Archivum Mathematicum

Sergey B. Karavashkin

Transformation of divergence theorem in dynamical fields

Archivum Mathematicum, Vol. 37 (2001), No. 3, 233--243

Persistent URL: http://dml.cz/dmlcz/107800

ARCHIVUM MATHEMATICUM (BRNO)
Tomus 37 (2001), 233 – 243

TRANSFORMATION OF DIVERGENCE THEOREM IN DYNAMICAL FIELDS

SERGEY B. KARAVASHKIN

Abstract. In this paper we will study the flux and the divergence of vector in dynamical fields, on the basis of conventional divergence definition and using the conventional method to find the vector flux. We will reveal that vector flux and divergence of vector do not vanish in dynamical fields. In terms of conventional EM field formalism, we will show the changes appearing in dynamical fields.

1. Introduction

"As long ago as in 19th century, the scientists began feeling about for the main mathematical 'body'. This 'body' contains the following substantial concepts: gradient, potential, flux, divergence, curl, circulation, and some others. The knowledge of these concepts is urgently needed when studying the physics, mechanics, and a number of engineering disciplines" [1, p.5].

In the number of mentioned basic concepts, the finding of divergence of vector is an unalienable part of the EM field theory formalism. Using it, we express the conservation laws of charge, current, flux, energy, etc. Using the theorems basing on it, we develop the methods to study the distribution, propagation, attenuation of EM processes.

It is thought to be absolutely proved that in a charge-free region

$$\operatorname{div} \overrightarrow{F}(\overrightarrow{r}, t) = 0 \,, \tag{1}$$

where $\overrightarrow{F}(\overrightarrow{r}, t)$ is some vector whose parameters depend on coordinates and time. Rather, in the initial formulation of Poisson theorem, $\overrightarrow{F}(\overrightarrow{r}, t)$ does not depend on time t, since "the operation $\overrightarrow{\nabla} \cdot \overrightarrow{U} \equiv \operatorname{div} \overrightarrow{U}$ (divergence of $\overrightarrow{U}$) relates to the sources of the vector field $\overrightarrow{U}(\overrightarrow{r})$" [2, p.29], not $\overrightarrow{U}(\overrightarrow{r}, t)$.

The more, the initial definition of vector flux, on whose basis the divergence concept is formulated, also means the field being stationary. Particularly, in [1,

---

2000 *Mathematics Subject Classification*: 76A02, 78A02, 78A25, 78A40.
Key words and phrases: theoretical physics, mathematical physics, wave physics, vector algebra.
Received December 1, 1999.

p.91], when formulating its definition, it is supposed that “the vector flux depends on the size of surface, on the value of vector $\overrightarrow{A}(P)$, and on the direction of this vector relatively to the perpendicular to the surface”. It means, in the initial definitions, the time dependence for the flux is absent too.

Necessity to take the divergence of flux in dynamical fields requires to broaden the domain, where the definition of divergence is true. Due to it, also for the vectors $\overrightarrow{F}(\overrightarrow{r}, t)$, by default, its main definition was recognised valid in the following form: “The divergence of vector function of the point $\overrightarrow{F}(\overrightarrow{r})$ is *scalar* function of the point, defined as

$$\operatorname{div} \overrightarrow{F}(\overrightarrow{r}, t) = \overrightarrow{\nabla} \cdot \overrightarrow{F} = \lim_{\delta \to 0} \frac{\int_{S_1} d\overrightarrow{s} \cdot \overrightarrow{F}(\overrightarrow{r_1})}{\int_{V_1} dV}, \tag{2}$$

where V_1 is the region containing the point $(\overrightarrow{r})$; S_1 is the closed surface bounding the region V_1; δ is the most distance from the point $(\overrightarrow{r})$ to the point on the surface S_1” [3, p.166]. With it, also by default, all the theorems basing on the definition of divergence were kept unchanged, in that number the mentioned Poisson theorem.

