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ACYCLIC CHROMATIC INDEX OF A SUBDIVIDED GRAPH

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We consider only simple graphs, i.e. finite graphs without loops and multiple edges. As a rule, we do not distinguish between isomorphic graphs.

By an acyclic chromatic index $a(G)$ of a graph G we mean the least number of colours of a regular edge colouring of G in which any adjacent edges have different colours and no cycle is 2-coloured. In [1] it is shown that $a(G)$ is finite for every integer $h(G)$, where $h(G)$ is the maximal vertex degree of G. In this paper we consider only graphs with $h(G) = 3$.

Notation and terminology. Below, fxy will denote the colour of an edge xy with end points x and y. A cycle (path), in which the edges are alternatively coloured by the colours i, j, will be denoted by $C_{ij}(P_{ij})$, respectively. Further concepts and notations not defined in the paper will be used as in standard textbooks, e.g. listed in [2].

We divide our results into two sections. In section 1. we fulfil a gap in the proof of Theorem in [2]. We prove the following.

Theorem 1. *If no connected component of a graph G with $h(G) = 3$ is isomorphic either with K_4 or with $K_{3,3}$, then $a(G) \leqq 4$, whereas $a(K_4) = a(K_{3,3}) = 5$.*

From Theorem 1 the following classification problem arises: which graphs G ($K_4 \neq G \neq K_{3,3}$) with $h(G) = 3$ have $a(G) = 3$ and which have $a(G) = 4$?

We say that a graph G with $h(G) = 3$ is of class one or two if $a(G) = 3$ or $a(G) = 4$, respectively. Using the notion of the subdivision of edges of a cubic graph we characterize below a set of graphs of class one (two). By the subdivision of an edge uv of a graph, we mean inserting a new vertex between the vertices u and v on the edge uv.

In section 2 we prove the following.

Theorem 2. (1) *Every cubic graph different from the graphs K_4 and $K_{3,3}$ is of class two.* (2) *If we subdivide at most two arbitrary edges in the cubic graph, we get a graph of class two.*

Theorem 3. *If we subdivide every edge in the cubic graph with at most two exceptions, we get a graph of class one.*

The upper bounds given in Theorems are strict. Bound four in Theorem 1 is attained for every cubic graph different from the graphs K_4 and $K_{3,3}$ (statment (1) of Theorem 2). Bound two of Theorems 2 and 3 is attained on the complete graph K_4 in which we subdivide or do not subdivide exactly three edges adjacent with the same vertex.

1. In the proof of Theorem in [2] it has been stated (p. 83) that $a(K_{3,3}) \leqq 4$. This is not true. Now we shall prove $a(K_{3,3}) = 5$. We show that $a(K_{3,3}) > 4$. Since $K_{3,3}$ has nine edges, then in regular colouring of edges by the set of colours $\{1, 2, 3, 4\}$ at least three of edges are coloured by the same colour, say by the colour 1. Without loss of generality we can put $fx_1x_4 = fx_2x_5 = fx_3x_6 = 1$ and $fx_1x_2 = 2$ according to Fig. 1a. The edges x_1x_6, x_2x_3 in the graph $K_{3,3}$ can be coloured either by the same colour or by different ones. In the first case let $fx_1x_6 = fx_2x_3 = 3$. The edge x_4x_5 is neither coloured by 2 nor by 3 (because of the cycles $C_{21} = x_1x_2x_5x_4x_1$ and $C_{31} = x_1x_6x_3x_2x_5x_4x_1$), consequently $fx_4x_5 = 4$. Thus $fx_3x_4 = fx_5x_6 = 2$ and we get a cycle $C_{12} = x_1x_4x_3x_6x_5x_2x_1$.

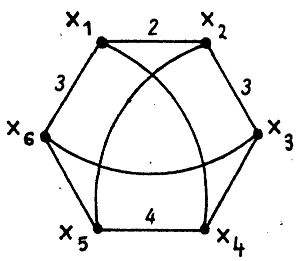

Fig. 1a

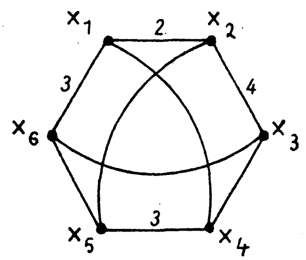

Fig. 1b

In the second case we put $fx_1x_6 = 3$ and $fx_2x_3 = 4$. If the edge x_4x_5 in Fig. 1b is coloured by the colour 3 or by the colour 4, then it must be $fx_5x_6 = fx_3x_4 = 2$. But in the graph $K_{3,3}$ we have a cycle C_{12} again. Hence $a(K_{3,3}) > 4$.

