Nikolaos Halidias; Nikolaos S. Papageorgiou
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SECOND ORDER MULTIVALUED BOUNDARY VALUE PROBLEMS

Nikolaos Halidias and Nikolaos S. Papageorgiou

Abstract. In this paper we use the method of upper and lower solutions to study multivalued Sturm–Liouville and periodic boundary value problems, with Caratheodory orientor field. We prove two existence theorems. One when the orientor field $F(t,x,y)$ is convex–valued and the other when $F(t,x,y)$ is nonconvex valued. Finally we show that the "convex" problem has extremal solutions in the order interval determined by an upper and a lower solution.

Introduction

C^2 , b

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Key words and phrases: upper solution, lower solution, order interval, usc multifunction, lsc multifunction, decomposable set, truncation map, penalty function, extremal solutions, Sturm–Liouville boundary conditions, periodic solutions.
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$-x'' \; t \quad f \; t, x \; t \;, x' \; t \;, u \; t \quad T$

$u \; t \; \in U \; t, x \; t \quad T \quad F \; t, x, y \quad f \; t, x, y, U \; t, x$

$f \; t, x \; t \;, x' \; t \;, x'' \; t \quad T$

$F \; t, x, y \quad \{-v \in \mathbb{R} \quad f \; t, x, y, v \quad \}$

$-x'' \; t \; \in f \; t, x \; t \;, x' \; t$

$-r \; t, x \; t \;, r \; t, x \; t \quad T \quad F \; t, x, y \quad f \; t, x, y \quad -r \; t, x \;, r \; t, x$

$-x'' \; t \quad f \; t, x \; t \;, x' \; t$

T

$$f_1 \; t, x, y \quad \underline{\lim_{\substack{x' \to x \\ y' \to y}}} f \; t, x', y' \qquad f_2 \; t, x, y \quad \overline{\lim_{\substack{x' \to x \\ y' \to y}}} f \; t, x', y'$$

N

$F \; t, x, y \quad f_1 \; t, x, y \;, f_2 \; t, x, y$

$-x'' \; t \; \in F \; t, x \; t \;, x' \; t \quad T \quad t \in T$

$f_1 \; t, \cdot, \cdot \quad f_2 \; t, \cdot, \cdot$

$F \; t, \cdot, \cdot$

Preliminaries

T $, b$

$$-x'' \; t \; \in F \; t, x \; t \; , x' \; t \qquad T$$
$$B_0 x \qquad k_0, \quad B_1 x \; b \quad k_1$$

$$-x'' \; t \; \in F \; t, x \; t \; , x' \; t \qquad T$$
$$x \qquad x \; b \; , \; x' \qquad x' \; b$$

$B_0 x$ $a_0 x$ $- c_0 x'$ $B_1 x \; b$ $a_1 x \; b$ $c_1 x' \; b$
$a_0, a_1, c_0, c_1 \geq$ $a_0 \; a_1 b \; c_1 \; c_0 a_1 \; /$
$x'' \; t$ $T \; B_0 x$ $B_1 x \; b$
$h \in L^1 \; T$ $x'' \; t$ $h \; t$
$T \; B_0 x$ $k_0 \; B_1 x \; b$ k_1

$$x \; t \quad u \; t \quad \int_0^b G \; t, s \; h \; s \; ds, \quad t \in T$$

$u \in C^2 \; , b$ $u'' \; t$ $t \in T \; B_0 u$ $k_0 \; B_1 u \; b$ k_1
$G \in C \; T \times T$

Definition. $x \in W^{2,1} \; T$
$v \in L^1 \; T$ $v \; t \; \in F \; t, x \; t \; , x' \; t$ $T \; -x'' \; t \quad v \; t$
T $B_0 x$ $k_0 \; B_1 x \; b$ k_1

Definition. $\phi \in W^{2,1} \; T$
$v_1 \in L^1 \; T$ $v_1 \; t \; \in F \; t, \phi \; t \; , \phi' \; t$
$T \; -\phi'' \; t \; \geq v_1 \; t$ T $B_0 \phi$ $\geq k_0 \; B_1 \phi \; b \; \geq k_1$
$\psi \in W^{2,1} \; T$ $v_0 \in L^1 \; T$
$v_0 \; t \; \in F \; t, \psi \; t \; , \psi' \; t$ $T \; -\psi'' \; t \; \leq v_0 \; t$ T
$B_0 \psi$ $\leq k_0 \; B_1 \psi \; b \; \leq k_1$

X

$P_{f(c)}\ X \quad \{A \subseteq X \quad A \quad \}.$

T $,b$ $F \quad T \to P_f\ X$
$x \in X$ $t \to d\ x, F\ t$ $\|x - y\|$ $y \in F\ t$
T σ $\mathcal{L}$
σ $B\ T$
$GrF \quad \{\ t, y\ \in T \times X \quad y \in$
$F\ t\ \} \in \mathcal{L} \times B\ X$ $B\ X$ σ X
$F\ \cdot$
$f_n \quad T \to X \quad n \geq$
$F\ t \quad \{\overline{f_n\ t}\}_{n \geq 1}$ $t \in T$

$F \quad T \to P_f\ X$ S^1_F
$F\ \cdot$ $L^1\ T, X$ $S^1_F \quad \{f \in$
$L^1\ T, X \quad f\ t\ \in F\ t$ $T\}$

$F\ \cdot \quad S^1_F\ /\ \varnothing$ $\{\|y\| \quad y \in F\ t\ \} \in L^1\ T$
$F\ \cdot \quad S^1_F$
$t \in T \quad F\ t$ S^1_F $L^1\ T$
$t \to \quad \|y\| \quad y \in F\ t$ $L^1\ T$ S^1_F
$f_1, f_2, A\ \in S^1_{F_1} \times S^1_{F_2} \times \mathcal{L}$
$\chi_A f_1 \quad \chi_{A^c} f_2 \in S^1_F$
$\chi_{A^c} \quad - \chi_A$ S^1_F

Y, Z $G \quad Y \to \ ^Z \setminus \{\varnothing\}$
$G\ \cdot$
$V \subseteq Z$ $G^+\ V \quad \{y \in Y \quad G\ y\ \subseteq V\}$ Y
$C \subseteq Z$ $G^-\ C \quad \{y \in Y$
$G\ y\ \cap C\ /\ \varnothing\}$ Y $G\ \cdot$ $y \in Y$
$G\ y\ \in P_f\ Z$ Z
$GrG \quad \{\ y, z\ \in Y \times Z \quad z \in G\ y\ \}$ $Y \times Z$
$\overline{G\ Y} \quad \overline{\bigcup_{y \in Y} G\ y}$ Z