As a result, accounting the electrical vector $\overrightarrow{E}$ and magnetic vector $\overrightarrow{H}$ time-dependent, Poisson equation in the form (1) was included to the Maxwell system for dynamical fields free of charges and currents. Particularly, Landau writes: “The electromagnetic field in vacuum is defined by Maxwell equations in which we have to put $\rho = 0$; $j = 0$ (here ρ is the charge density and j is the electric current density in the studied domain – S.K.). Write them down again:

$$\begin{aligned} \operatorname{curl} \overrightarrow{E} &= -\frac{1}{c}\frac{\partial \overrightarrow{H}}{\partial t}; \quad \operatorname{div} \overrightarrow{E} = 0\,; \\ \operatorname{curl} \overrightarrow{H} &= \frac{1}{c}\frac{\partial \overrightarrow{E}}{\partial t}; \quad \operatorname{div} \overrightarrow{H} = 0\,. \end{aligned} \tag{3}$$

Solutions of these equations can be non-zero. It means that an electromagnetic field can exist even in absence of whatever charges” [4, p.143].

However, the force lines of stationary and dynamical fields in general case essentially differ. This is well-known not only in case of electrodynamics, but e.g. in hydrodynamics: “In general case, the force lines do not coincide with the trajectories. The family of lines of current $x^i = x^i\left(c^1, c^2, c^3, \lambda, t\right)$ (where c^1, c^2, c^3 are the generalised parameters and $\lambda\,(s, L)$ is some function of the line of current L at the length of the arc s along the line of current – S.K.) is time-dependent and different at different moments. However, the parameter t is included to the right parts of the differential equations of lines of current and of the differential equations determining the regularity of motion or trajectories of particles - only in the case of unsteady motions. In case of steady motions, the difference between these equations disappears” [5, p.41].

This difference leads to such a fact that, for example, in the basic equations of the method used for studying the EM radiation attenuating above the Earth surface, the vertical dipole radiation is presented as [6, p.130]:

$$E_z\left(x,y,0\right)=w\left(r\right)\frac{e^{-jk_0r}}{r} \tag{4}$$

(where $w\left(r\right)$ is the 'attenuation factor' and k_0 is the wave coefficient – S.K.), in which the time dependence is deleted from a conventional expression for a travelling wave, and the radiation field is considered as a stationary one. Such a simplification allows the authors to express "the field at the arbitrary point of a plane with the help of Green function" [6, p.130].

At the same time, when studying the vector potential $\overrightarrow{A}\left(\overrightarrow{r},t\right)$ produced in the far of dipole having the dipole momentum $d\left(\tau_0\right)$ (where τ_0 is a time parameter), as a result of calculation, Levitch [7, p.107] obtains

$$\operatorname{div}\overrightarrow{A}\left(\overrightarrow{r},t\right)=-\frac{1}{c^2r}\overrightarrow{\ddot{d}}\left(\tau_0\right)\cdot\overrightarrow{n}=-\frac{1}{c}\frac{\partial\overrightarrow{A}}{\partial t}\cdot\overrightarrow{n} \tag{5}$$

(where c is the light velocity; r is the distance from dipole; $\overrightarrow{n}$ is the direction from the dipole to the studied point). The expression (5) basically differs from (1) by having a time-dependent right-hand part. This is due to the difference between dynamical and stationary fields shown above.

In some cases, as for example, when studying the attenuation of EM radiation from a stationary source, the difference is negligible. But in a number of problems, if the source amplitude varied in time or if the problem required knowing the momentary wave parameters value at some point or region, such an approximation becomes incorrect, and we need to take into account the dynamical pattern of process. The more that in the beginning of the 20th century Eichenwald noted that for Pointing vector also [8, p.123], "if we apply the equation

$$\frac{\partial}{\partial t}\int W d\tau=-\int f_n ds \tag{6}$$

(where f_n is the projection of vector f to the external normal to the surface element ds; W is the density of EM energy; τ is a picked out region – S.K.) to a finite region, then, generally speaking, its right part will be non-zero, and the electromagnetic energy within this region will vary in time".

This problem is solvable if introducing the correspondence between the conventional definition of divergence (2) and the formulation of theorems proved on the basis of this definition. This will generalise the divergence definition per se for the dynamical case and will refine the methods to investigate the EM fields in space.