By virtue of (i) of Theorem in [1] we have $a(K_{3,3}) = 5$.

Now we start with the proof of Theorem 1. There exist exactly two cubic graphs with six vertices. These graphs are $K_{3,3}$ and the graph on Fig. 3 in [2, p. 83]. If Johnson's construction, see for example [2, p. 82], is applied to arbitrary two non-adjacent edges of these graphs we obtain all cubic graphs with eight vertices. Regular and acyclic colouring of such graphs is shown on Fig. 2. The rest of the proof of Theorem 1 runs along the same lines as in [2].

Remark. It is necessary to read the last line of Lemma 2 in [2] as follows: "in such a fashion that we do not form a new cycle C_{12}^{+}."

2. The proof of Theorem 2 is indirect and will be divided into two parts. Let the edges of the considered graph G ($K_4 \neq G \neq K_{3,3}$) be assumed to be regularly and acyclically coloured by the set of colours $X = \{1, 2, 3\}$.

Proof of assertion (1). Let x be an arbitrary vertex of G and let $fxx_1 = 1$, $fx_1x_2 = 2, \ldots$ For even i, $fx_{i-1}x_i = 2$, for odd i, $fx_{i-1}x_i = 1$. As G is finite after finite number of steps we must form a cycle C_{12}, a contradiction.

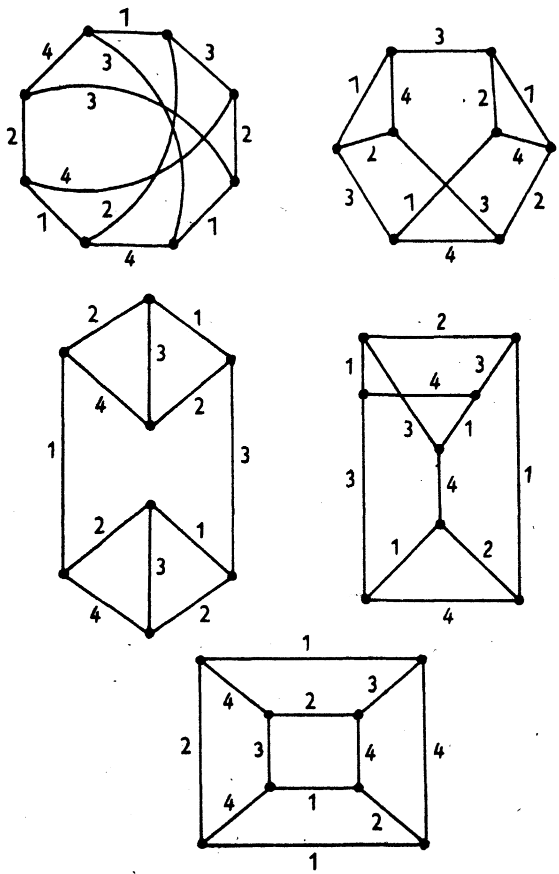

Fig. 2

Proof of assertion (2). We have two subcases.

a) Only one edge h of G is divided by the vertex x in two edges h_1, h_2 which are coloured, for example, $fh_1 = 1, fh_2 = 2$. The rest is the same as above.

b) Let $x(y)$ divide an edge of G in $h_1, h_2(h_3, h_4)$. Put $E_1 = \{fh_1, fh_2\}$, $E_2 = \{fh_3, fh_4\}$ and let $fh_1 = 1$, $fh_2 = 2$. If $E_1 = E_2$, we proceed as above. If $E_1 \neq E_2$, then card $(E_1 \cap E_2) = 1$. Let us suppose that $fh_3 = 1, fh_4 = 3$. Starting in the vertex y we construct an infinite path P_{12}, contradicting the fact that graph is finite.

Theorem 2 is proved.

