a| = n|b|$, then $|a| = |b|$.*

Proof. Let $a, b \in G$, $n \in N$ and let $n|a| = n|b|$. If $x \in |a|$, then $nx \in n|a| = n|b| = |nb|$. From this we obtain $nx \geqq nb$, $nx \geqq -nb$. Since G is isolated, we get $x \geqq b$, $x \geqq -b$. Thus $x \in |b|$. Therefore $|a| \subseteq |b|$.

Analogously, $|b| \subseteq |a|$.

9. Lemma. *Let f be a homogeneous (m, n)-transposition in G. For each $x \in G$ define $g(x) = m f(x)/n$. Then g is a homogeneous isometry in G.*

Proof. From 7 it follows that the mapping g is well defined. Let $x, y \in G$ and let $g(x) = g(y)$. Then $m f(x)/n = m f(y)/n$. Thus $m(f(x) - f(y)) = 0$. Since G is isolated, we have $f(x) = f(y)$. Hence $x = y$. Let $z \in G$. By 7, there exists nz/m in G. Let $u = f^{-1}(nz/m)$. Then $g(u) = z$. Hence g is a bijection. Clearly $g(0) = 0$. Further we have $n|g(x) - g(y)| = n|m f(x)/n - m f(y)/n| = |n(m f(x)/n - m f(y)/n)| = |m(f(x) - f(y))| = m|f(x) - f(y)| = n|x - y|$. By 8, we obtain $|g(x) - g(y)| = |x - y|$.

The isometry defined in Lemma 8 is called the isometry associated with the given homogeneous (m, n)-transposition.

If $C = A \times B$ is a direct decomposition of a po-group C, then for $x \in C$ we denote by $x(A)$ and $x(B)$ the components of x in the direct factors A and B, respectively.

10. Theorem. *Let G be an isolated abelian Riesz group.*

(i) *Let f be a homogeneous (m, n)-transposition in G. Then there exists a direct decomposition $G = A \times B$ of G such that $f(x) = nx(A)/m - nx(B)/m$ for each $x \in G$.*

(ii) *Let $m, n \in N$ and for each $x \in G$ let the element nx/m in G exist. Let $G = P \times Q$ be a direct decomposition of G. If we put $g(x) = nx(P)/m - nx(Q)/m$ for each $x \in G$, then g is a homogeneous (m, n)-transposition in G.*

Proof. (i) This is a consequence of 9 and Theorem 18 [6]. (ii) Clearly, g is a bijection and $g(0) = 0$. It is easy to verify that $|z| = |z(P)| + |z(Q)|$ for each $z \in G$. Let $x, y \in G$. Then $m|g(x) - g(y)| = m|nx(P)/m - nx(Q)/m - ny(P)/m + ny(Q)/m| = n|(x(P) - x(Q)) - (y(P) - y(Q))| = n(|x(P) - y(P)| + |-(x(Q) - y(Q))|) = n(|(x - y)(P)| + |(x - y)(Q)|) = n|x - y|$.

11. Lemma. *Let f be a homogeneous isometry in G, $m, n \in N$. For each $x \in G$ let nx/m in G exist. If we put $g(x) = nf(x)/m$ for each $x \in G$, then g is a homogeneous (m, n)-transposition in G.*

Proof. This is a consequence of Theorem 10.

12. Theorem. *Let f be an (m, n)-transposition in G. Then $f(U(L(x, y)) \cap L(U(x, y))) = U(L(f(x), f(y))) \cap L(U(f(x), f(y)))$ for each $x, y \in G$.*

Proof. If f is a translation, the assertion obviously holds. In view of 3 it suffices to consider the case when f is a homogeneous (m, n)-transposition in G.

Let g be the isometry associated with f. Then $g(z) = mf(z)/n$ for each $z \in G$. Let $x, y \in G$. Let $a \in U(L(x, y)) \cap L(U(x, y))$, $u' \in L(f(x), f(y))$, $v' \in U(f(x), f(y))$. By 7, the elements $u = mu'/n$, $v = mv'/n$ in G exist. Since G is isolated, we have $v \in U(g(x), g(y))$, $u \in L(g(x), g(y))$. By Theorem 22 [6], $g(U(L(x, y)) \cap L(U(x, y))) = U(L(g(x), g(y))) \cap L(U(g(x), g(y)))$. Thus $u \leqq g(a) \leqq v$. From this we obtain $u' = nu/m \leqq ng(a)/m = f(a) \leqq nv/m = v'$. Therefore $f(a) \in U(L(f(x), f(y))) \cap L(U(f(x), f(y)))$.