The investigation that we will carry out in this paper will be targeted to this problem, and the most important its results will be described below.

2. Preliminary analysis for the case of 1D flux

The divergence of vector in dynamical fields is convenient to be studied, beginning with a simplified model of 1D flux. Proceeding from the fact that the divergence definition "relates to any vector function, not only to the electrical field; to denote this function, we will use the symbol $F\left(x,y,z\right)$. In other words, in the meanwhile we will give a preference for mathematics before physics and will name F simply - a vector function in general form, meaning, of course, 3D space" [9, p.69].

Let in some bounded connective, source-free space region Ω propagate a plane-parallel wave, whose force vector $\overrightarrow{F}(x,t)$ has a conventional form

$$F(x,t) = F_0 \sin(\omega t - kx), \qquad (7)$$

where ω is the frequency of the given vector variation and k is the wave coefficient. Pick out of this region four surfaces a_0, a_1, a_2, a_3 perpendicular to the wave propagation direction $\overrightarrow{n}$, and form with their help three picked out regions V_{01}, V_{02}, V_{03} bounded by the corresponding surfaces and the lateral surface connecting them, as shown in Fig.1, above. Considering the 1D pattern of wave and that $\overrightarrow{F}(x,t)$ is parallel to the lateral surface of the picked out regions, further we will not take into account the lateral surfaces.

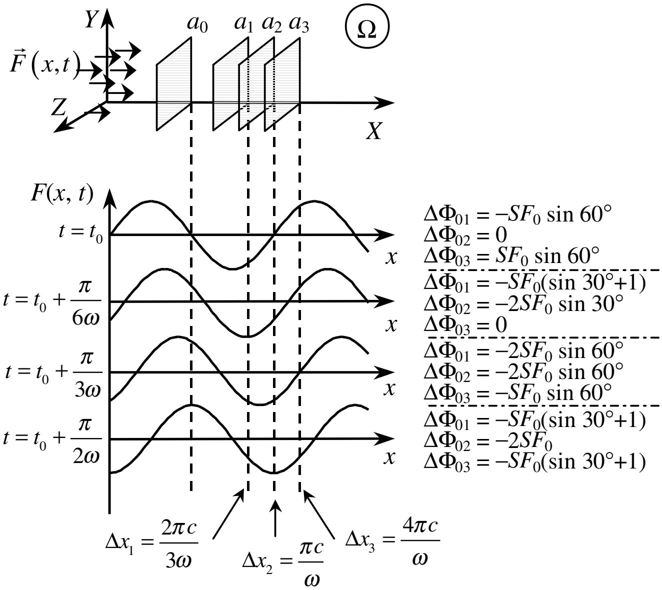

Fig. 1. The time-dependent diagram for the investigation of vector flux through the picked out space

Basing on this model and on the conventional divergence definition (2), determine the flux of vector $\Delta\Phi_{0i} = \Phi_i - \Phi_0$ and the specific flux of vector $G_i = \Delta\Phi_{0i}/V_{0i}$, where Φ_0 is the flux going through the surface a_0, and $i = 1, 2, 3$. We will use the conventional method described, e.g., in [1, pp.90-91] and [9, pp.70-71]. Since the picked out regions V_{01}, V_{02}, V_{03} are finite, then G_i has the form

$$G_i = \frac{\int_{S_1} d\overrightarrow{s} \cdot \overrightarrow{F}(s)}{\int_{V_1} dV} = \frac{\Delta\Phi_{0i}}{V_i}. \qquad (8)$$