P r o o f of Theorem 3. At first we outline the method of proof and introduce necessary notations. By Johnson's construction we extend the cubic graph L with $p \geqq 8$ vertices into cubic graph L^+ with $p + 2$ vertices by adding the set of the edges H (edges (2) in [2]) and we use induction on p vertices.

Below by $L_i(L_i^+)$, $i = 0, 1, 2$, we mean graph formed from $L(L^+)$ in which we subdivide every edge with the exception of i edges. As an inductive assumption, we assume that the edges of the graph L_i can be regular and acyclic coloured by the set of colours $X = \{1, 2, 3\}$. Our aim is to show that the edges of the graph L_i^+ can be coloured analogously. It suffices to colour only the set of the edges H_i formed from the set $H \subset L^+$ (the colouring of the rest of edges of the graph L_i^+ is induced by the colouring of the edges of the graph L_i).

According to the number of not subdivided edges i of the graph L_i^+ we divide the proof of Theorem 3 into three parts. In part 0. we will verify Theorem 3 in the case when in a cubic graph we have subdivided every edge. This enables us, in parts 1, 2 by the colouring of the edges of the graphs L_1^+, L_2^+ to consider only the cases when at least one not subdivided edge of mentioned graph is in the set of the edges H_1, H_2, respectively.

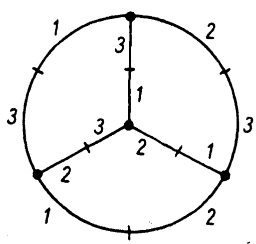

Fig. 3

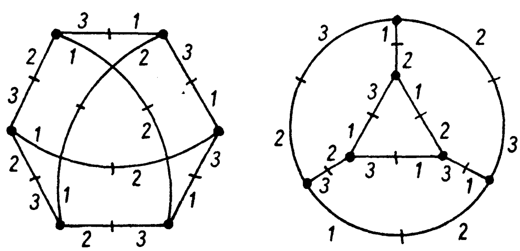

Fig. 4

Part 0. At first we show that Theorem 3 holds for graphs with $p = 4, 6$ vertices. One of the possible colouring of the edges of the graph K_4 with the set of colours X is shown on Fig. 3 in which the original vertices of a cubic graph are denoted by a small circle and new vertices on subdivided edges by a small line (this notation is used throughout the whole text).

For $p = 6$ vertices there exist exactly two cubic graphs. Required colouring of their subdivided edges is given on Fig. 4.

We start colouring the edges of the graph L_0^+. We delete from the graph L_0 the edges ak, kc; bg, gd, i.e. the dashed lines ac, $bd \notin L$ which are subdivided by the vertices k, g according to Fig. 5 (which illustrates all of our following considerations). Let the edges adjacent to the vertices $u, v \in H \subset L_0^+$ be subdivided by the vertices e, u_j, v_j, $j = 1, 2$ (edges of the set H subdivided in such a way form the set of the edges H_0).

We put

(1) $$u_1u,\ ue,\ ev,\ vv_2$$

(2) $$u_2u,\ vv_1$$

By the colouring of the edges of the graph L_0^+ it suffices to colour only the edges (1) and (2) (the colouring of the rest of the edges of the graph L_0^+ we induce by the colouring of the edges of the graph L_0). Let, for example, $fak = 1, fkc = 2$ in the graph L_0. We put $fau_1 = 1$, $fcv_2 = 2$, $fbv_1 = fbg$, $fdu_2 = fdg$. We colour the edges (1) by putting $fv_2v = feu = 1$, $fve = fuu_1 = 2$. From assumption that we have the edges of the graph L_0 acyclically coloured by the set of colours X it follows that in the graph L_0^+ we do not form a cycle $C_{12} = \dots au_1uevv_2c \dots$

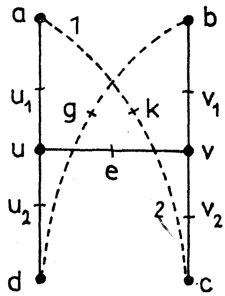

Fig. 5

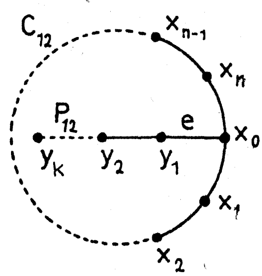

Fig. 6

Let $Y = \{1, 2\}$. If $\{fdu_2, fbv_1\} = Y$, then we colour the edges (2) by the colour $3 \in X$. In the opposite case either the edge du_2 or the edge bv_1 must be coloured by the colour 3 (we have $fdg \neq fgb$ in the graph L_0), say, for example, $fdu_2 = 3$. We colour the edges (2) by putting $fu_2u = 1$, $fue = fvv_1 = 3$.

