Archivum Mathematicum

Miroslav Bartušek

On proper oscillatory solutions of the nonlinear differential equations of the n-th order

Archivum Mathematicum, Vol. 24 (1988), No. 2, 89--98

Persistent URL: http://dml.cz/dmlcz/107315

ARCHIVUM MATHEMATICUM (BRNO)
Vol. 24, No. 2 (1988), 89–98

ON PROPER OSCILLATORY SOLUTIONS OF THE NONLINEAR DIFFERENTIAL EQUATIONS OF THE N-TH ORDER

MIROSLAV BARTUŠEK

(Received May 14, 1987)

Abstract. There are given sufficient conditions for the existence of unbounded proper oscillatory solutions or solutions tending to zero of the differential equation (1).

Key words. Ordinary differential equations, nonlinear oscillations, unbounded solutions.

MS Classification. 34 C 15, 34 C 10

I. Consider the differential equation

$$y^{(n)} = f(t, y, \dots, y^{(n-1)}), \qquad n \geqq 2. \tag{1}$$

In all the paper we shall suppose that $f: D \to R$, $D = R_+ \to R^n$ is continuous and there exists a number $\alpha \in \{0, 1\}$ such that

$$(-1)^\alpha f(t, x_1, \dots, x_n)\, x_1 \geqq 0 \qquad \text{in } D. \tag{2}$$

Let y be a solution of (1) defined on $[0, b)$, $b \leqq \infty$. Then y is called noncontinuable if either $b = \infty$ or $b < \infty$, $\limsup_{t \to b} \sum_{i=0}^{n-1} | y^{(i)}(t) | = \infty$. y is called proper if $b = \infty$ and for large t $\sup_{t \leqq s < \infty} | y(s) | > 0$ holds. y is called singular of the 1-st kind if $b = \infty$ and $y \equiv 0$ in some neighbourhood of $t = \infty$, it is called singular of the 2-nd kind if $b < \infty$ and $\limsup_{t \to b} | y^{(n-1)}(t) | = \infty$. The solution $y \equiv 0$ on R_+ is called trivial.

The proper solution is called oscillatory if there exists a sequence of its zeros tending to ∞.

Denote n_0 the entire part of $n/2$.

In the two last decades a great effort is devoted to the study of solutions of (1), (2) (see the monography [2] or [7], [11] and references there). One of the important problems consists in the study of asymptotic behaviour of proper solutions. In contradistinction to the nonoscillatory solutions the asymptotic behaviour of the oscillatory ones for $n > 2$ remains unexamined thoroughly (even for generalized Emden–Fowler equation, see [7]). Some considerations concerning the proper solutions tending to zero for $t \to \infty$ are given in [2] and [8]. But (1) has under certain assumptions the unbounded oscillatory solutions, too. Thus this paper is likely to contribute to the elucidation of this problem.

The equation (1) is said to have the property A if each proper solution y of this equation is oscillatory when n is even and it is either oscillatory or

$$\lim_{t\to\infty} | y^{(i-1)}(t) | \downarrow 0, \qquad i = 1, \dots, n,$$

when n is odd.

The equation (1) is said to have the property B^* if each proper solution y of (1) is either oscillatory or

$$y^{(i)}(t)\, y(t) > 0, \qquad i = 0, 1, \dots, n-1 \tag{3}$$

for large values of t holds when n is odd and is either oscillatory or bounded or (3) holds when n is even.

There are many conditions, under the validity of which (1) has the property A or B^*. With respect to our further considerations only the following are given here.

Lemma 1. ([6, 5, 1]). *Let $\alpha = 1$ and let one of the following assumptions hold:*

1° *Let for an arbitrary large positive number c there exist $\lambda_c \neq 1$ and a continuous function $a_c : R_+ \to R_+$ such that*

$$\int_1^\infty t^{(n-1)\lambda_c^*} a_c(t)\, dt = \infty$$

and

$$| f(t, x_1, \dots, x_n) | \geqq a_c(t) \, | x_1 |^{\lambda_c}$$

holds for $\frac{1}{c} \leqq | x_1 | \leqq ct^{n-1}$, $| x_i | \leqq c \, | x_1 |^{\frac{n-i}{n-1}}$, $i = 2, \dots, n$, *where* $\lambda_c^* = \lambda_c$ ($\lambda_c^* = 1$) *in case* $\lambda_c < 1$ ($\lambda_c > 1$);