If we consider f^{-1} instead of f, we can prove that $U(L(f(x), f(y))) \cap L(U(f(x), f(y))) \subseteq f(U(L(x, y)) \cap L(U(x, y)))$.

13. Theorem. *Let f be a homogeneous (m, n)-transposition in G and let $H \subseteq G$. Then H is a directed convex subset of G if and only if $f(H)$ is a directed convex subset of G.*

Proof. Let H be a directed convex subset of G. Let $f(x) \leqq f(y) \leqq f(z)$ for some $x, z \in H$, $y \in G$. By 7, the elements $m f(x)/n$, $m f(y)/n$, $m f(z)/n$ in G exist. Let g be the isometry associated with f. Since G is isolated, we have $g(x) \leqq g(y) \leqq g(z)$. By Lemma 26 [6], $g(H)$ is a directed convex subset of G. Then $g(y) \in g(H)$. From this we get $y \in H$. Thus $f(y) \in f(H)$. Hence $f(H)$ is a convex subset of G.

Let $f(a), f(b) \in f(H)$. Then the elements $m f(a)/n = g(a)$, $m f(b)/n = g(b)$ in G exist. Since $g(H)$ is a directed subset of G, there exist elements $u, v \in H$ such that $g(v) \in U(g(a), g(b))$, $g(u) \in L(g(a), g(b))$. Since G is isolated, we have $f(v) \in U(f(a), f(b))$, $f(u) \in L(f(a), f(b))$. Thus $f(H)$ is a directed subset of G.

If we consider f^{-1}, we can prove the sufficiency of the condition.

14. Lemma. *Let f be a homogeneous (m, n)-transposition in G and let g be the isometry associated with f. Let C be a directed convex subgroup of G. Then $f(C) = g(C)$.*

Proof. Let C be a directed convex subgroup of G. By 10, f and g are group homomorphisms. From this and from 13 it follows that $f(C)$, $g(C)$ are directed convex subgroups of G. Let $z \in g(C)$. Then there exist elements $u, v \in g(C)$ such that $v \in U(0, z)$, $u \in L(0, z)$. By 7, the elements mv/n, mz/n, mu/n in G exist. Since G is isolated, we have $mu/n \leqq mz/n \leqq mv/n$. From the relations $0 \leqq mv/n \leqq mv$, $mu \leqq mu/n \leqq 0$ and from the convexity of $g(C)$ we obtain that $mv/n, mu/n \in g(C)$. Hence $mz/n \in g(C)$. Let $z' = g^{-1}(mz/n)$. Then $z' \in C$, $f(z') = n\, g(z')/m = z \in f(C)$. Thus $g(C) \subseteq f(C)$.

Analogously we can prove can prove that $f(C) \subseteq g(C)$.

15. Theorem. *Let f be a homogeneous (m, n)-transposition in G and let C be a directed convex subgroup of G. Then $f(C) = C$.*

Proof. Let g be the isometry associated with f. Let $x \in C$. Then there exist $u, v \in C$ such that $u \in L(x, 0)$, $v \in U(x, 0)$. By 10, there exists a direct decomposition $G = A \times B$ of G such that $g(z) = z(A) - z(B)$ for each $z \in G$. Then we have $v(A) \geqq x(A)$, $v(B) \geqq x(B)$, $v(A) \geqq 0$, $v(B) \geqq 0$, $u(A) \leqq x(A)$, $u(B) \leqq x(B)$, $u(A) \leqq 0$, $u(B) \leqq 0$. This implies $v \geqq x(A) \geqq u$, $v \geqq x(B) \geqq u$. By the convexity of C, $x(A)$, $x(B) \in C$. Since $x(A) - x(B) \in C$ and $g(x(A) - x(B)) = x$, we have $C \subseteq g(C)$.

Let $y' \in g(C)$. Then $y' = g(y)$ for some $y \in C$. Since $y(A), y(B) \in C$, we obtain $y' = y(A) - y(B) \in C$. Thus $g(C) \subseteq C$.

Therefore $g(C) = C$. In view of 14 we obtain $f(C) = C$.