The colouring of the edges of the graph L_0^+ is finished.

Before proving part 1 of Theorem 3 we prove the following Lemma 1 which will be used in part 2, too.

Lemma 1. *Let the edges of a graph L_p^+, $p = 1, 2$ be regularly coloured by the set of colours X. If one edge of the cycle C_{12} adjacent to the vertex x_0 according to Fig. 6 is recoloured by the colour 3 and the edge e is recoloured by one from the colours 1, 2 in such a manner that we do not break the regularity of the colouring of the edges adjacent to the vertices y_1, x_i, $i = 0, 1, n$, then in the graph L_p^+ we do not create a new cycle C_{12}^+.*

Proof. Let, for example, by putting $fx_0x_1 = 3$, $fx_0y_1 = 1$ the regularity of the colouring of the edges coincides to the vertices y_1, x_0, x_1 be not broken. A cycle

C_{12}^+ can be formed only in the case if to the vertex x_0 there is attached a path $P_{12} = x_0 y_1 y_2 \dots y_k$, last vertex of which y_k coincides with some vertex x_i of the cycle C_{12} i.e. if $y_k = x_i$, contradicting our assumption that the edges of L_p^+ are regularly coloured. This proves the Lemma 1.

Part 1. We show that Theorem 3 holds in the case when exactly one arbitrary edge (denoted by dark line) in a cubic graph is not subdivided. Fig. 7 (Fig. 8) verifies Theorem 3 for $p = 4$ ($p = 6$) vertices.

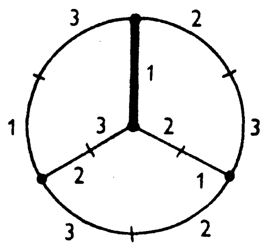

Fig. 7

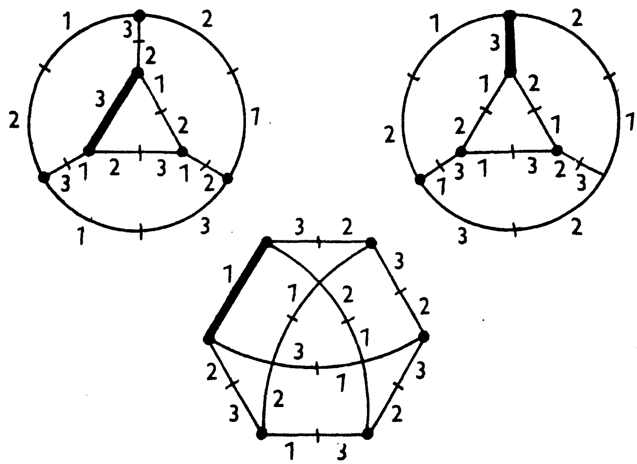

Fig. 8

We start with colouring the edges of the graph L_1^+. When colouring of the edges of H_1 using the symmetry of embedding of the not subdivided edge $h_1 = x_1 y_1$ in the set H_1 it is enough to consider only the case where the end vertices x_1, y_1 coincide either (i) with the vertices a, u according to Fig. 9 or (ii) with the vertices u, v according to Fig. 14.

Case (i). Without loss of generality we can suppose that the not subdivided edge bd in the graph L_1 is coloured by the colour 1 and the subdivided edge $ak\ kc$ is coloured by the colour $\alpha(\beta)$ ($\alpha \neq \beta$, $\alpha, \beta \in X$) according to Fig. 9 (considered edges are denoted by dashed lines). We induce the colouring of the end edges of the set H_1 (the edges adjacent to the vertices a, b, c, d) from the colouring of deleting the edges of the graph L_1, i.e. we put $fau = \alpha, fcv_2 = \beta, fbv_1 = fdu_2 = 1$. By the colouring of the edges

$$au, cv_2 \tag{3}$$

by the set of colours X we descern (because the edges (3) have different colours) the following cases:

1. $fau = 2(3)$, $fcv_2 = 1$;
2. $fau = 1$, $fcv_2 = 2(3)$;
3. $fau = 3(2)$, $fcv_2 = 2(3)$.