2° *Let a continuous function $a : R_+ \to R_+$ exist such that*

$$| f(t, x_1, \dots, x_n) | \geqq a(t) \, | x_1 | \qquad \text{in } D$$

holds and $\limsup_{t\to\infty} t \int_t^\infty s^{n-2} a(s)\, ds > (n-1)!$;

3° *Let continuous functions $a : R_+ \to R_+$, $g : R_+^2 \to R_+$ exist such that* $\int_0^\infty a(t)\, dt = \infty$, $g(s_1, s_2) > 0$ *for* $s_1 > 0$, *g is nondecreasing with respect to the 1-st argument and* $| f(t, x_1, \dots, x_n) | \geqq a(t)\, g(| x_1 |, | x_n |)$ *in D hold. Then the equation* (1) *has the property A.*

Lemma 2. ([8]). *Let $n \geqq 3$, $\alpha = 0$ and let continuous functions $a : R_+ \to R_+$, $g : R_+ \to R_+$ and a number λ exist such that $\int_1^\infty t^\mu a(t)\,\mathrm{d}t = \infty$, $g(0) = 0$, $g(x) > 0$ for $x > 0$, $\liminf_{x\to\infty} \frac{g(x)}{x^\lambda} > 0$, $|f(t, x_1, \dots, x_n)| \geqq a(t)\,g(|x_1|)$ in D hold where*

$$\mu = 1 + (n-2)\lambda \quad \text{for } \lambda < 1, \qquad \mu < n - 1 \quad \text{for } \lambda = 1, \tag{4}$$

$$\mu = n - 1 + \frac{1}{2}(1 + (-1)^n)(\lambda - 1) \qquad \text{for } \lambda > 1.$$

Then the equation (1) *has the property B^*.*

In [2] the conditions are given under which the oscillatory solutions of (1) are unbounded if

$$\text{either } n \text{ is even, } n_0 + \alpha \text{ odd} \qquad \text{or} \qquad n \text{ is odd.} \tag{5}$$

From this and with respect to the existence theorems for oscillatory solutions ([8, 3]) the following theorem is valid.

Theorem 1. *Let* (5) *be valid and let $n \geqq 4$ in case $\alpha = 0$. Let the equation* (1) *have the property $A(B^*)$ if $\alpha = 1 (\alpha = 0)$. Let there exist a constant $M > 0$ and continuous functions $g : R_+ \to R_+$, $g_1 : R_+ \to (0, \infty)$* such that $\int_0^\infty \frac{\mathrm{d}t}{g_1(t)} = \infty$, $g(x) > 0$ for $x > 0$, $\liminf_{x\to\infty} g(x) > 0$,

$$(n - 2n_0)\,g(|x_1|) \leqq |f(t, x_1, \dots, x_n)| \leqq t^{\frac{n_0\sigma}{n_0 - 1}} g_1(|x_1|) \qquad \text{in } [M, \infty) \times R^n$$

hold where

$$\sigma = 1 - (n - 2n_0) \tag{6}$$

Then there exists the continuum of unbounded oscillatory solutions of (1).

Consider the generalized Emden–Fowler's equation

$$y^{(n)} = a(t)\,|y|^\lambda \operatorname{sign} y, \qquad n \geqq 2, \tag{7}$$

where $\lambda > 0$, $a : R_+ \to R$ is continuous and there exists $\alpha \in \{0, 1\}$ such that $(-1)^\alpha a(t) \geqq 0$, $t \in R_+$ holds.

Corollary 1. *Let* (5) *be valid, $n \geqq 4$ in case of $\alpha = 0$ and let σ, μ be given by* (4), (6), *respectively. Suppose that there exist positive constants K, K_1, M such that*

$$K_1(n - 2n_0) \leqq (-1)^\alpha a(t) \leqq K t^{\frac{n_0\sigma}{n_0 - 1}} \qquad \text{for } t \in [M, \infty)$$

holds. Let one of the following conditions be valid

a) $\alpha = 0, \lambda \leqq 1$, $\int_0^\infty t^\mu |a(t)|\,\mathrm{d}t = \infty$;

b) $\alpha = 1, \lambda < 1, \int_0^\infty t^{(n-1)\lambda} |a(t)| \,\mathrm{d}t = \infty$;

c) $\alpha = 1, \lambda = 1, \limsup_{t\to\infty} t \int_t^\infty s^{n-2} |a(s)| \,\mathrm{d}s > (n-1)!$.