Plot the diagram of $\overrightarrow{F}(x,t)$ space-time variation, remembering the progressive pattern of wave (7). Basing on the diagrams of $F(x,t)$ shown in Fig.1, centre, we can easy determine $\Delta\Phi_{0i}$, since the integration amounts to a simple multiplying of $\overrightarrow{F}(x,t)$ parameters at a picked out moment of time t at a studied surface into the size of this surface. The obtained results calculated for the moment fixed in Fig.1, left are shown in Fig.1, right. As we can see from calculation, despite we used the conventional definition and method, the obtained value $\Delta\Phi_{0i}$ in general case does not vanish at the boundaries of the picked out regions. It is different both for all the surfaces a_1, a_2, a_3 and all the moments of time, though $\Delta\Phi_{0i}$ was calculated relatively to the surface a_0 common for all the regions and simultaneously for all the picked out regions. The regularity $\Delta\Phi_{0i}(t)$ shown in Fig.2 reflects this peculiarity; it is plotted on the basis of calculation of Fig.1. As we see from Fig.2, both amplitude and phase of $\Delta\Phi_{0i}(t)$ are different for the picked out regions, the same as these parameters are different for $G_i(t)$ whose regularity is shown in Fig.3. The more, when diminishing the size of picked out region, the amplitude of G_i increases. It confirms that the flux through a picked out region is time-inconstant in dynamical fields, and the fact of $\Delta\Phi_{0i}$ time-variation is conditioned not by the space parameters of flux, but namely by the progressive pattern of wave, which we can easy prove mathematically.

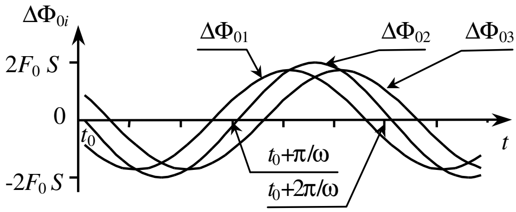

Fig.2. The time dependencies of the flux through the picked out space on the size of given region

Actually, for any region picked out at a moment t we have

$$\begin{aligned}\Delta\Phi_{0i} &= [F(x_i,t) - F(x_0,t)]\,S = \\ &= F_0 S\,[\sin(\omega t - kx_i) - \sin(\omega t - kx_0)] = \\ &= -2F_0 S \sin k\frac{\Delta x_i}{2}\cos\left(\omega t - kx_i - k\frac{\Delta x_i}{2}\right)\end{aligned} \tag{9}$$

(where $\Delta x_i = x_i - x_0$ is the distance between the surfaces a_i and a_0). Considering that in the studied case $V_i = \Delta x_i S$, we obtain (8) for G_i as

$$G_i = -\frac{2F_0 S \sin k\frac{\Delta x_i}{2}}{\Delta x_i}\cos\left(\omega t - kx_i - k\frac{\Delta x_i}{2}\right). \tag{10}$$

Because the regularity $G_i(\Delta x_i)$ is conditioned by the finite velocity of the wave space-propagation, we can express Δx_i through the time characteristic of the wave delay Δt_i:

$$k\Delta x_i = \omega\Delta t_i\ . \tag{11}$$

Substituting (11) into (10), we obtain

$$G_i = \frac{\omega F_0}{c} \frac{\sin \omega \frac{\Delta t_i}{2}}{\omega \frac{\Delta t_i}{2}} \cos\left(\omega t - kx_0 - \omega \frac{\Delta t_i}{2}\right) \tag{12}$$

(where c is the wave propagation velocity). As we see from (12), both amplitude and phase of G_i depend on Δt_i; it completely corresponds to the plot shown in Fig.3. With it the specific flux amplitude depends on the ratio of sine of argument $\omega \Delta t_i/2$ to this argument, determining the first significant limit. It backgrounds that the inequality of flux to zero in dynamical fields is the objective fact.

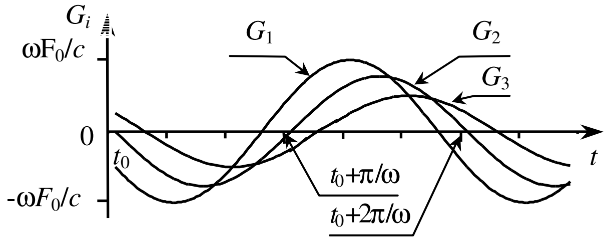

Fig.3. The time dependencies of the specific flux through the picked out space on the size of given region