It is enough to desire colouring all the rest edges of H_1 (edges coincide to the vertices u, v) only in the cases if one edge from the edges (3) is coloured by the colour 2 (the case when it is coloured by the colour 3 is analogous).

Before starting colouring the edges of H_1 we state the following Lemma 2 (which immediately follows from the regular colourability of edges).

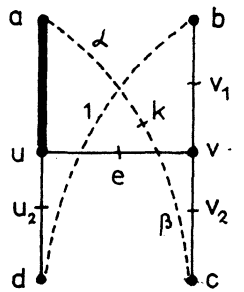

Fig. 9

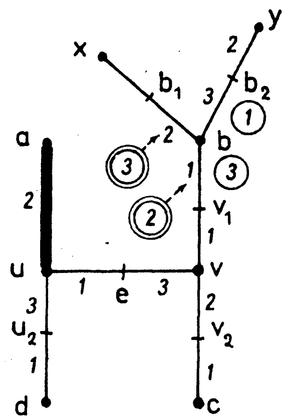

Fig. 10

Lemma 2. *Let P be a path in the graph L^+ and let P_j, $j = 1, 2$ be a path in L_j^+ obtained from P by the subdivision of the edges. Let the edges of P_j be alternatively coloured by the colours α, β ($\alpha \neq \beta$, $\alpha, \beta \in X$) starting with the colour α. Then the last edge of the path P_j has not the colour α, hence it cannot be incident to another edge coloured by α.*

Subcase 1). From the regular edge colouring of the graph L_1 by the set of the colours X, it follows that the edge b_2y on Fig. 10 is coloured either A. by the colour 2 or B. by the colour 1.

In the case A. we recolour the edge $v_1b(bb_2)$ by the colour 3(1). We colour all the rest edges of the set H_1 according to Fig. 10 (in which the recolouring edges are given in small circle). If such a recolouring does not create the cycle $C_{12} = bb_2y \dots xb_1b$ in the graph L_1^+, then the edges of the set H_1 (and the edges of the whole graph L_1^+) are coloured in desirable way. In the opposite case we eliminate the cycle C_{12} from the graph L_1^+ by recolouring two edges, adjacent to the vertex b, by the colours given in double circles according to Fig. 10 (the colouring of the rest of the edges in the graph L_1^+ is not changed; in this case $fbb_2 = fb_1x = 1$). By virtue of Lemma 1 in the graph L_1^+ we do not create a new cycle $C_{12}^+ = \dots cv_2vv_1bb_2y \dots$

In the case B. we consider the path P_{31}^+ in the graph L_1^+ formed from the path $P_{13} = v_1 b b_2 y y_1 \ldots y_{k-4} y_{k-3} y_{k-2} y_{k-1} y_k$ which is attached to the vertex v_1 according to Fig. 11 by recolouring their edges, i.e. we put $f v_1 b = 3, f b b_2 = 1, f b_2 y = 3, \ldots,$ $f y_{k-1} y_k = 1$ (the edges of path P_{31}^+ are given in small circles on Fig. 11) in P_{13}. If all the rest edges of H_1 are coloured according to Fig. 10, then in the graph L_1^+ we can create only a cycle $C_{12} = y_{k-1} y_k z z^1 \ldots y_{k-1}^1 y_{k-1}$ (the last vertex y_k of the path P_{13} cannot coincide on Fig. 10 with the vertex d because $\deg y_k \neq \deg d$). For the elimination of a cycle C_{12} it is enough to recolour two edges adjacent to the vertex y_{k-1} by the colours given in double circles by Fig. 11 (the colouring of the rest of the edges in L_1^+ is not changed). By virtue of Lemma 1 we do not create a new cycle $C_{12}^+ = y_{k-3} y_{k-2} y_{k-1} y_k z z^1 \ldots y_{k-3}^1 y_{k-3}$.