Then there exists the continuum of unbounded oscillatory solutions of (7).

II. Next, we shall study the case when (5) is not valid for the special differential equation (1):

$$y^{(n)} = h(t, y), \qquad n \geqq 2 \quad \text{is even.} \tag{8}$$

We shall suppose that $h : D_1 \to R$, $D_1 = R_+ \times R$ is continuous, for any $x \in R$, $h(., x)$ is absolutely continuous on each finite segment, $\frac{\partial h}{\partial t}$ satisfies the local Carathéodory conditions and theie exists $\alpha \in \{0, 1\}$ such that $\alpha + n_0$ is even,

$$(-1)^\alpha h(t, x)\, x \geqq 0 \qquad \text{in } D_1. \tag{9}$$

Let y be a proper solution of (8). Then denote

$$F(t, x) = (-1)^\alpha \int_0^x h(t, s) \,\mathrm{d}s \qquad \text{in } D_1,$$

$$Z(t; y) = \sum_{i=1}^{n_0-1} (-1)^{i+\alpha} y^{(n-i)}(t)\, y^{(i)}(t) + \frac{1}{2} [y^{(n_0)}(t)]^2.$$

$$E(t; y) = F(t, y(t)) + Z(t; y), \tag{10}$$

$$J_m(t; v) = \int_0^t \int_0^{\tau_m} \dots \int_0^{\tau_2} v(\tau_1) \,\mathrm{d}\tau_1 \dots \mathrm{d}\tau_m, \qquad J_0(t; v) = v(t)$$

for $v : R_+ \to R$ continuous,

$$w(t; y) = \int_0^t \dots \int_0^t F(t; y) \underbrace{\mathrm{d}t \dots \mathrm{d}t}_{n-2} + \sum_{i=0}^{n_0-1} (-1)^{\alpha+i+1} c_i J_{2i}(t; [y^{(i+1)}]^2), \tag{11}$$

$$t \in R_+, \qquad c_i = \binom{n-1-i}{i} \frac{n-1}{2(n-1-i)} > 0.$$

From this it can be easily seen that

$$F(t, y(t)) \geqq 0, \tag{12}$$

$$E(t; y) = E(t_0; y) + \int_{t_0}^t \frac{\partial F(\tau, y(\tau))}{\partial t} \,\mathrm{d}\tau. \tag{13}$$

Further in [2] the following statement is proved

$$\overset{(n-2)}{w}(t; y) = E(t; y), \qquad t \in R_+. \tag{14}$$

Remark. The identity (13) is used by Kiguradze [8] in case $n = 4$.
With respect to (13) we have the following

Lemma 3. *Let y be a proper solution of (8) and let $(-1)^{\alpha} \dfrac{\partial h(t,x)}{\partial t} x \geqq 0 (\leqq 0)$ in D_1. Then the function $E(t; y)$ is nondecreasing (nonincreasing) with respect to the first argument.*

Lemma 4. *Let $t_0 \in R_+$ and y be a proper solution of (8) such that $E(t_0; y) < 0$. Let $n \geqq 4$,*

$$(15) \qquad (-1)^{\alpha} \frac{\partial h(t,x)}{\partial t} x \leqq 0 \qquad in\ D_1.$$

Then for arbitrary constants $K \in R_+$, $\beta \in [0, 1)$ we have

$$(16) \qquad \limsup_{t\to\infty} (|y^{(n_0-1)}(t)| - Kt^{\beta}) = \infty,$$

$$(17) \qquad \limsup_{t\to\infty} |y^{(n_0)}(t)| > 0.$$

Proof. According to Lemma 3

$$E(t; y) \leqq E(t_0; y) < 0, \qquad t \in [t_0, \infty).$$

Thus, with respect to (14) there exists a number $t_1 \geqq t_0$ such that

$$(18) \qquad w(t; y) \leqq E(t_0; y)\, t^{n-2}, \qquad t \in [t_1, \infty).$$

We prove (16) by the indirect proof. Thus suppose that

$$|y^{(n_0-1)}(t)| - Kt^{\beta} \leqq M < \infty, \qquad t \in [t_0, \infty).$$

Then by virtue of (10), (11), (12) there exists a number $t_2 > t_0$ such that

$$w(t; y) \geqq -\sum_{i=0}^{n_0-2} c_i J_{2i}(t; [y^{(i+1)}]^2) \geqq -M_1 t^{n-4+2\beta}, \qquad t \in (t_2, \infty)$$

holds with suitable positive constant M_1. But this inequality contradicts to (18). The inequality (17) may be proved similarly. The lemma is proved.