To determine the divergence on the basis of (12) in accord to (2), it is sufficient to find the limit of G_i at $\Delta t_i \to 0$. Taking it for any of picked out regions at the moment t, we obtain

$$\begin{aligned} \operatorname{div} \vec{F}(x,t) &= -\frac{\omega F_0}{c} \lim_{\Delta t_i \to 0} \frac{\sin \omega \frac{\Delta t_i}{2}}{\omega \frac{\Delta t_i}{2}} \cos\left(\omega t - kx - \omega \frac{\Delta t_i}{2}\right) = \\ &= -\frac{\omega F_0}{c} \cos\left(\omega t - kx\right) . \end{aligned} \tag{13}$$

Thus, similarly to the flux, the divergence of $\vec{F}(x,t)$ does not vanish too, and Levitch's expression (5) in case of 1D flux and harmonic time-dependence of longitudinal vector $\vec{A}(\vec{r},t)$ will completely correspond to (13). On one hand, it corroborates the result obtained by Levitch for the vector potential of dipole radiator, and on the other hand, it shows more general pattern of the result.

So we can state that at least for any 1D flux in dynamical fields the divergence of its vector does not vanish, being inconsistent with the conventional concepts basing on Poisson theorem.

We should note again, as we mentioned in the introduction, the difference between the obtained results and conventional concept is conditioned by the fact that in all the theorems proved before on the basis of divergence definition, in fact only stationary fields and stable fluxes were considered. While in dynamical fields, not only the density of space-distribution of the force lines, but also the wave propagation velocity, being finite and causing the phase delay, have effect on the value of divergence. The presence of this phase delay causes that the momentary values of $\overrightarrow{F}(x,t)$ amplitude at the opposite surfaces are different. Because of it, the flux becomes dependent on the size of picked out region. It evidences that, when finding the divergence in dynamical fields, in general case, one should take into account the time characteristic of the flux. And when passing to the stationary fields, i.e. at $\omega \to 0$ and/or $c \to \infty$, the right part of (13) automatically vanishes, coming to the complete accord with conventional concept (1).

3. The complete proof of the divergence theorem in dynamical fields

Generalising the above consideration of 1D flux, consider a general case of arbitrary flux of vector $\overrightarrow{F}(\overrightarrow{r},t)$. Let in some connective source-free space Ω propagate some flux whose vector $\overrightarrow{F}(\overrightarrow{r},t)$ coincides with the direction $\overrightarrow{n}$ of flux, as shown in Fig.4; its time dependence is

$$F\,(r,t) = F_0 \sin\left(\omega t - kr\right)\,. \tag{14}$$

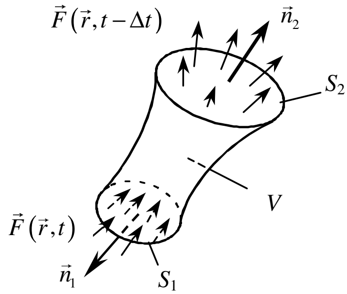

Fig.4. General form of the time-variable field tube of the flux

To find the divergence of this vector, we will use, as before, a conventional definition (2) and the same technique to find the flux through the picked out region, but taking into account the presence of phase of the wave space-delay. Pick out of Ω some region V, so that its ends coincide with the equiphase surfaces of flux, and its lateral surfaces - with the force tubes of current. Then, accounting that the wave propagation velocity is finite, we can write

$$\Delta l = c\Delta t = \text{const}, \tag{15}$$

where Δl is the length of the picked out force tube; Δt is the wave phase delay. Thus, basing on (15) and taking into account the particular case considered above, we have set up a relation between the length of the picked out region and the phase delay. We will take this fact into account in the further studying.