$G\ \cdot$
$G \quad Y \to \ ^Z \setminus \{\varnothing\}$
$V \subseteq Z$ $G^-\ V \quad \{y \in Y \quad G\ y\ \cap V\ /\ \varnothing\}$ Y
$C \subseteq Z$ $G^+\ C \quad \{y \in Y \quad G\ y\ \subseteq C\}$ Y
Y Z
$G\ \cdot$
$y_n \to y \quad Y \quad n \to \infty$ $G\ y\ \subseteq \underline{\quad} G\ y_n \quad \{z \in Z \quad d_Z\ z, G\ y_n \quad \}$
$d_Z\ \cdot, \cdot$ Z $G\ \cdot$

$z \in Z \quad y \to d_Z \; z, G \; y$

$\mathbb{R}_+$ $G \; \cdot$

Y, Z $G \quad Y \to {}^Z \setminus \{\varnothing\}$

Y, Z

$A \quad Y \to Z$

Y

Z

Existence results

ϕ

ψ $\psi \le \phi$

$K \quad \psi, \phi \quad \{x \in W^{2,1} \; T \quad \psi \; t \; \le x \; t \; \le \phi \; t \qquad t \in T\}$

H F 1 $F \quad T \times \mathbb{R} \times \mathbb{R} \to P_{f_c} \; \mathbb{R}$

$x, y \; \in \mathbb{R} \times \mathbb{R} \; \; t \to F \; t, x, y$

$t \in T \quad x, y \; \to F \; t, x, y$

$r >$ $\gamma_r \in L^1 \; T$ $t \in T$

$|x|, |y| \le r$ $|F \; t, x, y \; |$ $|v| \quad v \in F \; t, x, y \; \; \le \gamma_r \; t$

H0 $\phi \in W^{2,1} \; T$ $\psi \in W^{2,1} \; T$

$\psi \; t \; \le \phi \; t$ $t \in T$ $h \in C \; \mathbb{R}_+, \quad , \quad \infty$

$|F \; t, x, y \; | \; \le \; h \; |y|$ $t \in T \quad x \in \quad \psi \; t \; , \phi \; t \qquad y \in \mathbb{R}$

$\int_\theta^\infty \frac{r}{h \; r} dr > \; _{t \in T} \; \phi \; t \; - \; _{t \in T} \; \psi \; t \qquad \theta \quad \frac{}{b} \qquad |\psi \quad - \phi \; b \; |, |\psi \; b \; - \phi \quad |$

Remark. H_0

$-x'' \; t \; \in F \; t, x \; t \; , x' \; t \qquad T$

Lemma 1.

If H_0 $x \in W^{2,1} \; T$

$-x'' \; t \; \in F \; t, x \; t \; , x' \; t \qquad T \qquad t \in T$

$\psi \; t \; \le x \; t \; \le \phi \; t$

then N_1 ϕ, ψ, h $|x' \; t \; | \le N_1$

$t \in T$

Proof. H_0 $N_1 > \theta$

$$_{t \in T} \; \phi \; t \; - \; _{t \in T} \; \psi \; t \; < \int_\theta^{N_1} \frac{r}{h \; r} dr.$$

$t \in T$ $|x'| \leq N_1$

$t \in T$ $|x' \; t \;| > N_1$

$t_0 \in \; , b$ $x' \; t_0$ $\frac{x(b)-x(0)}{b} \Rightarrow$

$|x' \; t_0 \;|$ $\frac{1}{b}|x \; b \; - x \;\;| \leq \frac{1}{b}$ $|\psi \;\; - \phi \; b \;|, |\psi \; b \; - \phi \;\;|$ $\psi \; b \; - \phi \;\; \leq$

$x \; b \; - x \;\; \leq \phi \; b \; - \psi$

$$x' \; t_0 \;\; \leq \theta < N_1 \qquad x' \; t \;\; > N_1 .$$

$x' \; \cdot$ $W^{2,1} \; T$

$C^1 \; T$ $t_1, t_2 \; \subseteq T$

$$x' \; t_1 \quad \theta, \; x' \; t_2 \quad N_1 \qquad \theta < x' \; t \; < N_1 \qquad t \in \; t_1, t_2$$

$$x' \; t_1 \quad N_1, \; x' \; t_2 \quad \theta \qquad \theta < x' \; t \; < N_1 \qquad t \in \; t_1, t_2$$

$$x' \; t_1 \quad -\theta, \; x' \; t_2 \quad -N_1 \qquad -N_1 < x' \; t \; < -\theta \qquad t \in \; t_1, t_2$$

$$x' \; t_1 \quad -N_1, \; x' \; t_2 \quad -\theta \qquad -N_1 < x' \; t \; < -\theta \qquad t \in \; t_1, t_2 \; .$$

$$-x'' \; t \; \in F \;\; t, x \; t \;, x' \; t \qquad T$$

$$\Rightarrow \quad x'' \; t \;\; x' \; t \; \leq \; F \;\; t, x \; t \;, x' \; t \;\;\; x' \; t \; \leq h \;\;\; x' \; t \;\;\; x' \; t \qquad T$$

$$\Rightarrow \quad \frac{x'' \; t \;\; x' \; t}{h \; |x' \; t \;|} \leq x' \; t \qquad T \qquad h \; \cdot \quad \mathbb{R}_+ \setminus \{ \; \} \qquad .$$

t_1, t_2

$$\int_{t_1}^{t_2} \frac{x'' \; t \;\; x' \; t}{h \; |x' \; t \;|} dt \leq \int_{t_1}^{t_2} x' \; t \; dt \quad x \; t_2 \; - x \; t_1 \; \leq \; {}_{t \in T} \; \phi \; t \; - \; {}_{t \in T} \; \psi \; t \; .$$

$r \quad x' \; t$ $dr \quad x'' \; t \; dt$ $x' \; t_1 \quad \theta \;\; x' \; t_2 \quad N_1$

$$\int_{\theta_1}^{N_1} \frac{r}{h \; r} dr \leq \; {}_{t \in T} \; \phi \; t \; - \; {}_{t \in T} \; \psi \; t \; ,$$

N_1

□

$K \quad \psi, \phi \quad \{x \in W^{2,1} \; T \quad \psi \; t \; \leq x \; t \; \leq \phi \; t$

$t \in \}$ K

Theorem 2.