By Kiguradze [8] the conditions of the existence of oscillatory solutions of (1) are given in case $\alpha = 0$. We use the following consequence of his Lemmas 2.1 and 2.3.

Lemma 5. *Suppose, that $\alpha = 0$, the equation (8) has the property B^* and the set of singular solutions of the 2-nd kind of (8) is empty. Further, let $\Phi_i : R_+ \to R$, $i = 1, \dots, n-2$ be continuous functions and r be a number $r \in (0, \infty)$ such that $\Phi_i(x)\, x \geqq 0$ for $|x| \geqq r$, $i = 1, 2, \dots, n-2$. Then there exists a solution y of (8) satisfying the conditions $y(0) = 0$, $y^{(i-1)}(0) = \Phi_{i-1}(y^{(n-1)}(0))$, $i = 2, \dots, n-1$*

such that y is either oscillatory or singular of the 1-st kind or trivial or $(-1)^i y^{(i)}(t)\, y(t) > 0$, $i = 0, \ldots, n-1$ *holds.*

Theorem 2. *Let $n > 4$ and let the equation* (8) *have the property $B^*(A)$ in case of $\alpha = 0$ ($\alpha = 1$). Let* (15) *be valid and there exist continuous function $b: R_+ \to (0, \infty)$ and $M \in R_+$ such that*

$$(19) \qquad |h(t, x)| \leqq b(t)\,|x| \qquad \text{in } R_+ \times ((-\infty, -M] \cup [M, \infty))$$

holds. Then there exists the continuum of unbounded oscillatory solutions of (8) *with properties* (16) *and* (17).

Proof. According to (19) and the generalized Wintner's Theorem (see [6]) singular solutions of the 2-nd kind do not exist.

Let $\alpha = 0$. Let $\Phi_i: R_+ \to R$, $i = 2, \ldots, n-2$, $\Phi_i = 0$ for $i \neq 3, n-3$, $\Phi_3(x) = x - 1$, $\Phi_1(x) = 4 - x^2$ for $-2 \leqq x \leqq 2$, $\Phi_1(x) = 0$ for other x, $\Phi_{n-3}(x) = x$. Then according to Lemma 5 there exists a solution y of (8) with the following initial conditions $y^{(i)}(0) = 0$, $i = 0, 1, \ldots, n-2$, $i \neq 3$, $i \neq n-3$, $y^{(j)}(0) = \Phi_j(y^{(n-1)}(0))$, $j = 1, 3, n-3$. By virtue of (10) $E(0; y) = -K^2 < 0$. From this and according to Lemmas 3 and 5 y is oscillatory and the statement of the theorem follows from Lemma 4.

Further, let $\alpha = 1$ and let y be the noncontinuable solution of (8) with the property $E(0; y) < 0$. Then with respect to Lemma 3 y is not singular of the 1-st kind. Thus y is proper and as (8) has the property A it is oscillatory. The statement follows from Lemma 4. The theorem is proved.

Remark. The conclusions of Theorem 2 are not valid for $n = 2, 4$ as it is seen from linear equations $h(t, x) = (-1)^\alpha x$. Moreover, the way used in Lemma 4 can not be applied for $n = 2, 4$. It is clear that for $n = 2$ the condition $E(t_0; y) < 0$ is never fulfilled. For $n = 4$ the situation is similar as it is shown in