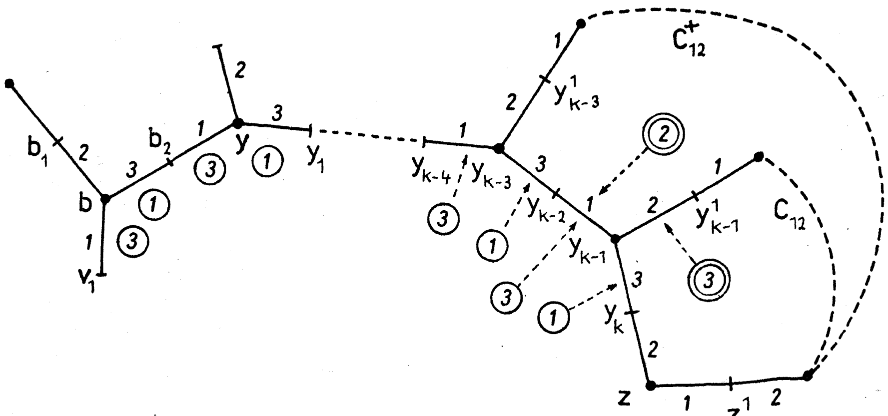

Fig. 11

The colouring of the edges of H_1 in the subcase 1. is finished.

In the subcase 2. we colour by Fig. 12 (recolouring the edges given in small circles is considered below) the edges of the set H_1 which are adjacent to the vertices u, v. Since the vertices a, d cannot be connected by a path P_{21} (by virtue of Lemma 2) in the graph L_1 we do not create a cycle $C_{12} = \ldots a u u_2 d \ldots$ in the graph L_1^+.

Subcase 3. If by colouring of the edges of the set H_1 according to Fig. 13 we do not form a cycle $C_{12} = \ldots d u_2 u e v v_2 c \ldots$, then the edges of the graph L_1^+ are coloured in desirable way. In the opposite case it is enough to change two colours of the edges incident to the vertex v by Fig. 13. By Lemma 1 we do not create a new cycle $C_{12}^+ = \ldots c v_2 v v_1 b \ldots$

The colouring of the edges of the graph L_1^+ in the case (i) is finished.

Case (ii). In Fig. 14 we put $f u v = 1$ and consider all possibilities of colouring the edge $c v_2 = p$ by colours from the set X. To simplify the illustration in Fig. 14 there is mentioned the necessary colouring of the set of the edges H_1 for $fp = 1, 2, 3$ at vertices u, v. (Recall that end edges $d u_2$, $b v_1$ of the set H_1 in Fig. 14 are coloured by the colour 1 which is induced from Fig. 9, where $fak \neq fkc$; hence $fau_1 \in \in X - \{fp\}$ in Fig. 14). By colouring the edges of the set H_1 according to Fig. 14 we do not create a cycle $C_{12}(C_{13})$ in the graph L_1^+. (These cycles can be formed only in the case if in Fig. 14 the path $P_{21}(P_{31})$ is attached to the vertices ***a, b, d, b*** and *d, c* which is impossible by Lemma 2).

Colouring the edges of a graph L_1^+ is finished.

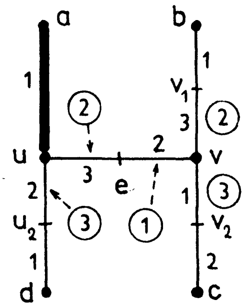

Fig. 12

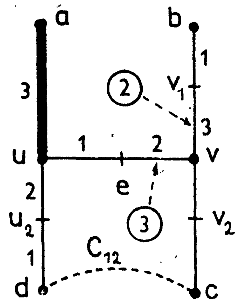

Fig. 13

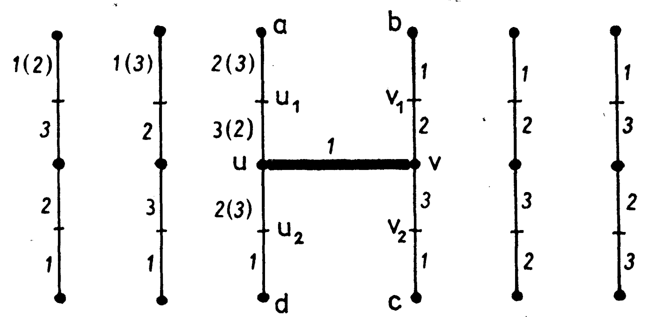

Fig. 14

Part 2. We show that Theorem 3 holds in the case when exactly two arbitrary edges (denoted by dark lines) in a cubic graph are not subdivided. Fig. 15 (Fig. 16) verifies Theorem 3 for $p = 4$ ($p = 6$) vertices.