16. Theorem. *Let f be an (m, n)-transposition in an isolated abelian po-group F, $a, c \in F$, $a \leqq c$.*

(i) *If $f(a) \leqq f(c)$, then $f([a, c]) = [f(a), f(c)]$.*

(ii) *If $f(a) \geqq f(c)$, then $f([a, c]) = [f(c), f(a)]$.*

Proof. (i) From the assumption we have $c - a \geqq 0$, $f(c) - f(a) \geqq 0$. Since $n|a - c| = m|f(a) - f(c)|$ we have $nU(c - a) = mU(f(c) - f(a))$. Thus $n(c - a) = m(f(c) - f(a))$. Hence $-mf(c) + nc = -mf(a) + na$.

Let $b \in [a, c]$. Since $b - a \geqq 0$, from $n|b - a| = m|f(b) - f(a)|$ we get $n(b - a) = d_1 + \ldots + d_m$, where $d_1, \ldots, d_m \in |f(b) - f(a)|$. Then $d_i \geqq f(b) - f(a)$ for $i = 1, \ldots, m$. Thus $n(b - a) \geqq m(f(b) - f(a))$. This implies $-mf(b) + nb \geqq -mf(a) + na = -mf(c) + nc$. Hence $m(f(c) - f(b)) \geqq n(c - b) \geqq 0$. Since F is isolated, we have $f(c) \geqq f(b)$. The relations $c - b \geqq 0$, $n|c - b| = m|f(c) - f(b)|$ imply that $n(c - b) = m(f(c) - f(b))$. Hence $-mf(b) + nb = -mf(c) + nc = -mf(a) + na$. Thus $0 \leqq n(b - a) = m(f(b) - f(a))$. Hence $f(b) \geqq f(a)$. Therefore $f([a, c]) \subseteq [f(a), f(c)]$.

Let $b' \in [f(a), f(c)]$, $b = f^{-1}(b')$. Since $f(b) - f(a) \geqq 0$, the relation $n|b - a| = m|f(b) - f(a)|$ yields $m(f(b) - f(a)) \geqq nb - na$. Then $-mf(b) + nb \leqq -mf(a) + na = -mf(c) + nc$. From this we get $0 \leqq mf(c) - mf(b) \leqq nc - nb$. Since F is isolated, we have $c \geqq b$. Analogously we can prove that $a \leqq b$. Hence $b \in [a, c]$. Therefore $[f(a), f(c)] \subseteq f(([a, c])$.

The assertion (ii) can be proved analogously.

17. Theorem. *Let f be a homogeneous (m, n)-transposition in G, $m > 1$, $n > 1$. Let g be the isometry associated with f and for each $x \in G$ let x/n or x/m in G exist. Then there exist a homogeneous $(1, n)$-dilation f_1 and a homogeneous $(m, 1)$-contraction f_2 such that $f(x) = f_2(f_1(g(x)))$ for each $x \in G$.*

Proof. Let $y \in G$. From 7 it follows that y/m exists in G if and only if y/n exists in G. Put $f_1(x) = nx$ and $f_2(x) = x/m$ for each $x \in G$. Since the identical mapping is a homogeneous isometry, 11 implies that f_1 is a homogeneous $(1, n)$-dilation and f_2 is a homogeneous $(m, 1)$-contraction in G. Finally, we have $f_2(f_1(g(x))) = f_2(ng(x)) = ng(x)/m = f(x)$.

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Súhrn

ON DILATIONS AND CONTRACTIONS IN RIESZ GROUPS

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V článku je zavedený pojem (m, n)-transpozície v čiastočne usporiadanej grupe (m, n sú kladné celé čísla). Pre $n > m$ ($n < m$) je (m, n)-transpozícia v izolovanej čiastočne usporiadanej grupe dilatáciou (kontrakciou).

Hlavný výsledok stanovuje vzťahy medzi (m, n)-transpozíciami v izolovanej abelovskej Rieszovej grupe G a priamymi rozkladmi G. Ďalej je ukázané, že (m, n)-transpozície v G zachovávajú určité konvexné podmnožiny G.

Резюме

О ДИЛАТАЦИЯХ И СЖАТИЯХ В ГРУППАХ РИССА

MILAN JASEM

В статье вводится понятие (m, n)-транспозиции в частично упорядоченной группе (m и n — положительные целые числа). Если $n > m$ ($m > n$), то (m, n)-транспозиция в изолированной частично упорядоченной группе является дилатацией (сжатием).

Главный результат устанавливает соотношения между (m, n)-транспозициями в изолированной абелевой группе Рисса G и прямыми разложениями G. Кроме того показано, что транспозиции в G сохраняют некоторые выпуклые подмножества в G.

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