If *hypotheses* $H \;\; F \;\; _1$ *and* H_0 *hold,*

then *problem* *has a solution* $x \in W^{2,1}\ T$ *in* K ψ, ϕ .

Proof. $x \in W^{2,1}\ T$

K $|x^{'}\ t\ | \leq N_1$ $t \in T$ N_1 ϕ, ψ, h

N_1 $N, \|\phi^{'}\|_\infty, \|\psi^{'}\|_\infty$

$\tau \in W^{1,1}\ T\ \rightarrow W^{1,1}\ T$

$$\tau\ x\ \ t \quad \begin{matrix} \phi\ t & \phi\ t\ \leq x\ t \\ x\ t & \psi\ t\ \leq x\ t\ \leq \phi\ t \\ \psi\ t & x\ t\ \leq \psi\ t\ . \end{matrix}$$

$x \in W^{1,1}\ T$ $\tau\ x\ \in$ $W^{1,1}\ T$

$$\tau\ x\ ^{'}\ t \quad \begin{matrix} \phi^{'}\ t & \phi\ t\ < x\ t \\ x^{'}\ t & \psi\ t\ \leq x\ t\ \leq \phi\ t \\ \psi^{'}\ t & x\ t\ < \psi\ t\ . \end{matrix}$$

N q_N $L^1\ T\ \rightarrow L^1\ T$

$$q_N\ h\ \ t \quad \begin{matrix} N & N \leq h\ t \\ h\ t & -N \leq h\ t\ \leq N \\ -N & h\ t\ \leq -N, \end{matrix}$$

$u\ \ T \times \mathbb{R} \rightarrow \mathbb{R}$

$$u\ t, x \quad \begin{matrix} x - \phi\ t & \phi\ t\ \leq x \\ & \psi\ t\ \leq x \leq \phi\ t \\ x - \psi\ t & x \leq \psi\ t\ . \end{matrix}$$

$N_F\ x$ $S^1_{F(\cdot,\tau(x)(\cdot),q_N(\tau(x)')(\cdot))}$ $x \in W^{1,1}\ T$ G

$W^{1,1}\ T\ \rightarrow\ ^{L^1(T)}$

$$G\ x\quad \{v \in N_F\ x\quad v\ t\ \geq v_0\ t \qquad x < \psi < \phi$$

$$v\ t\ \leq v_1\ t \qquad \psi < \phi < x\ \}.$$

$x \in W^{1,1}\ T$ $G\ x$

$G\ x$

$G\ x$ $t \rightarrow F\ t, \tau\ x\ \ t\ , q_N\ \tau\ x\ ^{'}\ t$

$H\ F\ _1$ $t \rightarrow F\ t, \tau\ x\ \ t\ , q_N\ \tau\ x\ ^{'}\ t$

$t \rightarrow$

$F\ t, \tau\ x\ \ t\ , q_N\ \tau\ x\ ^{'}\ t$

$\{s_n\}_{n\geq 1}$ $\{r_n\}_{n\geq 1}$ $|s_n\ t\ | \leq |\tau\ x\ \ t\ |$

$|r_n\ t\ | \leq |q_N\ \tau\ x\ ^{'}\ t\ |$ $s_n\ t\ \rightarrow \tau\ x\ \ t$ $r_n\ t\ \rightarrow q_N\ \tau\ x\ ^{'}\ t$ T

$n \rightarrow \infty$ H_0 $n \geq$ $t \rightarrow F\ t, s_n\ t\ , r_n\ t$

$v_n \; T \to \mathbb{R}$ $v_n \; t \; \in F \; t, s_n \; t \;, r_n \; t$
T $H \; F \; _1$ $|v_n \; t \;| \leq \gamma_r \; t$ $T \; n \geq$
$r >$ $\|\psi\|_\infty, \|\phi\|_\infty, N$ $\gamma_r \in L^1 \; T$

$$v_n \xrightarrow{w} v \quad L^1 \; T \quad n \to \infty$$

$$v \; t \; \in \overline{\{v_n \; t\;\}_{n\geq 1}} \subseteq \overline{F \; t, s_n \; t \;, r_n \; t} \subseteq$$

$F \; t, \tau \; x \; t \;, q_N \; \tau \; x \;' \; t$ T
$H \; F \; _1$ $v \in N_F \; x$ v $\chi_{A_1} v$ $\chi_{A_2} v_0$ $\chi_{A_3} v_1$
A_1 $\psi \leq x \leq \phi$ A_2 $x < \psi < \phi$ A_3 $\psi < \phi < x$
$v \in G \; x$

$G \; \cdot$ $W^{1,1} \; T$ $L^1 \; T$

$C \subseteq L^1 \; T$ $G^- \; C$ $\{x \in$
$W^{1,1} \; T$ $G \; x \; \cap C \; / \; \emptyset\}$ $\{x_n\}_{n\geq 1} \subseteq G^- \; C$
$x_n \to x$ $W^{1,1} \; T$ $n \to \infty$ $W^{1,1} \; T$ $C \; T$
$x_n \to x$ $C \; T$ $n \to \infty$ $v_n \in G \; x_n \; \cap C$
$n \geq$ $n \geq$ $|v_n \; t \;| \leq \gamma_r \; t$ T $r >$ $\|\psi\|_\infty, \|\phi\|_\infty, N$

$$v_n \xrightarrow{w} v \quad L^1 \; T \quad n \to \infty$$

$$v \; t \; \in \overline{\{v_n \; t\;\}_{n\geq 1}} \qquad t \in T \setminus S_1$$

$\lambda \; S_1$ λ T
$\tau \; W^{1,1} \; T \; \to W^{1,1} \; T$ $q_N \; L^1 \; T \; \to L^1 \; T$

$$\tau \; x_n \; t \; \to \tau \; x \; t \qquad q_N \; \tau \; x_n \;' \; t \; \to q_N \; \tau \; x \;' \; t \qquad T$$

$H \; F \; _1$
$v \in N_F \; x$ $t \in \; x < \psi < \phi \; \setminus S_1$ $t \in \; x_n < \psi < \phi$ $n \geq n_0$
$v_n \; t \; \geq v_0 \; t$ $v \; t \; \geq v_0 \; t$ $t \in \; \psi < \phi < x$
$t_n \in \; \psi < \phi < x$ $n \geq n_1$ $v_n \; t \; \leq v_1 \; t$
$v \; t \; \leq v_1 \; t$ $v \in G \; x \; \cap C$
D $\{x \in W^{2,1} \; T$ $B_0 x$ $k_0,$ $B_1 x \; b$ $k_1\}$
$L \; D \subseteq L^1 \; T \; \to L^1 \; T$ $L \; x$ $-x''$ $L \; I \; L$

