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A PARALLELOGRAM CONFIGURATION CONDITION IN NETS

Jitka Markvartová

Abstract. After describing a (general and special) coordinatization of k-nets there are found algebraic equivalents for the validity of certain quadrangle configuration conditions in k-nets with small degree k.

k

k § §

k

§ § k

§

§1 Definition of a k-net

k net $k \geq$ $\mathcal{N}$ $\mathcal{P}, \mathcal{L}, \|, \mathrm{I}$ $\mathcal{P}$ $\mathcal{L}$
I $\mathcal{P}$ $\mathcal{L}$ $\mathrm{I} \subseteq \mathcal{P} \times \mathcal{L}$ $\|$
$\mathcal{L}$ $\mathcal{L}_1, \dots, \mathcal{L}_k$

$P \in \mathcal{P}$ $i \in \{\ , \dots, k\}$ $h_i \in \mathcal{L}_i$
$P \ \mathrm{I} \ h_i$

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$i, j \in \{\ , \dots, k\}$ i / j $h_i \in \mathcal{L}_i$ $h_j \in \mathcal{L}_j$
$\{X \in \mathcal{P} \mid X \,\mathrm{I}\, h_i \quad X \,\mathrm{I}\, h_j\}$
$h \in \mathcal{L}$ $\{X \in \mathcal{P} \mid X \,\mathrm{I}\, h\} >$
$\mathcal{L}_1 \quad \cdots \quad \mathcal{L}_k$
order $\mathcal{N}$ k $\mathcal{N}$

A, B *joinable* h $A \,\mathrm{I}\, h$ $B \,\mathrm{I}\, h$
A, B *non-joinable*

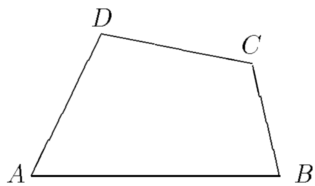
A, B $A\bullet \quad \bullet B$
$A\bullet \quad \bullet B$ A, B $\mathcal{L}_i$ $A \bullet^{\,i} \bullet B$
h h $\{X \in \mathcal{P} \mid X \,\mathrm{I}\, h\}$ P
P $\{x \in \mathcal{L} \mid P \,\mathrm{I}\, x\}$
h h

A, B C
$A \,\mathrm{I}\, C$ $B \,\mathrm{I}\, C$ *join line* A, B $A \sqcup B$
A $i \in \{\ , \dots, k\}$ a $a \in \mathcal{L}_i$ $A \,\mathrm{I}\, a$
$A \sqcup i$

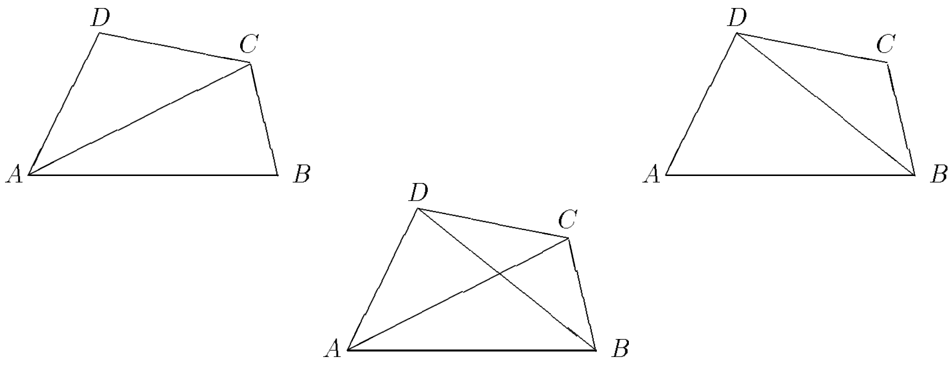
a, b $\mathcal{L}_i$ $\mathcal{L}_j$ C
$C \,\mathrm{I}\, a$ $C \,\mathrm{I}\, b$ *intersection point* $a \sqcap b$