According to the statement of problem, the surface S consists of three components: $S = S_1 + S_2 + S_l$ (where S_1, S_2 are the end surfaces, and S_l is the lateral surface of the picked out region). Taking into account the statement of problem and the results of previous investigation, the complete flux through the surface S is

$$\int_S d\overrightarrow{s}\cdot\overrightarrow{F}(\overrightarrow{r},t) = \int_{S_1} d\overrightarrow{s}\cdot\overrightarrow{F}(\overrightarrow{r},t)+\int_{S_2} d\overrightarrow{s}\cdot\overrightarrow{F}(\overrightarrow{r},t-\Delta t) = \\ = \int_{S_1+S_2} d\overrightarrow{s}\cdot\overrightarrow{F}(\overrightarrow{r},t)+\int_{S_2} d\overrightarrow{s}\cdot\Delta\overrightarrow{F}(\overrightarrow{r},t)\ , \tag{16}$$

where $\Delta\overrightarrow{F}(\overrightarrow{r},t)=\overrightarrow{F}(\overrightarrow{r},t)-\overrightarrow{F}(\overrightarrow{r},t-\Delta t)$. In (16) we accounted at once the time shift of vector function, as well as the absence of flux through the lateral surface.

The first integral of the right-part sum in (16) has not a phase shift Δt. In the source-free field it vanishes, since in this case the condition for the divergence of stationary vector function becomes true. The second right-hand integral in (16) in general case is non-zero and can be easy transformed into the integral over the space. For it, we will divide the picked out region into p small regions whose height (along the lines of current) is

$$\Delta h=\frac{\Delta l}{p}=c\frac{\Delta t}{p}=c\delta t\,.$$

After it, we can write the under-integral expression $\Delta\overrightarrow{F}(\overrightarrow{r},t)$ as

$$\Delta\overrightarrow{F}(\overrightarrow{r},t) = \sum_p \delta_i\overrightarrow{F}(\overrightarrow{r},t)=\sum_p\frac{\delta_i\overrightarrow{F}(\overrightarrow{r},t)}{\Delta h}\Delta h = \\ = \frac{1}{c}\sum_p\frac{\delta_i\overrightarrow{F}(\overrightarrow{r},t)}{\delta t}\Delta h\ , \tag{17}$$

where $\delta_i\overrightarrow{F}(\overrightarrow{r},t)=\overrightarrow{F}(\overrightarrow{r},t-(i-1)\,\delta t)-\overrightarrow{F}(\overrightarrow{r},t-i\delta t)$; in this case $1\le i\le p$. Taking the limit $\delta t\to 0$ in (17), we come to the integral

$$\Delta\overrightarrow{F}(\overrightarrow{r},t)=\frac{1}{c}\int_{\Delta l}\frac{\partial\overrightarrow{F}(\overrightarrow{r},t)}{\partial t}dl\,. \tag{18}$$

Substituting (18) into (16) and knowing that, according to Fig.4, at the boundary S_2 the vector $d\overrightarrow{s}=ds\cdot\overrightarrow{n_2}$, and $\overrightarrow{n_2}$ coincides with the vector of flux $\overrightarrow{n}$, we obtain the required

$$\int_S d\overrightarrow{s}\cdot\overrightarrow{F}(\overrightarrow{r},t) = -\frac{1}{c}\int_{S_2} d\overrightarrow{s}\cdot\int_{\Delta l}\frac{\partial\overrightarrow{F}(\overrightarrow{r},t)}{\partial t}dl = \\ = -\frac{1}{c}\int_V d\overrightarrow{s}\cdot\frac{\partial\overrightarrow{F}(\overrightarrow{r},t)}{\partial t}dl=-\frac{1}{c}\int_V\overrightarrow{n}\cdot\frac{\partial\overrightarrow{F}(\overrightarrow{r},t)}{\partial t}dV\,. \tag{19}$$

Substituting (19) into (2), we come to the finite expression for the divergence of vector:

$$\operatorname{div}\overrightarrow{F}(\overrightarrow{r},t)=-\frac{1}{c}\overrightarrow{n}\cdot\frac{\partial\overrightarrow{F}(\overrightarrow{r},t)}{\partial t}\,. \tag{20}$$

In the most general case of dynamical vector flux, (20) coincides with the result obtained by Levitch in particular case for the vector potential. At the same time, we identified $\overrightarrow{F}(\overrightarrow{r}, t)$ with no any specified physical value, and studied the vector in the most general form. So we can state that we have proved the

Theorem 1. *When the wave flux propagating in the source-free space, the divergence of flux of vector is proportional to the scalar product of derivative of this vector with respect to time into the unit vector of direction of the flux.*