$L^{-1} \; L^1 \; T \; \to D \subseteq W^{1,1} \; T$

$R \; L$ $L^1 \; T$ L
$h \in L^1 \; T$

$$-x'' \; t \quad x \; t \quad h \; t \qquad T$$
$$B_0 x \quad k_0, \quad B_1 x \; b \quad k_1$$

$x \in W^{2,1} \; T$ $h \in C \; T$

$h \in L^1 \; T$
$\{h_n\}_{n\geq 1} \subseteq C \; T$ $h_n \to h$ $L^1 \; T$ $n \to \infty$
$x_n \in W^{2,1} \; T$ $n \geq$

h_n $$x_n \; t \quad u \; t \quad \int_0^b G \; t, s \; h_n \; s \; ds$$

$u \in W^{2,1} \; T$ u'' $B_0 u$ k_0 $B_1 u \; b$ k_1

$G \in C\ T \times T$

$|x_n\ t\ | \leq |u\ t\ |\quad \|G\|_\infty \|h_n\|_1 \leq u\ t\quad M_1\quad n \geq\quad t \in T$

$\{x_n\}_{n\geq 1}\quad \|x_n\|_1\quad \|x_n^{''}\|_1$

$W^{2,1}\ T\quad \{x_n\}_{n\geq 1}$

$W^{2,1}\ T\quad W^{2,1}\ T\quad W^{1,1}\ T\quad \{x_n^{''}\}_{n\geq 1}$

$x_n \to x\quad W^{1,1}\ T\quad x_n^{''} \xrightarrow{w} v\quad L^1\ T\quad n \to \infty\quad v\quad x^{''}$

$x \in W^{2,1}\ T\quad -x^{''}\ t\quad x\ t\quad h\ t\quad T\quad B_0 x\quad k_0$

$B_1 x\quad b\quad k_1\quad x \in W^{2,1}\ T\quad h \in L^1\ T$

$R\ L\quad L^1\ T$

$x_1, x_2 \in D\quad x\quad x_1 - x_2$

$$T_+\quad \{t \in T\quad x\ t\ >\ \}\qquad T_-\quad \{t \in T\quad x\ t\ <\ \}$$

$\lambda >$

$$\int_0^b x\ t\ - \lambda x^{''}\ t\quad dt \geq \int_{T_+} x\ t\ - \lambda x^{''}\ t\quad dt \quad \int_{T-} x\ t\ - \lambda x^{''}\ t\quad dt$$

$$\geq \int_{T_+} x\ t\ - \lambda x^{''}\ t\quad dt - \int_{T_-} x\ t\ - \lambda x^{''}\ t\quad dt$$

$$\int_{T_+} x\ t\ dt - \int_{T_-} x\ t\ dt - \lambda \int_{T_+} x^{''}\ t\ dt\quad \lambda \int_{T_-} x^{''}\ t\ dt$$

$$\int_0^b |x\ t\ |\, dt - \lambda \int_{T_+} x^{''}\ t\ dt - \int_{T_-} x^{''}\ t\ dt\ .$$

$T_k\quad T_+\quad T_k\quad a, c\quad < a < c < b\quad T_k$

$, a\quad < a < b\quad T_k\quad c, b\quad < c < b\quad T_k\quad a, c$

$x\ a\quad x\ c\quad x\ t\ >\quad t \in\ a, c\quad x^{'}\ a\ \geq\quad x^{'}\ c\ \leq$

$\int_a^c x^{''}\ t\ dt\quad x^{'}\ c\ - x^{'}\ a\ \leq\quad T_k\quad , a$

$x\ a\quad x^{'}\ a\ \leq\quad \int_0^a x^{''}\ t\ dt\quad x^{'}\ a\ - x^{'}\quad \leq\quad c_0\ /$

$x\quad \frac{a_0}{c_0} x^{'}\quad B_0 x\quad x\quad x_1 - x_2\quad c_0$

$x^{'}\quad \int_{T_+} x^{''}\ t\ dt \leq$

$$\int_{T_-} x^{''}\ t\ dt \geq$$

$$\int_0^b x\ t\ - \lambda x^{''}\ t\quad dt \geq \int_0^b |x\ t\ |\, dt$$

$$\Rightarrow\quad x_1\quad \lambda L x_1 - \quad x_2\quad \lambda L x_2\quad {}_1 \geq \|x_1 - x_2\|.$$

$I \quad L^{-1} \quad L^{-1} \quad L^1 \; T \;\to D \subseteq L^1 \; T$

m L

$\theta >$

$$E_\theta \qquad x \in D \quad \|x\|_1 \quad \|x''\|_1 \leq \theta \quad .$$

$\|x\|_1 \quad \|x''\|_1$ $W^{2,1} \; T$

E_θ $W^{2,1} \; T$

$L^1 \; T$ E_θ $L^1 \; T$

$L^{-1} \quad L^1 \; T \;\to D \subseteq L^1 \; T$

$V \subseteq L^1 \; T$ $h \in V$ $x \quad L^{-1} \; h$

$-x'' \quad x \quad h$

$$\|x\|_1 \leq \| - x'' \quad x\|_1 \leq \qquad \|h\|_1 \quad h \in V \qquad |V| < \infty$$
$$\Rightarrow \quad \|x''\|_1 \leq \quad |V|.$$

$L^{-1} \; V$ $I \quad L^{-1} \; V$ $W^{2,1} \; T$

$W^{1,1} \; T$ $I \quad L^{-1} \; V$

$W^{1,1} \; T$ $h_n \to h \quad L^1 \; T \quad n \to \infty \quad x_n$

$I \quad L^{-1} \; h_n \quad n \geq$ $x_n \to x \quad I \quad L^{-1} \; h$

$L^1 \; T \quad n \to \infty$ $\{x_n\}_{n \geq 1}$ $W^{2,1} \; T$

$W^{2,1} \; T$ $W^{1,1} \; T$ $x_n \to x \quad W^{1,1} \; T$

$n \to \infty$ $L^{-1} \quad L^1 \; T \;\to D \subseteq W^{1,1} \; T$

$U \quad L^1 \; T \;\to L^1 \; T$ $U \; x \; \cdot \quad u \; \cdot, x \; \cdot$ $G_1 \; x$