§2 A configurational condition

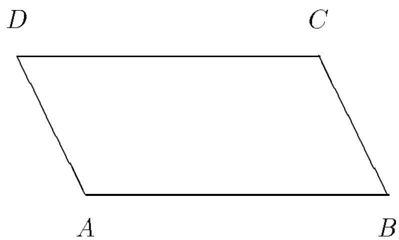
Definition 2.1. A, B, C, D
k *quadrangle* $A\bullet \ \bullet B$ $B\bullet \ \bullet C$ $C\bullet \ \bullet D$ $D\bullet \ \bullet A$
A, B, C, D



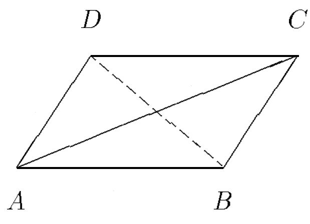
A, B, C, D *to have the first* *the second diagonal*
$A\bullet \ \bullet B$ $B\bullet \ \bullet C$ $C\bullet \ \bullet D$ $D\bullet \ \bullet A$ $A\bullet \ \bullet C$ $A\bullet \ \bullet B$ $B\bullet \ \bullet C$ $C\bullet \ \bullet D$
$D\bullet \ \bullet A$ $B\bullet \ \bullet D$ *to be without the first* *the second diagonal*
$A\bullet \ \bullet C$ $B\bullet \ \bullet D$ *to have both diagonals*
$A\bullet \ \bullet B$ $B\bullet \ \bullet C$ $C\bullet \ \bullet D$ $D\bullet \ \bullet A$ $A\bullet \ \bullet C$ $B\bullet \ \bullet D$



A, B, C, D $ABCD$ $A \sqcup B$
$B \sqcup C$ $C \sqcup D$ $D \sqcup A$ *sides* $A \sqcup C$ $B \sqcup D$
diagonals
$ABCD$ *parallelogram* $A \sqcup D \| B \sqcup C$ $A \sqcup B \| C \sqcup D$



$\mathcal{N}$ $\mathcal{P}, \mathcal{L}, \|, \mathrm{I}$ 1234
parallelogram "letter" condition $ABCD$ $A \bullet \overset{1}{} \bullet D$ $A \bullet \overset{2}{} \bullet B$
$A \sqcup C$ $A \bullet \overset{3}{} \bullet C$ $B \sqcup D$
$B \bullet \overset{4}{} \bullet D$



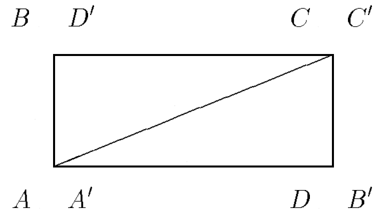
LP_{ijkh} i, j, k, h $\{\ ,\ ,\ ,\ \}$

§3 An elementary property of given configuration condition

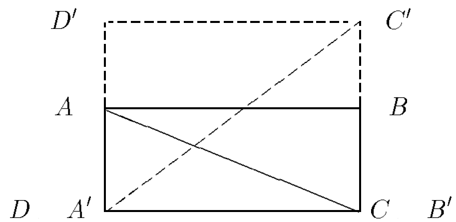
Lemma 3.1. *In a 4-net $\mathcal{N}$ let there hold the condition LP_{1234}. Then in $\mathcal{N}$ there holds the condition LP_{ijkh} for* $i, j \in \{ \; , \quad , \; \}$, $k, h \in \{ \; , \quad , \; \}$.