Lemma 6. *Let $n = 4$, $\alpha = 0$ and y be oscillatory solution of* (8) *such that $\sum_{i=0}^{3} |y^{(i)}(C)| \neq 0$ holds for an arbitrary large C. Then there exists $t_0 \in R_+$ such that*

$$Z(t; y) > 0, \quad E(t; y) > 0 \qquad \text{for } t \in [t_0, \infty).$$

Proof. In [4] the existence of numbers t_k^i, $i = 0, 1, 2, 3$, $\bar{t}_k^3$, $k = 1, 2, \ldots$ is proved such that

$$(20) \qquad \begin{gathered} y^{(i)}(t_k^i) = 0,\ y^{(i)}(t) \neq 0 \quad \text{for } t \in (t_k^i, t_{k+1}^i),\ i = 0, 1, 2, \\ y'''(t) = 0 \quad \text{for } t \in [t_k^3, \bar{t}_k^3], \qquad y'''(t) \neq 0 \quad \text{for } t \in (\bar{t}_k^3, t_{k+1}^3), \\ t_k^i < t_k^j < t_{k+1}^i, \qquad t_k^i < \bar{t}_k^3 < t_{k+1}^i, \qquad i = 0, 2;\ j = 1, 3 \end{gathered}$$

and

(21) $$y^{(s-1)}(t_k^s)\, y^{(s+1)}(t_k^s) < 0, \qquad s = 1, 2, \; k = 1, 2, \ldots$$

Put $t_0 = t_1^0$. According to (21) for $s = 2$, $Z(t; y)$ need not be positive only in the intervals $[\sigma_k, \tau_k]$ where $\sigma_k = \min(t_k^1, t_k^3)$, $\tau_k = \max(t_k^1, t_k^3)$, $k = 1, 2, \ldots$ But it is clear that

(22) $$Z(t_k^1; y) = \frac{1}{2} y''^2(t_k^1) > 0, \qquad Z(t_k^3; y) = \frac{1}{2} y''^2(t_k^3) > 0$$

and $Z'(t; y) = -y^{(4)}(t)\, y'(t)$ do not change the sign on $[\sigma_k, \tau_k]$ (see (20)). This fact with (22) proves that $Z(t; y) > 0$, $t \in [t_0, \infty)$. The rest of the statement follows from (10) and (12). Lemma is proved.

Corollary 2. *Let $n = 2n_0 > 4$, $0 < \lambda \leqq 1$ and there exist an absolutely continuous function $a : R_+ \to R$ such that a' is locally integrable. Let $\alpha \in \{0, 1\}$ be such that $n + \alpha$ is even,*

$$(-1)^\alpha a(t) \geqq 0, \quad (-1)^\alpha a'(t) \leqq 0 \qquad \text{in } D_1$$

and one of the assumptions a, b, c of Corollary 1 is valid.

Then there exists the continuum of unbounded oscillatoıy solutions of (7) with the properties (16) and (17).

Lemma 7. *Let $t_0 \in R_+$, y be a proper solution of (8) with the property $E(t_0; y) > 0$ and let*

(23) $$(-1)^\alpha \frac{\partial h(t, x)}{\partial t} x \geqq 0 \qquad \text{in } D_1.$$

Further, let a continuous function $h_1 : R_+ \to R_+$ exist such that

(24) $$|h(t, x)| \leqq h_1(|x|) \qquad \text{in } D_1$$

holds. Then

(25) $$\limsup_{t\to\infty} |y(t)| > 0.$$

Proof. We prove the lemma by the indirect proof. Thus suppose that $\lim_{t\to\infty} y(t) = 0$. From this, (24) and Kolmogorov–Horny inequality (see [6, p. 167]) $\lim_{t\to\infty} y^{(i)}(t) = 0$, $i = 0, 1, 2, \ldots, n-1$ follows and thus $\lim_{t\to\infty} E(t; y) = 0$.

But this fact contradicts (13) and Lemma 3. The lemma is proved.

Theorem 3. *Let the equation (8) have the property $B^*(A)$ in case of $\alpha = 0$ ($\alpha = 1$). Suppose that (23) is valid, there exist constans $K \in R_+$, $M \in R_+$ such that $|h(t, x)| \leqq K|x|$ in $R_+ \times ((-\infty, -M] \cup [M, \infty))$. Then there exists the continuum of oscillatory solutions of (8) with the property (25).*

Proof. The statement can be proved in the similar way as Theorem 2.