$G \; x \; - U \; x \quad x$ $x \in W^{1,1} \; T$ $G_1 \; \cdot$

$L^1 \; T$ $G_1 \; \cdot$

$W^{1,1} \; T$ $L^1 \; T$

$u \; t, x$ $x \in W^{1,1} \; T$

$|G_1 \; x \;|$ $\|g\|_1 \quad g \in G_1 \; x \quad \leq \eta^*$ $\eta^* \quad \|\gamma\|_1 \quad b \; \dfrac{\|\psi\|_\infty, \|\phi\|_\infty}{L^{-1} G_1 \; W^{1,1} \; T}$

$r >$ $\|\psi\|_\infty, \|\phi\|_\infty, N$

$L^{-1} G_1 \quad W^{1,1} \; T \;\to D \subseteq W^{1,1} \; T$

$x \in D$ $x \in L^{-1} G_1 \; x$ $x \in D$

$L^{-1} G_1 \; \cdot$ $S \subseteq D$

$S \subseteq K \quad \psi, \phi$

$x \in S$

$$-x'' \; t \qquad v \; t \; - u \; t, x \; t \qquad\qquad T$$
$$B_0 x \qquad k_0, \quad B_1 x \;\; b \quad k_1$$

$v \in G \; x$ $v \; t \;\in F \; t, \tau \; x \;\; t \;, q_N \; \tau \; x \;' \;\; t$ $T \;\; v \; t \;\geq v_0 \; t$

$x < \psi < \phi$ $v \; t \;\leq v_1 \; t$ $\psi < \phi < x$

$\psi \in W^{2,1} \quad T$

$$-\psi^{''} \quad t \quad \leq v_0 \; t \qquad T$$
$$B_0\psi \qquad \leq k_0, \quad B_1\psi \quad b \quad \leq k_1$$

$v_0 \in L^1 \; T \quad v_0 \; t \quad \in F \quad t, \psi \; t \; , \psi^{'} \; t \qquad T$

$$\psi^{''} \; t \; - x^{''} \; t \; \geq v \; t \; - v_0 \; t \; - u \; t, x \; t \qquad T$$

$\psi - x \;_+ \; t \qquad T \quad ,b$

$$\int_0^b \psi^{''} - x^{''} \quad t \quad \psi - x \;_+ \; t \; dt \qquad \int_0^b v - v_0 \quad t \quad \psi - x \;_+ \; t \; dt$$

$$- \int_0^b u \; t, x \; t \quad \psi - x \;_+ \; t \; dt.$$

$$\int_0^b \psi^{''} - x^{''} \quad t \quad \psi - x \;_+ \; t \; dt$$
$$\psi^{'} - x^{'} \quad b \quad \psi - x \;_+ \; b \; - \; \psi^{'} - x^{'} \qquad \psi - x \;_+$$
$$- \int_0^b \psi^{'} - x^{'} \quad t \quad \psi - x \;^{'}_+ \; t \; dt.$$

$$c_1\psi^{'} \; b \; \leq k_1 - a_1\psi \; b \qquad c_1 x^{'} \; b \qquad k_1 - a_1 x \; b \; .$$

$c_1 \qquad a_1\psi \; b \; \leq k_1 \qquad c_1 x \; b \qquad k_1 \qquad \psi \; b \; \leq x \; b$
$\psi - x \;_+ \; b \qquad c_1 > \qquad \psi^{'} \; b \; \leq \frac{k_1}{c_1} - \frac{a_1}{c_1}\psi \; b \qquad -x^{'} \; b \qquad -\frac{k_1}{c_1} \quad \frac{a_1}{c_1} x \; b$
$\psi^{'} - x^{'} \quad b \; \leq \frac{a_1}{c_1} \; x - \psi \quad b \qquad \psi^{'} - x^{'} \quad b \quad \psi - x \;_+ \; b \; \leq \frac{a_1}{c_1} \; x - \psi \quad b \quad \psi -$
$x \;_+ \; b \; \leq \qquad \psi^{'} - x^{'} \quad b \quad \psi - x \;_+ \; b \; \leq$
$\psi^{'} - x^{'} \qquad \psi - x \;_+ \qquad \geq$

$$\int_0^b \psi^{''} - x^{''} \quad t \quad \psi - x \;_+ \; t \; dt \leq - \int_0^b \psi^{'} - x^{'} \quad t \quad \psi - x \;^{'}_+ dt$$

$$- \int_0^b \psi - x \;^{'}_+ \; t \quad ^2 dt \leq \quad .$$

$$\int_0^b v - v_0 \quad t \quad \psi - x \;_+ \; t \; dt \qquad \int_{\{\psi > x\}} v - v_0 \quad t \quad \psi - x \quad t \; dt \geq$$

$v \in G\ x$

$$- \int_0^b u\ t, x\ t \quad \psi - x\ _+\ t\ dt \leq$$

$$\Rightarrow \int_0^b \psi - x\ ^2_+\ t\ dt \leq \qquad u\ ,$$

$$\Rightarrow \psi\ t\ \leq x\ t \qquad t \in T.$$

$x\ t\ \leq \phi\ t$ $t \in T$ $S \subseteq K$ ψ, ϕ $x \in S$ $\tau\ x$ x $q_N\ \tau\ x\ '$ $q_N\ x'$ x' $x \in D \subseteq W^{2,1}\ T$ □

$F\ t, x, y$

H F 2 $F\ \ T \times \mathbb{R} \times \mathbb{R} \to P_f\ \mathbb{R}$

$t, x, y\ \to F\ t, x, y$

$t \in T\ \ x, y\ \to F\ t, x, y$

$r >$ $\gamma_r \in L^1\ T$ $t \in T$

$|x|, |y| \leq r$ $|F\ t, x, y\ |$ $|v|\ \ v \in F\ t, x, y\ \ \leq \gamma_r\ t$

Theorem 3.