Proof. $\mathcal{N}$

A, B, C, D $\mathcal{N}$ $A \bullet^2 \bullet D$ $A \bullet^1 \bullet B$ $A \bullet^3 \bullet C$ $B \bullet^2 \bullet C$ $D \bullet^1 \bullet C$ $_{2134}$



A' A B' D C' C D' B A', B', C', D' $_{1234}$ $D' \sqcup B'$ $D' \bullet^4 \bullet B'$ $D \sqcup B$ $D \bullet^4 \bullet B$ $_{2134}$ A, B, C, D $\mathcal{N}$ $A \bullet^1 \bullet D$ $A \bullet^2 \bullet B$ $A \bullet^4 \bullet C$ $B \bullet^1 \bullet C$ $C \bullet^2 \bullet D$



A, B, C, D $_{1243}$ A' D B' C C' $A' \sqcup$ $\sqcap$ $B' \sqcup$ D $C' \sqcup$ $\sqcap$ $A' \sqcup$ A', B', C', D' $_{1234}$ $B' \sqcup D'$ $B' \bullet^4 \bullet D'$ D' A C' B $D \sqcup B$ $D \bullet^3 \bullet B$ $_{1243}$ $_{2143}$ □

§4 Coordinatization of a k-net

Definition 4.1. $S_1, \ldots, S_k$ $\subseteq S_1 \times \cdots \times S_k$ *admissible relation*

$$\forall \alpha, \beta \in \{ \; , \ldots, k\}, \alpha \;/\; \beta \; \forall x \in S_\alpha, \forall y \in S_\beta \exists \; x_1, \ldots, x_k \; \in \quad x \quad x_\alpha, y \quad y_\beta .$$

Remark 4.2. $\subset S_1 \times \cdots \times S_k$
$\alpha, \beta, \gamma \in \{\ ,\ldots,k\}$ $\alpha \;/\; \beta \;/\; \gamma \;/\; \alpha$ $\alpha\beta\gamma$ $a_\alpha, a_\beta \mapsto$
a_γ $\alpha\beta\gamma$ a_α, a_β $a_\gamma \Leftrightarrow \exists\ x_1,\ldots,x_k \in$ x_α a_α x_β a_β x_γ a_γ

$$\frac{k}{k-}$$

Definition 4.3. $\mathcal{N}$ $\mathcal{P}, \mathcal{L},\ \mathrm{I}\ , \mathcal{L}_1 \ldots \mathcal{L}_k$ k σ $\mathcal{P} \to$
$\mathcal{L}_1 \times \cdots \times \mathcal{L}_k$

$$P \mapsto\ a_1,\ldots,a_k \quad a_1 \in \mathcal{L}_1,\ldots,a_k \in \mathcal{L}_k,$$

coordinatization $\mathcal{N}$ $\mathcal{R}_\mathcal{N}$ $\{\sigma\ P \mid P \in \mathcal{P}$
coordinatizing relation $\mathcal{N}$ k

Lemma 4.4. *Let* $\mathcal{P}, \mathcal{L}, \mathcal{L}_1,\ldots,\mathcal{L}_k,\ \mathrm{I}$ *be a k-net. Then $\mathcal{R}_\mathcal{N}$ is an admissible relation.*

Proof.
a_i, a_j $a_i \in \mathcal{L}_i$ $a_j \in \mathcal{L}_j$ $i,j \in \{\ ,\ldots,k\}$ $i \;/\; j$ $a_i \sqcap a_j$
$h \in \{\ ,\ldots,k\}$ $h \;/\; i,j$ a_h $a_1,\ldots,a_k \in \mathcal{P}$
$a_1 \in \mathcal{L}_1,\ldots,a_k \in \mathcal{L}_k$ □

Lemma 4.5. *Let $\mathcal{R} \subset S_1 \times \cdots \times S_k$ be an admissible relation. Then there exists a k-net $\mathcal{N}$ $\mathcal{P}, \mathcal{L}, \mathcal{L}_1,\ldots,\mathcal{L}_k,\ \mathrm{I}$ such that $\mathcal{R}$ is isotopic*[1] *with $\mathcal{R}_\mathcal{N}$.*