Now we pass to colouring the edges of the graph L_2^+. Because of embedding of not subdivided edges $h_1 = x_1y_1$, $h_2 = x_2y_2$ in the set of the edges H_2 we distinquish the following two cases:

a) $h_1 \in H_2$, h_2 non $\in H_2$;

b) $h_1, h_2 \in H_2$.

Case a) Consider how the upper procedure of the colouring of the edges must be changed if we have two not subdivided edges h_1, h_2. In the subcase 1 the

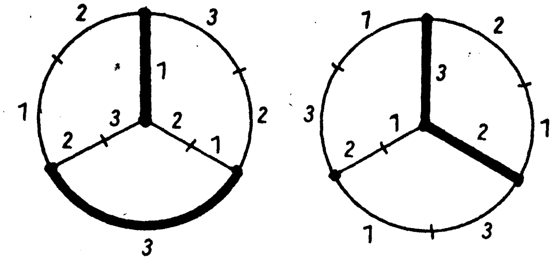

Fig. 15

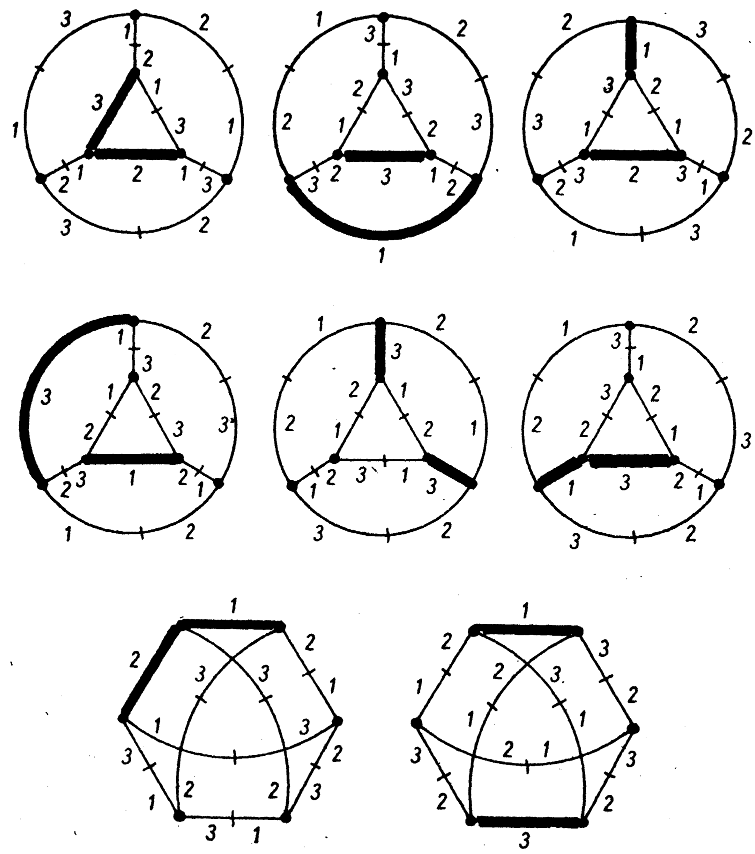

Fig. 16

edge h_2 produces an influence of given construction only in the case if it is in the path P_{13}. Then the last vertex y_k of the path P_{13} can be identified with the vertex d or with the vertex c. Take the path P_{31}^{+} attached in the graph L_2^{+} to two vertices v_1, d

according to Fig. 17. From a regular colouring of the edges of L_2 and by Lemma 2 it follows that by the colouring of the edges of H_2 according to Fig. 17 we create neither cycle $C_{13} = \dots du_2uevv_2c \dots$ nor a cycle C_{12} containing the edge h_2 coloured in the path P_{31}^+ by the colour 1 (in the path P_{13} the edge h_2 is coloured by the colour 3).