If *hypotheses* $H\ F\ _2$ *and* H_0 *hold,*
then *problem* *has a solution* $x \in W^{2,1}\ T$ *in the order interval* $K\ \ \psi, \phi$

Proof. $x \in W^{1,1}\ T$ $N_F\ x$ $S^1_{F(\cdot,\tau(x)(\cdot),q_N(\tau(x)')(\cdot))}$ $G\ \ W^{1,1}\ T\ \to\ ^{L^1(T)}$

$$G\ x\ \ \{v \in L^1\ T \quad \begin{array}{ll} v\ t\ \ f\ t & \psi \leq x \leq \phi\ \cap\ \psi < \phi\ , \\ v\ t\ \ f\ t\ , v_0\ t & x < \psi < \phi\ , \\ v\ t\ \ f\ t\ , v_1\ t & \psi < \phi < x\ , \\ v\ t\ \ \psi''\ t & \psi\ \ \phi\ , \\ f \in N_F\ x\ ,\ \ f\ t\ \geq v_0\ t & x\ \ \psi < \phi \\ \quad f\ t\ \leq v_1\ t & \psi < \phi\ \ x\ \}. \end{array}$$

$x'\ t\ \ \psi'\ t$ $x\ \ \psi$ $x'\ t\ \ \phi'\ t$ $x\ \ \phi$ $H\ F\ _2$ $x \in W^{1,1}\ T\ \ G\ x\ \ / \varnothing$

$G\ \ W^{1,1}\ T\ \ \to P_f\ L^1\ T$

$x_n \to x\ \ W^{1,1}\ T\ \ n \to \infty$ $G\ x\ \subseteq \underline{\quad} G\ x_n$ $\{v \in L^1\ T\ \ v\ \ v_n\ \ L^1\ T\ ,\ v_n \in G\ x_n\ ,\ n \geq\ \}$ $\{v \in L^1\ T$

$d_{L^1(T)}\ v, G\ x_n \quad \}$ $\quad v \in G\ x$
$f \in N_F\ x$ $\quad v\ t \quad f\ t$ $\quad \psi \leq x \leq \phi \ \cap\ \psi < \phi$
$f\ t \ \geq v_0\ t$ $\quad x \quad \psi < \phi$ $\quad f\ t \ \leq v_1\ t$ $\quad \psi < \phi \quad x$
$g_n \in N_F\ x_n \quad n \geq$ $\quad g_n \to f$
$L^1\ T \quad n \to \infty$ $\quad f_n$ $\quad g_n, v_0\ \chi_{A_1^n}$ $\quad g_n, v_1\ \chi_{A_2^n} \quad g_n \chi_{A_3^n}$
$A_1^n \quad x_n \quad \psi < \phi \quad A_2^n \quad \psi < \phi \quad x_n \quad A_n^3 \quad T \setminus A_1^n \cup A_2^n$
$f_n \in N_F\ x_n \quad f_n \to f \quad L^1\ T$
$f_n\ t \ \to f\ t \quad t \in T \setminus S_1 \ \ \lambda\ S_1$

$$v_n\ t \quad \begin{matrix} f_n\ t & \psi \leq x_n \leq \phi \ \cap\ \psi < \phi \\ f_n\ t\ , v_0\ t & x_n < \psi < \phi \\ f_n\ t\ , v_1\ t & \psi < \phi < x_n \\ \psi''\ t & \psi \quad \phi\ . \end{matrix}$$

$$v_n \in G\ x_n \quad n \geq$$

$$v_n\ t \ \to v\ t \qquad t \in T \setminus S_1 \quad n \to \infty$$

$t \in \ \psi < x < \phi \ \setminus S_1$ $\quad t \in \ \psi < x_n < \phi \ \setminus S_1$ $\quad n \geq n_0$
$x_n \to x \quad C\ T \quad n \to \infty$ $\quad v_n\ t \quad f_n\ t \ \to f\ t \quad v\ t \quad n \to \infty$

$t \in \ x < \psi < \phi \ \setminus S_1$ $\quad t \in \ x_n < \psi < \phi \ \setminus S_1$ $\quad n \geq n_1$
$v_n\ t \quad f_n\ t\ , v_0\ t \ \to \quad f\ t\ , v_0\ t \quad v\ t \quad n \to \infty$

$t \in \ \psi < \phi < x \ \setminus S_1$ $\quad t \in \ \psi < \phi < x_n \ \setminus S_1$ $\quad n \geq n_2$
$v_n\ t \quad f_n\ t\ , v_1\ t \ \to \quad f\ t\ , v_1\ t \quad v\ t \quad n \to \infty$

$t \in \ x \quad \psi < \phi \ \setminus S_1$ $\quad \{m\} \quad \{n\}$ $\quad x_m\ t \ <$
$x\ t \quad v_m\ t \quad f_m\ t\ , v_0\ t \ \to \quad f_m\ t\ , v_0\ t \quad v\ t \quad f\ t$
$\{k\} \quad \{n\}$ $\quad x\ t \ < x_k\ t \ < \phi\ t$ $\quad v_k\ t \quad f_k\ t \ \to$
$f\ t \quad v\ t \quad k \to \infty$

$t \in \ \psi < \phi \quad x \ \setminus S_1$

$t \in \ \psi \quad \phi \ \setminus S_1$ $\quad v_n\ t \quad \psi''\ t \ \to \psi''\ t \quad v\ t \quad n \to \infty$

$\{v_n\}_{n \geq 1}$
$v \quad L^1\ T \quad v_n \to v \quad L^1\ T \quad n \to \infty \quad v_n \in G\ x_n$
$n \geq$ $\quad G\ x \ \subseteq \underline{\quad} G\ x_n$ $\quad G\ \cdot$

$G_1\ x \quad G\ x \ - U\ x \quad x \quad U\ x \ \cdot$
$u\ \cdot, x\ \cdot$ $\quad G_1\ \cdot$

$g_1 \quad W^{1,1}\ T \ \to L^1\ T$ $g_1\ x \ \in G_1\ x$
$x \in W^{1,1}\ T$ L $S_0 \quad \{x \in D \quad x$
$L^{-1} g_1\ x\ \}$
$S_0 \ / \ \varnothing$ $S_0 \subseteq K$ ψ, ϕ
$x \in S_0$ □

Extremal solutions

S $K \quad \psi, \phi$
K
$x_*, x^* \in S$ $x \in S$ $x_* \leq x \leq x^*$

Theorem 4.