Proof. $S_1,\ldots,S_k$ $S_i \geq$ $i \in \{\ ,\ldots,k\}$ $\mathcal{R} \subset$
$S_1 \times \cdots \times S_k$ $S'_1,\ldots S'_k$

$$\alpha_i \quad S_i \to S'_i \quad i \in \{\ ,\ldots,k\}.$$

$S'_1,\ldots,S'_k$

$$\mathcal{P} \qquad S'_1 \times \cdots \times S'_k$$
$$\mathcal{L}_i \qquad S'_i, i \in \{\ ,\ldots,k\}$$
$$\mathcal{L} \qquad \mathcal{L}_1 \cup \cdots \cup \mathcal{L}_k$$

$P \in h$ $P \in \mathcal{P}$ $h \in \mathcal{L}$ $P\ \mathrm{I}\ h$
$\mathcal{P}, \mathcal{L}, \mathcal{L}_1,\ldots,\mathcal{L}_k,\ \mathrm{I}$ k
$A \in \mathcal{P}$ $i \in \{\ ,\ldots,k\}$ $h_i \in \mathcal{L}_i$
$A\ \mathrm{I}\ h_i$
$h'_i \in \mathcal{L}_i$ $h'_j \in \mathcal{L}_j$ $i,j \in \{\ ,\ldots,k\}$ $i \;/\; j$
h'_i $a'_1,\ldots,a'_{i-1}, h'_i, a'_{i+1},\ldots,a'_{j-1}, h'_j, a'_{j+1},\ldots,a'_k$

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[1]Two relations $\mathcal{A} \subseteq A_1 \times \cdots \times A_k$, $\mathcal{B} \subseteq B_1,\ldots,B_k$ are said to be *isotopic*, if there exist bijections $\gamma_1 : A_1 \to B_1,\ldots,\gamma_k : A_k \to B_k$ such that for every $(x_1,\ldots,x_k) \in A_1 \times \cdots \times A_k$ it holds $(x_1,\ldots,x_k) \in \mathcal{A} \leftrightarrows (\gamma_1(x_1),\ldots,\gamma_k(x_k)) \in \mathcal{B}$

h'_j $b'_1, \dots, b'_{i-1}, h'_i, b'_{i+1}, \dots, b'_{j-1}, h'_j, b'_{j+1}, \dots, b'_k$ $a'_1, \dots, a'_k$
$b'_1, \dots, b'_k$ k $S'_1 \times \dots \times S'_k$ $a'_1, \dots, a'_k$
$b'_1, \dots, b'_k$

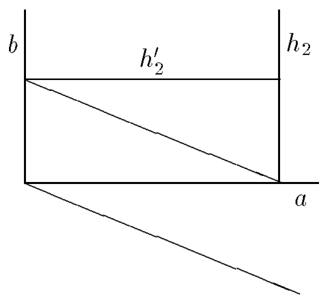
$$\{X \in \mathcal{P} \mid X \mathrm{I}\ h'_i \quad X \mathrm{I}\ h'_j\} \quad .$$

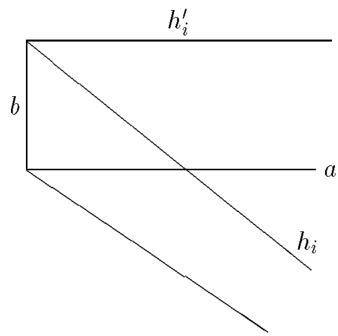
h $A' \mathrm{I}\ h$ $h \in$
$\mathcal{L}_m$ $m \in \{\ , \dots, k\}$ A' $a'_1, \dots, a'_k$ $a'_i \in S'_i$ $i \in \{\ , \dots, k\}$
$S'_i \geq$ B' $b'_1, \dots, b'_k$ $\in h$ $b'_i \in S'_i$ $i \in \{\ , \dots, k\}$
$a'_i \ / \ b'_i$ $i \in \{\ , \dots, k\}$ $i \ / \ m$ $A' \ / \ B'$ $\{X \in \mathcal{P} \mid X \mathrm{I}\ h\} \geq$ □

admissible algebra S, $S \geq$
$\in S$ φ_i $_{i \in \{3, \dots, k\}}$ S
$_i$ $_{i \in \{3, \dots, k\}}$