III. In the last part of the paper we shall deal with oscillatory solutions of (1) tending to zero for $t \to \infty$.

Theorem 4. *Let $n + \alpha$ be odd, n_0 even, let positive numbers ε, M, M_1, M_2, M_3 and continuous functions $g : [0, \varepsilon] \to R_+$, $a : R_+ \to R_+$ exist such that one of the following assumptions is valid:*

1^0 $\alpha = 1$, $s = n_0 - 2$, $a(t) = \dfrac{1}{t}$ *for* $t \in [M, \infty)$, $g(x) > 0$,

for $x > 0$, *g is nondecreasing*;

2^0 $s = n_0 - 1$, $\lambda \in [1, \infty)$, $g(x) = x^\lambda$, $\lim\limits_{t \to \infty} t^{n - \frac{\lambda - 1}{2}(1+\varepsilon)} a(t) = \infty$;

3° $\alpha = 0$, $s = n - 1$, $a(t) = M_3$ *for* $t \in R_+$, $g(x) > 0$ *for* $x > 0$.

Further, suppose that

$$a(t)\, g(|x_1|) \leqq |f(t, x_1, \ldots, x_n)| \leqq M_1 \sum_{i=1}^{n} |x_i| \qquad \text{on } R_+ \times [-\varepsilon, \varepsilon]^n, \tag{26}$$

$$|f(t, x_1, \ldots, x_n| \leqq M_2 \sum_{i=1}^{s+1} |x_i| \qquad \text{on } R_+ \times [-\varepsilon, \varepsilon]^{s+1} \times R^{n-s-1}. \tag{27}$$

Then there exists an oscillatory solution y of (1) defined in some neighbourhood of ∞ such that

$$\lim_{t \to \infty} y^{(i)}(t) = 0, \qquad i = 0, 1, \ldots, n. \tag{28}$$

Proof. Let $\alpha = 1$. Let us define the continuous functions $\bar{f} : D \to R$ and $\bar{g} : R_+ \to R_+$: $\bar{x}_i = x_i$ for $|x_i| \leqq \varepsilon$, $\bar{x}_i = \varepsilon \operatorname{sign} x_i$ for $|x_i| > \varepsilon$, $i = 1, 2, \ldots, n$;

$$\bar{f}(t, x_1, \ldots, x_n) = f(t, \bar{x}_1, \ldots, \bar{x}_n) \qquad \text{on } R_+ \times [-\varepsilon, \varepsilon] \times R^{n-1}; \tag{29}$$

if 1° is valid, then (29) holds on D and $\bar{g}(x) = g(\bar{x})$, $x \in R_+$;
if 2° is valid, then

$$\bar{f}(t, x_1, \ldots, x_n) = a(t)\,(|x_1|^\lambda - \varepsilon^\lambda) \operatorname{sign} x_1 + f(t, \varepsilon \operatorname{sign} x_1, \bar{x}_2, \ldots, \bar{x}_n)$$

for $|x_1| > \varepsilon$, $\bar{g}(x) = x^\lambda$, $x \in R_+$. From this and according to (26), (27) we have in both cases

$$a(t)\,\bar{g}(|x_1|) \leqq |\bar{f}(t, x_1, \ldots, x_n)| \leqq M_2 \sum_{i=1}^{s+1} |x_i| + a(t)\,|x_1|^\lambda \qquad \text{on } D. \tag{30}$$

Thus, with respect to [10] there exists a non-trivial solution $y : R_+ \to R$ of the differential equation

$$y^{(n)} = \bar{f}(t, y, \ldots, y^{(n-1)}) \tag{31}$$

satisfying

(32) $$\int_0^\infty t[y^{(n_0)}(t)]^2\,dt < \infty \qquad \text{for } 1^0$$

(33) $$\int_0^\infty t^{2j}[y^{(j)}(t)]^2\,dt < \infty,\ j = 0, 1, \dots, n_0 \qquad \text{for } 2^0.$$

It can be easily seen that by virtue of (26), (29) y is proper. Further, with the respect to (30), 1°, 2° and Lemma 1 the equation (31) has the pıoperty A. Thus y is oscillatory. By use of (32) and some results of [2] ((33) and [9]) the relation $\lim_{t\to\infty} y^{(i)}(t) = 0$, $i = 0, 1, \dots, s$ holds in case 1° (2°). But from this and from (27), (29), (31) $\lim_{t\to\infty} y^{(n)}(t) = 0$ holds and the validity of (28) can be proved from Kolgomorov–Horny inequality (see [6]); it is clear that y is the solution of (1), too.

For $\alpha = 0$ the proof is similaı. We must use the property B (see [6, 5, 1]) instead of A and [3] instead of [2]. The theorem is proved.