4. Application to the EM field

Basing on the above results, we can obtain the refined values for the divergence of vectors of electrical and magnetic fields in Maxwell system. Substituting the electrical strength $\overrightarrow{E}(\overrightarrow{r}, t)$ into (20) instead $\overrightarrow{F}(\overrightarrow{r}, t)$, we obtain

$$\text{div}\, \overrightarrow{E}(\overrightarrow{r}, t) = -\frac{1}{c}\, \overrightarrow{n} \cdot \frac{\partial \overrightarrow{E}(\overrightarrow{r}, t)}{\partial t}\,. \tag{21}$$

Similarly, for the magnetic strength $\overrightarrow{H}(\overrightarrow{r}, t)$

$$\text{div}\, \overrightarrow{H}(\overrightarrow{r}, t) = -\frac{1}{c}\, \overrightarrow{n} \cdot \frac{\partial \overrightarrow{H}(\overrightarrow{r}, t)}{\partial t}\,. \tag{22}$$

For the vector potential $\overrightarrow{A}(\overrightarrow{r}, t)$

$$\text{div}\, \overrightarrow{A}(\overrightarrow{r}, t) = -\frac{1}{c}\, \overrightarrow{n} \cdot \frac{\partial \overrightarrow{A}(\overrightarrow{r}, t)}{\partial t}\,. \tag{23}$$

Knowing that the scalar potential $\varphi(r, t)$ relates with the vector potential (see e.g. [5, p.106]) as

$$\varphi(r, t) = \overrightarrow{A}(\overrightarrow{r}, t) \cdot \overrightarrow{n}\,, \tag{24}$$

we come to the known expression

$$\text{div}\, \overrightarrow{A}(\overrightarrow{r}, t) + \frac{1}{c}\frac{\partial \varphi(r, t)}{\partial t} = 0 \tag{25}$$

establishing the integrity of relations in the EM field theory. The fact is important that (25) has been obtained for dynamical fields. It follows from it that it is incorrect to equal the scalar potential to zero in general case. More completely it was proved in [10, pp.25-26].

At the same time, for transverse $\overrightarrow{E}$ and $\overrightarrow{H}$, (21)-(23) coincide with the known results (3), since the scalar product of perpendicular vectors in the right part vanishes. The more, zero result of divergence of curl will also keep valid. In fact, in accord with item 3,

$$\overrightarrow{\nabla} \cdot \overrightarrow{F}(\overrightarrow{r}, t) = -\frac{1}{c}\, \overrightarrow{n} \cdot \frac{\partial \overrightarrow{F}(\overrightarrow{r}, t)}{\partial t}\,. \tag{26}$$

If we take $\overrightarrow{F}(\overrightarrow{r}, t)$ as

$$\overrightarrow{F}(\overrightarrow{r}, t) = \text{curl}\, \overrightarrow{G}(\overrightarrow{r}, t)\ , \tag{27}$$

we obtain for transverse field, where $\vec{G} \perp \vec{n}$

$$\vec{\nabla} \cdot \left[\vec{\nabla} \times \vec{G}\right] = -\frac{1}{c}\vec{n} \cdot \frac{\partial \left[\vec{\nabla} \times \vec{G}\right]}{\partial t} = 0\,, \tag{28}$$

since we know that the curl of transverse vector $\vec{G}(\vec{r}, t)$ is perpendicular both to the vector and to the field propagation direction $\vec{n}$. The similar result will be for the longitudinal vector, since the curl of longitudinal vector is zero.

Thus, for any inclined vector of a field, the divergence of curl of vector remains zero, since the curl of vector is perpendicular to the wave propagation. But it does not concern the results obtained for the divergence of longitudinal vector of the field. In (21) and (22) the derivatives of $\vec{E}$ and $\vec{H}$ with respect to time are the consequence of wave space-delay and cannot vanish if these vectors have a dynamical pattern. The changes in (21) and (22) give them the wave pattern, since, despite these equations are the first-order, expressions like

$$\begin{aligned} \vec{E}(\vec{r}, t) &= \vec{E}\,(\omega t - kr) \\ \vec{H}(\vec{r}, t) &= \vec{H}\,(\omega t - kr) \end{aligned} \tag{29}$$

are their solutions.