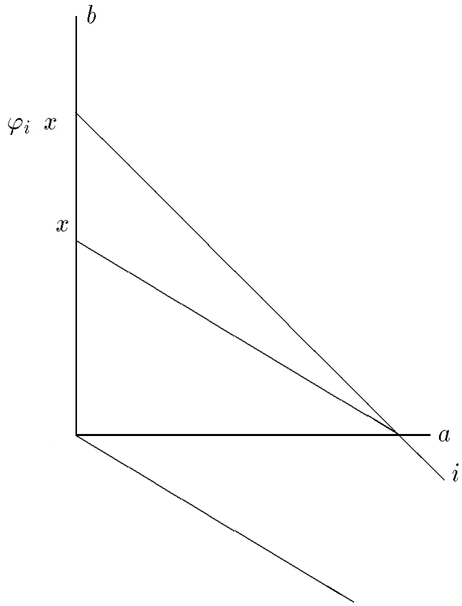
φ_3 $_S$
$\forall c_1, c_2 \in S,\ i_1, i_2 \in \{\ , \dots, k\}$ $i_1 \ / \ i_2$
$\exists\ x, y \ \in S \times S$ $\varphi_{i_1}\ x$ $_{i_1}\ y$ c_1 $\varphi_{i_2}\ x$ $_{i_2}\ y$ c_2

Construction 4.6. k $\mathcal{N}$ $\mathcal{P}, \mathcal{L}, \|, \mathrm{I}$
$a \in \mathcal{L}_1$ $b \in \mathcal{L}_2$ $a \sqcap b$
S $\mathcal{L}_1$
λ_2 $\mathcal{L}_2 \to S$ $h_2 \mapsto h'_2$ $\{\ h_2 \sqcup a \ \sqcup \ \ \sqcap b\} \sqcup$

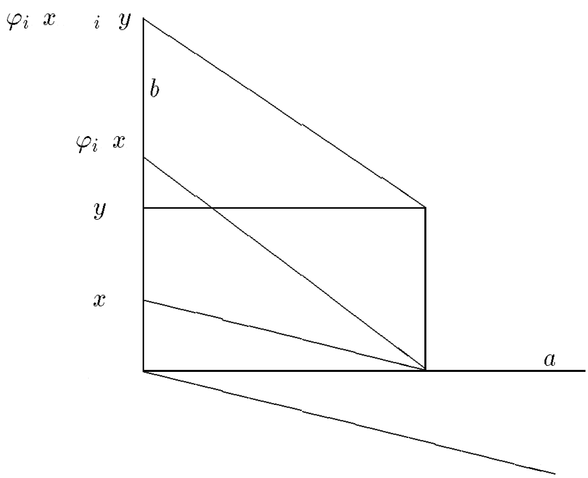




λ_i $\mathcal{L}_i \to S$ $h_i \mapsto h'_i$ $h_i \sqcap b \ \sqcup$ $i \in \{\ , \dots, k\}$
φ_i $S \to S$ $x \mapsto \varphi_i\ x$ $\{\ x \sqcup \ \ \sqcup a \ \sqcup i\} \sqcap b$ $i \in \{\ , \dots, k\}$
φ_3 $_S$



$_i\ S \times S \to S\quad x, y \mapsto \varphi_i\ x\quad {}_i\ y\qquad i\quad x, y\qquad i \in \{\ , \ldots, k\}$



3 $S,\ \ ,\ \varphi_i\ {}_{i\in\{3,\ldots,k\}},$
$i\ {}_{i\in\{3,\ldots,k}$ *coordinatizing algebra* $\mathcal{N}$

Remark 4.7. k $\mathcal{N}$

$$S,\ \ ,\ \varphi_i\ {}_{i\in\{3,\ldots,k\}},\quad i\ {}_{i\in\{3,\ldots,k\}}\ .$$

$$\mathcal{L}_1 \times \dots \mathcal{L}_k \to \overbrace{S \times \cdots \times S}^{k} \quad h_1, \dots, h_k \;\mapsto\; x_1, \dots, x_k$$