Theorem 5. *Let $n + \alpha$ be odd, n_0 even and let positive numbers ε, M, M_1, p and continuous functions $g : R_+ \to R_+$, $g_1 : R_+^{n_0} \to R_+$, $a : R_+ \to R_+$, $a_1 : R_+ \to R_+$ exist such that $p \in (1, \infty)$, g_1 is nondecreasing with respect to the all arguments and either the assumption 1° or 2° of Theorem 4 holds. Further, let*

$$|f(t, x_1, \dots, x_n)| \leqq a_1(t)\sum_{i=1}^{n}|x_i| \qquad \text{on } R_+\times[-\varepsilon, \varepsilon]^n,$$

$$a(t)\,g(|x_1|) \leqq |f(t, x_1, \dots, x_n)| \leqq$$

$$\leqq a_1(t)\,g_1(|x_1|, \dots, |x_{n_0}|)\left(1 + \sum_{i=n_0+1}^{n}|x_i|^{\frac{2n-2n_0+1}{2i-2n_0-1} - \frac{2}{(2i-2n_0-1)p}}\right) \qquad \text{on } D.$$

Then there exists an oscillatory solution of (1) *such that* $\lim_{t\to\infty} y^{(i)}(t) = 0$, $i = 0, 1, \dots, s$ *holds.*

This theorem can be proved similarly to Theorem 4, only we use (1) instead of (31.).

Corollary 3. *Let $n + \alpha$ be odd, n_0 even and $\lambda \in [1, \infty)$, $\varepsilon > 0$. Let $a : R_+ \to R$ be continuous, $(-1)^\alpha a(t) \geqq 0$ and $\lim_{t\to\infty} t^{n-\frac{\lambda-1}{2}(1+\varepsilon)}|a(t)| = \infty$. Then there exists an oscillatory solution of* (7) *such that* $\lim_{t\to\infty} y^{(i)}(t) = 0$, $i = 0, 1, \dots, n_0 - 1$ *holds.*

REFERENCES

[1] M. Bartušek, *On Oscillatory Solutions of the System of Differential Equations with Deviating Arguments*, Czech. Math. J., 35 (110), 1985, 529–532.

[2] M. Bartušek, *On Oscillatory Solution of the Differential Equation of the n-th Order*, Arch. Math., XXII, No. 3 (1986), 145–156.

[3] M. Bartušek, *On Properties of Oscillatory Solutions of Non-linear Differential Eguations of the n-th Order,* Differential Equations and Their Application, Equadiff 6, Proc. 6th Int. Conf. Brno, Czech. 1985, Lecture Notes Math., 1192, 107–113.

[4] М. Бартушек, *О свойствах колеблющихся решений обыкновенных дифференциальных неравенств и уравнений,* Дифф. урав., 23, № 2 (1987), 187—191.

[5] Г. А. Чантурия, *Об осцилляционных свойствах систем нелинейных обыкновенных дифференциальных уравнений,* Труды Института прикладной математики (Тбилиси) 14 (1983), 163—204.

[6] И. Т. Кигурадзе, *Некоторые сингулярные краевые задачи для обыкновенных дифференциальных уравнений,* Изд. Тбилисского у-та, Тбилиси, 1975.

[7] I. T. Kiguradze, *On the Oscillatory and Monotone Solutions of Ordinary Differential Equations,* Arch. Math., XIV, No. 1 (1978), 21–44.

[8] I. T. Kiguradze, *On Asymptotic Behaviour of Solutions of Nonlinear Non-autonomous Ordinary Differential Equations.* Colloquia Math. Soc. J. Bolyai, 30, Qualitative Theory of Differential Equations, Szeged, 1979, 507–554.

[9] I. T. Kiguradze, *On Vanishing at Infinity of Solutions of Ordinary Differential Equations,* Czech. Math. J., 33 (108), 1983, 613–646.

[10] И. Т. Кигурадзе, *Об одной краевой задаче с условием на бесконечности для обыкновенных дифференциальных уравнений высших порядков,* Дифференциальные уравнения в частных производных и их приложения, Труды всесоюзного симпозиума, Изд. Тбилисского у-та, Тбилиси 1986.

[11] J. S. Wong, *On the Generalized Emden-Fowler Equation,* SIAM Review, 17, No. 2 (1975), 339–360.

Miroslav Bartušek
Department of Mathematics
Faculty of Science, J. E. Purkyně University,
Janáčkovo nám. 2a,
662 95 Brno,
Czechoslovakia