If *hypotheses* $H\ F\ _1$ *and* H_0 *hold,*
then *problem* *has extremal solutions in the order interval* $K \quad \psi, \phi$

Proof. $S \ / \ \varnothing$

S

$x_1, x_2 \in S$ x_3 x_1, x_2
$x_3 \in W^{1,1}\ T$ $\tau_3\ x$ $u_3\ t, x$
$\{x_3, \phi\}$ $\tau_3\ x$
$u_3\ t, x$

$$\tau_3\ x \quad t \qquad \begin{matrix} \phi\ t & \phi\ t \ \leq x\ t \\ x\ t & x_3\ t \ \leq x\ t \ \leq \phi\ t \\ x_3\ t & x\ t \ \leq x_3\ t \end{matrix}$$

$$u_3\ t, x \qquad \begin{matrix} x - \phi\ t & \phi\ t \ \leq x \\ & x_3\ t \ \leq x \leq \phi\ t \\ x - x_3\ t & x \leq x_3\ t\ . \end{matrix}$$

$x \ \in \ W^{1,1}\ T$ $N_F\ x$ $S^1_{F(\cdot,\tau_3(x)(\cdot),q_N(\tau_3(x)')(\cdot))}$ G
$W^{1,1}\ T \ \to \ {}^{L^1(T)}$

$$G\ x \quad \{v \in N_F\ x \qquad \begin{matrix} v\ t \ \geq w_1\ t & x < x_1\ \cap int\ x_1 \geq x_2\ , \\ v\ t \ \geq w_2\ t & x < x_2\ \cap int\ x_2 \geq x_1\ , \\ v\ t \ \leq v_1\ t & \phi < x\ \}, \end{matrix}$$

$w_1 \ \in \ S^1_{F(\cdot,x_1(\cdot),x_1'(\cdot))}$ $w_2 \ \in \ S^1_{F(\cdot,x_2(\cdot),x_2'(\cdot))}$ $-x_1''\ t$
$w_1\ t$ T $-x_2''\ t$ $w_2\ t$ T
$G\ \cdot$
$W^{1,1}\ T$ $L^1\ T$
$L^1\ T\ _w$ $G_1\ x \quad G\ x \ - U_3\ x \quad x$ $U_3\ x \quad \cdot \quad u_3\ \cdot, x\ \cdot$ $G_1\ \cdot$
$W^{1,1}\ T$ $L^1\ T\ _w$
$L \quad I \quad L$

$$S_f \quad \{x \ \in \ W^{1,1}\ T \quad x \ \in \ L^{-1} G_1\ x\ \}$$

$S_f \subseteq x_3, \phi$ $\overline{A_1 \cup A_2}$ T $x \in S_f$ A_1 $int\, x_1 \geq x_2$ A_2 $int\, x_2 \geq x_1$

$$x_1^{''} t - x^{''} t \quad v\, t - w_1\, t - u_3\, t, x\, t \qquad T$$

$$x_2^{''} t - x^{''} t \quad v\, t - w_2\, t - u_3\, t, x\, t \qquad T$$

$$x_1 - x_+\, t \qquad A_1$$

$$\int_{A_1} x_1^{''} - x^{''}\; t\;\; x_1 - x_+\, t\, dt \qquad \int_{A_1} v - w_1\;\; t\;\; x_1 - x_+ dt$$

$$- \int_{A_1} u_3\, t, x\, t \quad x_1 - x_+ dt$$

α, β $A_1 \cap \{x_1 > x\}$

$$\int_\alpha^\beta x_1^{''} - x^{''}\;\; t\;\; x_1 - x\;\; t\, dt \quad x_1^{'} - x^{'}\;\; b\;\; x_1 - x\;\; b - x_1^{'} - x^{'}\;\; a\;\; x_1 - x\;\; a$$

$$- \int_\alpha^\beta x_1^{'} - x^{'\,2} dt$$

$< \alpha < \beta < b$ $x_1 - x\;\; \alpha$ $x_1 - x\;\; \beta$

$$\int_\alpha^\beta x_1^{''} - x^{''}\; t\;\; x_1 - x\;\; t\, dt \quad - \int_\alpha^\beta x_1 - x^{\,2} dt \leq$$

$$\Rightarrow \int_{A_1} x_1^{''} - x^{''}\;\; t\;\; x_1 - x\;\; t\, dt \leq$$

$G\, x$ $v \in G\, x$

$$\int_{A_1} v - w_1\;\; t\;\; x_1 - x_+\, t\, dt \qquad \int_{A_1 \cap \{x_1 > x\}} v - w_1\;\; t\;\; x_1 - x\;\; t\, dt \geq$$

$$\geq - \int_{A_1} u_3\, t, x\, t \quad x_1 - x_+\, t\, dt \qquad \int_{A_1} x_1 - x\;\; t\;\; x_1 - x_+\, t\, dt$$

$$\Rightarrow \int_{A_1} x_1 - x_+^2\, t\, dt \leq\; , \qquad x\, t \geq x_1\, t \qquad t \in A_1.$$

$$x_2 - x_+\, t \qquad A_2$$

$$\int_{A_2} x_2 - x_+^2\, t\, dt \leq\; , \qquad x\, t \geq x_2\, t \qquad t \in A_2.$$

$$x_3\ t\ \le x\ t \qquad t \in A_1 \cup A_2$$

$$\Rightarrow x_3\ t\ \le x\ t \qquad t \in T \qquad .$$

$x \in\ x_3, \phi$ $S_f \subseteq\ x_3, \phi$ $S_f \subseteq S$ S

C S S

$\{x_n\}_{n\ge 1} \subseteq C$ C x_n $n\ge 1$ $\{x_n\}_{n\ge 1}$

$$-x_n^{''}\ t \quad v_n\ t \qquad T$$
$$B_0 x_n \qquad k_0, \quad B_1 x_n\ \ b \qquad k_1$$

$v_n \in S^1_{F(\cdot,x_n(\cdot),x_n'(\cdot))}$ $x_n \in\ \psi, \phi\ \ n \ge$ $|v_n\ t\ | \le \gamma_r\ t$
T $r >$ $\|\psi\|_\infty, \|\phi\|_\infty, N_1$ $\{x_n^{''}\}_{n\ge 1}$
$\|x_n\|_1$ $\|x_n^{''}\|_1$ $W^{2,1}\ T$
$\{x_n\}_{n\ge 1}$ $W^{2,1}\ T$ $W^{2,1}\ T$
$W^{1,1}\ T$
$x_n^{''} \xrightarrow{w} w$ $L^1\ T$ $x_n \to x$ $W^{1,1}\ T$ $x_n\ t\ \to x\ t$
$x_n^{'}\ t\ \to x^{'}\ t$ $t \in T$ $n \to \infty$ w $x^{''}$ $x_n \xrightarrow{w} x$
$W^{2,1}\ T$ $n \to \infty$ $H\ F$ 1