$\mathcal{L}_1 \quad S, \lambda_2 \quad \mathcal{L}_2 \to S,\ h_2 \mapsto h_2' \quad \{\ h_2 \sqcap a \sqcap \quad \sqcap b\} \sqcup \ , \quad a, b \in S,$

$\lambda_i \quad \mathcal{L}_i \to S, h_i \mapsto h_i' \quad \{h_i \sqcap b\} \sqcup \ ,$

$\mathcal{R}_{\mathcal{N}}$

Lemma 4.8. *Every admissible algebra is a coordinatizing algebra of a suitable k-net.*

Proof. $S, \ , \varphi_i\ {}_{i \in \{3,\dots,k\}},\ t_i\ {}_{i \in \{3,\dots,k\}}$ $\mathcal{P}$ $S \times S$ $\mathcal{L}_i$ $\{\{\ a, y \mid y \in S\} \mid a \in S\}$, $\mathcal{L}_2$ $\{\{\ x, b \mid x \in S\} \mid b \in S\}$ $\mathcal{L}_i$ $\{\{\ x, y \mid y \quad \varphi_i\ x \quad {}_i\ y \quad c_i\} \mid c_i \in S\}$ $i \in \{\ , \dots, k\}$ $\mathcal{L}$ $\mathcal{L}_1 \cup \mathcal{L}_2 \cup \mathcal{L}_3 \cup \cdots \cup \mathcal{L}_k$ $\mathrm{I} \subseteq \mathcal{P} \times \mathcal{L}$ $\mathcal{P}, \mathcal{L}, \mathcal{L}_1, \dots, L_k, \mathrm{I}$ k □

§5 Algebraic properties of k-nets satisfying given configurational conditions

Theorem 5.1. *Let $\mathcal{N}$ $\mathcal{P}, \mathcal{L}, \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4, \mathrm{I}$ be a net and $S, \ , \varphi_3, \varphi_4$, ${}_3,\ {}_4$ its coordinatizing algebra with respect to the origin . Then in $\mathcal{N}$ there holds condition LP_{1234} if and only if*

a) ${}_3 \quad {}_4$ *where* $S,$ *is a commutative group,*

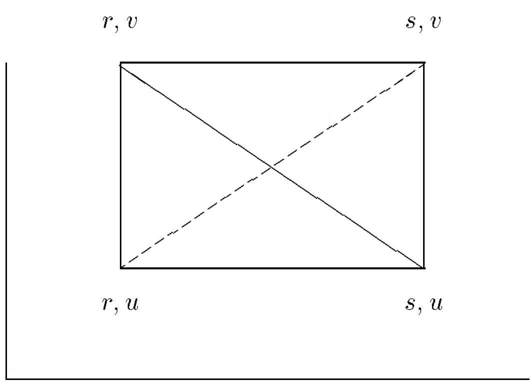
b) $\varphi_4\ x \quad -x\ \mathrm{I}\ \forall x \in S$, *where $-x$ is the opposite element of $x \in S$ in* $S,$.

Proof. $\mathcal{L}$

${}_{1234}$ $\mathcal{N}$ $\mathcal{N}$

${}_{1243}$ ${}_3,\ {}_4$ ${}_{1234}$

$$\forall r, u, s, v \in S \quad r \quad {}_3\, v \quad s \quad {}_3\, u \Rightarrow \varphi\ r \quad {}_4\, u \quad \varphi\ s \quad {}_4\, v$$