As follows from it, the longitudinal component of EM field has the wave properties too, irrespectively to the conventional concept that in the longitudinal field "the motion of energy is absent, there takes place only a periodic exchange of the energy between the electrical and magnetic components of a field" [11, p.99]. The only point we should mark is, the longitudinal component of a field has its properties that basically distinguish it from a transverse wave. Conventional symbols of the vortex vector $\vec{H}(\vec{r}, t)$ do not allow us to describe the magnetic field generating around the longitudinal dynamical E-field, because with it, at any point of the space, not vector but some circulation around the electrical vector will correspond to the magnetic field [10, p.42]. The divergence of this circulation will not vanish, as in (28), since its direction coincides with the flux direction. So it also will have the wave properties.

The fact that longitudinal EM waves (LEMWs) in a free space do exist was corroborated experimentally at the laboratory SELF in 1990. The portable device radiating/receiving the directed LEMW at 30 kHz range has been constructed and multiply demonstrated. But this is the subject of a wide consideration being out of frames of this paper.

5. Conclusions

As a result of investigation that we carried out, basing on the conventional definition of divergence and using the conventional method to find the flux of vector, we have revealed that:

- in dynamical fields, in general case, the flux and divergence of vector are non-zero;

- for the vector of flux directed along the wave propagation, the divergence is proportional to the scalar product of the particular derivative of this vector with respect to time into the wave propagation direction. Particularly, for this component of the field, the pair of Maxwell equations describing the flux of vector acquires the wave pattern;

- for the transverse component of wave, the divergence of vector remains zero, and consequently, Maxwell equations remain valid.

References

[1] Ignazius, G. I., *The field theory (mathematical analysis of multivariable functions),* Znanie, Moscow, 1971, 112 pp. (Russian).

[2] Rossel, J., *General physics,* Mir, Moscow, 1964, 506 pp. (Russian; from edition: *Physique generale,* Par Jean Rossel. Editions du Griffon - Neuchatel diffusion dunod - Paris; edited in Suisse, 1960).

[3] Korn, G. A., Korn, T. M., *Mathematical handbook for scientists and engineers,* Nauka, Moscow, 1968, pp.720 (Russian; from edition by McGraw-Hill Book Company, Inc., New York - Toronto - London, 1961).

[4] Landau, L. D., Lifshitz, E. M., *Theoretical physics, vol.II. The field theory,* Nauka, Moscow, 1973, 504 pp. (Russian).

[5] Sedov, L. I., *Mechanics of continuum, vol. 1,* Nauka, Moscow, 1973, 536 pp. (Russian).

[6] Alpert, Ya. L., Ginzburg, V. L., Feinberg, E. L., *The radio waves propagation,* The State publishing of technical and theoretical literature, Moscow, 1953, 863 pp. (Russian).

[7] Levitch, V. G., *The course of theoretical physics, vol. 1,* The State publishing of physical and mathematical literature, Moscow, 1962, 695 pp. (Russian).

[8] Eichenwald, A. A., *Selected works,* The State publishing of technical and theoretical literature, Moscow, 1956, 266 pp. (Russian).

[9] Purcell, E. M., *Electricity and magnetism. Berkeley physics course, vol. 2,* Nauka, Moscow, 1971, 447 pp. (Russian; from edition by McGraw-Hill Book Company).

[10] Karavashkin, S. B., *To the question of longitudinal EM waves. Chapter 1. Taking off the bans,* In the book *SELF. Transactions, vol. 1,* Eney Ltd., Ukraine, 1994, 118 pp. (English).

[11] Kugushev, A. M., Golubeva, N. S., *The foundations of radiolectronics. The linear electromagnetic processes,* Energia, Moscow, 1969, 880 pp. (Russian).

Special Laboratory for Fundamental Elaboration SELF
187 apt., 38 bldg., Prospect Gagarina
Kharkov 61140, UKRAINE
E-mail: sbkarav@altavista.com