$$-x^{''}\ t\ \in F\ t, x\ t\ , x^{'}\ t \qquad T$$
$$B_0 x \qquad k_0, \quad B_1 x\ \ b \qquad k_1$$

x $C \in S$
S $x^* \in S$
S x^* K ψ, ϕ
x_* K ψ, ϕ $\square$

Periodic solutions

Definition. $\phi \in W^{2,1}\ T$
$-\phi^{''}\ t\ \ge v_1\ t$ T $v_1 \in L^1\ T$ $v_1\ t\ \in F\ t, \phi\ t\ , \phi^{'}\ t$ T
ϕ $\phi\ b$ $\phi^{'}$ $\le \phi^{'}\ b$ $\psi \in W^{2,1}\ T$
$-\psi^{''}\ t\ \le v_0\ t$ T $v_0 \in L^1\ T$ $v_0\ t\ \in F\ t, \psi\ t\ , \psi^{'}\ t$
T ψ $\psi\ b$ $\psi^{'}$ $\ge \psi^{'}\ b$

$L \quad D \subseteq L^1 \ T \ \to L^1 \ T$ $\quad L \ x \quad -x^{''} \quad x \in D$
$\{x \in W^{2,1} \ T \quad x \quad x \ b \ , \ x^{'} \quad x^{'} \ b \ \}$

H_0 ϕ
ψ

Theorem 5.

If *hypotheses* $H \ F \ _1$ *and* H_0 *hold,*
then *problem has extremal solutions in the order interval* $K \quad \psi, \phi$

Theorem 6.

If *hypotheses* $H \ F \ _2$ *and* H_0 *hold,*
then *problem has a solution* $x \in W^{2,1} \ T$ *in the order interval* $K \quad \psi, \phi$

Remark.

References

[1] Bernfeld, S., Lakshmikantham, V., *An Introduction to Nonlinear Boundary Value Problems*, Academic Press, New York (1974).

[2] Bressan, A., Colombo, G., *Extensions and selections of maps with decomposable values*, Studia Math. 90 (1988), 69–85.

[3] Brezis, H., *Analyse Fonctionelle*, Masson, Paris (1983).

[4] Cabada, A., Nieto, J., *Extremal solutions of second order nonlinear boundary value problems*, Appl. Math. Comp. 40 (1990), 135–145.

[5] Chang, K.C. *The obstacle problem and partial differential equations with discontinuous nonlinearities*, Comm. Pure Appl. Math. 33 (1980), 117–146.

[6] DeBlasi, F.S., Myjak, J., *On continuous approximations for multifunctions*, Pacific J. Math. 123 (1986), 9–31.

[7] Dunford, N., Schwartz, J., *Linear Operators I*, Wiley, New York (1958).

[8] Erbe, L., Krawcewicz, W., *Boundary value problems for differential inclusions* $y^{''} \in F \ \ t, y, y^{'}$, Annales Polon Math. 56 (1990), 195–226.

[9] Gaines, R., Mawhin, J., *Coincidence Degree and Nonlinear Differential Equations*, Springer Verlag, Berlin (1977).

[10] Gao, W., Wang, J., *On nonlinear second order periodic boundary value problem with Caratheodory functions*, Annales Polon Math. 62 (1995), 283–291.

[11] Gilbarg, D., Trudinger, N., *Elliptic Partial Differential Equations of Second Order*, Springer Verlag, New York (1977).

[12] Himmelberg, C., *Fixed points of compact multifunctions*, J. Math. Anal. Appl. 38 (1972).

[13] Kandilakis, D., Papageorgiou, N.S., *Dirichlet and periodic problems for second order differential inclusions*, Houston J. Math. – to appear.

[14] Klein, E., Thompson, A., *Theory of Correspondences*, Wiley, New York (1984).

[15] Kravvaritis, D., Papageorgiou, N.S., *Boundary value problems for nonconvex differential inclusions*, J. Math. Anal. Appl. 185 (1994), 146–160.

[16] Leela, S., *Monotone method for second order periodic boundary value problems*, Nonl. Anal. – TMA 7 (1983), 349–355.

[17] Marano, S., *Existence theorems for multivalued boundary value problems*, Bull. Austr. Math. Soc. 45 (1992), 249–260.

[18] Mönch, H., *Boundary value problems for nonlinear ordinary differential equations of second order in Banach spaces*, Nonl. Anal. – TMA 4 (1980), 985–999.

[19] Nieto, J., *Nonlinear second order periodic value problems with Caratheodory functions*, Appl. Anal. 34 (1989), 111–128.

[20] Nieto, J., Cabada, A., *A generalized upper and lower solutions method for nonlinear second order ordinary differential equations*, J. Appl. Math. Stoch. Anal. 5 (1992), 157–166.

[21] Nkashama, M. N., *A generalized upper and lower solutions method and multiplicity results for nonlinear first-order ordinary differential equations*, J. Math. Anal. Appl. 140 (1989), 381–395.

[22] Omari, P., *A monotone method for constructing extremal solutions of second order scalar boundary value problems*, Appl. Math. Comp. 18 (1986), 257–275.

[23] Omari, P., Trombetta, M., *Remarks on the lower and upper solutions method for second and third order periodic boundary value problems*, Appl. Math. Comp. 50 (1992), 1–21.

[24] Papageorgiou, N. S., *Convergence theorems for Banach space valued integrable multifunctions*, Intern. J. Math.. Sci. 10 10 (1987), 433–442.

[25] Papageorgiou, N. S., *On measurable multifunctions with applications to multivalued equations*, Math. Japonica 32 (1987), 437–464.

[26] Papageorgiou, N. S. "Decomposable sets in the Lebesgue–Bochner spaces", Comm. Math. Univ. Sancti Pauli 37 (1988), pp. 49–62.

[27] Papageorgiou, N. S., Papalini, F., *Periodic and boundary value problems for second order differential equations*, Trans. AMS – to appear.

[28] Vrabie, I., *Compactness Methods for Nonlinear Evolutions*, Longman Scientific and Technical, Essex, U.K. (1987).

[29] Wagner, D., *Survey of measurable selection theorems*, SIAM J. Control Optim. 15 (1977), 859–903.

NATIONAL TECHNICAL UNIVERSITY
DEPARTMENT OF MATHEMATICS
ZOGRAFOU CAMPUS
ATHENS 157 80, GREECE
E-MAIL: NPAPG@MATH.NTUA.GR