1243

$$\forall r, u, s, v \in S \quad \varphi\ r \quad {}_4\, u \quad \varphi\ s \quad {}_4\, v \Rightarrow r \quad {}_3\, v \quad s \quad {}_3\, u.$$

r , s v u φ v ${}_4\, v$ $\forall u, v \in S$ v v ${}_3\, u$
u

$$\varphi\ v \quad {}_4\, v$$

$v \in S$
s v r ${}_3\, v$ v ${}_3\, u$ φ r ${}_4\, u$ φ v ${}_4\, v$
$u, r, v \in S$
φ r ${}_4\, u$ r u
u ${}_3\, v$ v ${}_3\, u$ $u, v \in S$ $S,$ ${}_3$
r φ s u v φ^{-1} u ${}_u$ u u ${}_4\, v$
$u, v \in S$
v

$$\varphi^{-1}\ u \quad {}_3\, u$$

$u \in S$
r v s ${}_3\, u$
$u, s \in S$ u φ s

$$s \quad {}_3\, \varphi\ s$$

$s \in S$
$a \in S$ φ^{-1} a ${}_3\, a$
a ${}_3\, \varphi$ a
$S,$ ${}_3$

$$\varphi\ a \quad \varphi^{-1}\ a$$

$a \in S$
u φ r φ s ${}_4\, v$ $r, s, v \in S$
r ${}_3\, v$ s

$$\varphi\ r \quad \varphi\ r \quad {}_3\, v \quad {}_4\, v$$

$r, v \in S$
s r ${}_3\, v$ u φ r ${}_4\, u$ v
$r, u, v \in S$

$$\varphi\ r \quad {}_4\ r \quad {}_3\, v \quad v$$

$r, v \in S$

$$\varphi\ r\ \ _3 v\ \ _4 v\ \ _4\ r\ \ _3 v\ \ v$$

$r, v \in S$ $S,$ 3 $z \in S$ z r $_3 v$

$$\varphi\ z\ \ _4 v\ \ _4 z\ \ v$$

$z, v \in S$

$d \in S$ z $\varphi\ d$

$$d\ \ _4 v\ \ _4 \varphi\ d\ \ v$$

$d, v \in S$

c d $_4 v$

$$c\ \ _4 \varphi\ d\ \ v\,.$$

a b $x \in S$ b a $_4 x$

a d b c

$$x\ \ v\ \ c\ \ _4 \varphi\ d\ \ b\ \ _4 \varphi\ a\,.$$

$$a\ \ _4 x\ \ b$$

$$x\ \ b\ \ _4 \varphi\ a$$

$a, b \in S$

$$r\ \ _3 v\ \ v\ \ _4 \varphi\ r$$

$r, v \in S$ φ^2

$$r\ \ _3 v\ \ v\ \ _4 r$$

$r, v \in S$

$S,$ 3

$$v\ \ _3 r\ \ v\ \ _4 r$$

$r, v \in S$ 3 4 3

$a, b, c \in S$

$$a \quad c \quad b \quad \varphi b \quad a \quad c\ .$$

$\mathcal{N}$ 1234

$$\varphi a \quad \varphi b \quad a \quad c \quad \varphi b \quad c.$$

a

$$a \quad \varphi a \quad \varphi b \quad a \quad c \quad a \quad \varphi b \quad c\ .$$

$$\varphi b \quad a \quad c \quad a \quad \varphi b \quad c$$

$a, b, c \in S$
$a \quad x \quad c \quad y \quad b \quad \varphi z$

$$x \quad y \quad z \quad a \quad c \quad \varphi b\ .$$

$$a \quad c \quad \varphi b \quad a \quad \varphi b \quad c \quad a \quad c \quad \varphi b \quad x \quad y \quad z\ .$$

$x \quad y \quad z \quad x \quad y \quad z \quad x \quad y \quad z \in S$
$S,$

$\mathcal{N}$
$r \quad u \quad s \quad v \in S$

$$r \quad v \quad s \quad u\ .$$

$S,$

$$r \quad v - r \quad s \quad u - r\ ,$$

$v \quad s \quad u - r$ $-s$

$$-s \quad v \quad u - r,$$

$$r \quad v \quad s \quad u$$

$$-r \quad u \quad -s \quad v.$$

$\varphi x \quad -x \quad x \in S$

$$r \quad v \quad s \quad u \qquad \varphi r \quad u \quad \varphi s \quad v$$

$r \quad s \quad u \quad v \in S$ 1234 □

Theorem 5.2. *Let* $\mathcal{N}$ $\mathcal{P}, \mathcal{L}, \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4, \mathcal{L}_5, \mathcal{L}_6,$ I *be a net and* $S,\ , \varphi_3, \varphi_4, \varphi_5, \varphi_6,\ _3,\ _4,\ _5,\ _6$ *its coordinatizing algebra with respect to the origin* . *Then in* $\mathcal{N}$ *there holds condition* LP_{1256} *if and only if*

a) $_5$ $_6$ $\oplus$ *and* $S, \oplus$ *is a commutative group*

b) $_6\ x$ $\ominus\varphi_5\ x$ $\forall x \in S$. *Here the symbol* $\ominus$ *designates the subtraction in* $S, \oplus$.

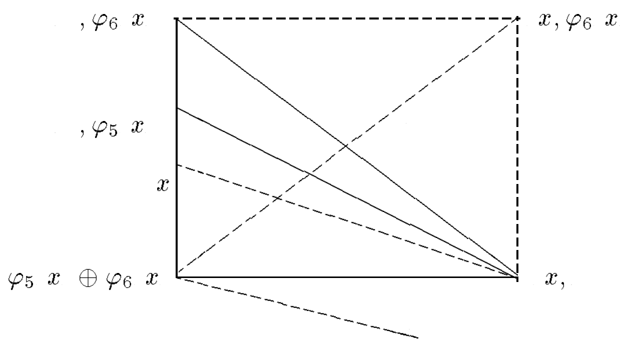
Proof. $\mathcal{N}$

$\mathcal{N}$ $_{1256}$

$$a \quad _i\, b \qquad i\ \varphi_i^{-1}\ a\ , b \quad \forall a, b \in S \quad \forall i \in \{\ ,\ ,\ ,\ \}.$$

$_5$ $_6$ $_5$ $\oplus$ $S, \oplus$

$_{1256}$ $\varphi_5\ x\ \oplus$ $\varphi_6\ x$ $x \in S$



$\mathcal{N}$

$x_1, x_2, y_1, y_2 \in S$

$$\ominus\varphi_5\ x_1\ \oplus y_1 \quad \varphi_5\ x_2 \quad y_2.$$

$$\varphi_5\ x_1\ \oplus y_2 \quad \ominus\varphi_5\ x_2\ \oplus y_1.$$

$$\varphi_6\ x \quad \ominus\varphi_5\ x$$

$$\varphi_6\ x_1\ \oplus y_2 \quad \varphi_6\ x_2\ \oplus y_1.$$

$$\varphi_5\ x_1\ \oplus y_1 \quad \varphi_5\ x_2\ \oplus y_2 \Rightarrow \varphi_6\ x_1\ \oplus y_2 \quad \varphi_6\ x_2\ \oplus y_1$$

$_{1256}$ □

Corollary 5.3. *Let $\mathcal{N}$ be a net with even degree $k \geq$ and $S, \ , \ \varphi_i \ {}_{i\in\{3,\dots,k\}},$ ${}_i \ {}_{i\in\{3,\dots,k\}}$ its coordinatizing algebra with respect to the origin .*
Then in $\mathcal{N}$ there holds condition $LP_{1,2(2h+1),(2h+2)}$ $\forall h \in \{ \ , \ , \dots, \frac{k}{2} - \ \}$ if and only if

a) ${}_{2h+2} \quad {}_{2h+1} \quad \oplus_h$ *and* $S, \oplus_h$ *is a commutative group.*

b) $\varphi_{2h+2} \ x \quad \ominus_h \varphi_{2h+1} \ x \quad \forall x \in S$ *where the symbol* $\ominus_h$ *designates the subtraction in* $S, \oplus_h$, $h \in \{ \ , \dots, \frac{k}{2} - \ \}$.

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