If the path P_{21} is attached to the vertices a, d and the edge h_2 belongs to it, then in the subcase 2 when colouring the edges according to Fig. 12 we can form a cycle $C_{12} = \dots auu_2d \dots$ We can eliminate it from the graph L_2^+ by recolouring the edges of H_2 according to Fig. 12 with the colours in small circles. By Lemma 1 we do not create a new cycle $C_{12}^+ = \dots auevv_1b \dots$ in the graph L_2^+. Since in the graph L_2 we have exactly one not subdivided edge h_2 (which is coloured by the colour 2 in the path P_{21}), then we do not form a cycle $C_{13} = \dots auu_2d \dots$

In the subcase 3 the edge h_2 does not influence the given construction.

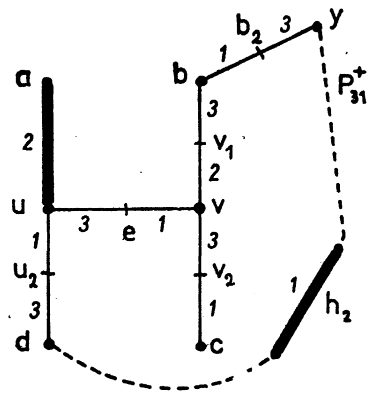

Fig. 17

It remains to consider case (ii) from part 1. Notice, there is no path P_{21} attached to the vertices b, d in the graph L_2^+ (in the graph L_2 there would be the cycle $C_{12} = P_{21} \cup \{bd\}$ which contradicts acyclic colouring of the edges of the graph L_2). If the not subdivided edge h_2 belongs to the path $P_{31}(P_{21})$ attached to the vertices d, c (a, b, resp.) from the set H_2, then if $fp = 1, 2, 3$ by the colouring of the set of the edges H_2 according to Fig. 14 we can form cycles $C_{13}^1 = \dots du_2uvv_2c \dots$, $C_{12} = \dots au_1uvv_1b \dots$ and $C_{13}^2 = \dots au_1uvv_1b \dots$ The cycle C_{13}^1 will be eliminated by interchanging the colours of the edges incident to the vertex v, i.e. in Fig. 14 for $fp = 1$ we put $fvv_1 = 3, fvv_2 = 2$ keeping the colouring of the rest of the edges of L_2^+ unchanged. The cycles C_{12}, C_{13}^2 can be eliminated in the following way. We interchange the colours of the edges incident to the vertex u (see Fig. 14). We know there is neither path P_{21} nor path P_{31} attached to the vertices b, d in the graph L_2. Therefore by this recolouring no new cycle C_{13}^+ nor a cycle C_{12}^+ will be created in the graph L_2^+.

The colouring of the edges of L_2^+ is finished for case a).

Case b). When we want to colour the edges of the set of H_2 in L_2^+ we can see by symmetry that it is sufficient to consider the cases if the non subdivided edges h_1, h_2 are situated in the set H according to Figures 18, 19 and 20 (edges h_1, h_2

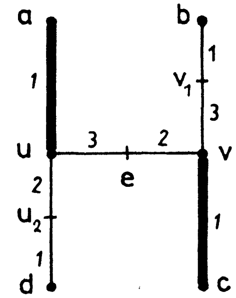

Fig. 18

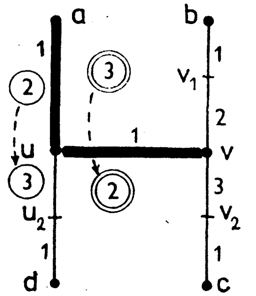

Fig. 19

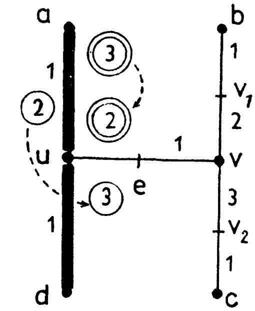

Fig. 20

are denoted by dark lines). Let ac, bd (in [2] dashed edges on Fig. 1) be the edges deleted by Johnson's construction from the graph L_2. In the sequel H_2 is the set of the edges incident to the vertices u or v. We shall discern two cases: (i) $fac = fbd$ (ii) $fac \neq fbd$.

Subcase (i). Let $fac = 1$. The Figures 18, 19 and 20 show the induced colouring of the end edges of the set of the edges H_2 from the graph L_2. If we colour the edges of the set H_2 by Fig. 18, then by Lemma 2 we form neither cycle $C_{12} = \ldots auu_2d \ldots$ nor cycle $C_{13} = \ldots bv_1vc \ldots$

In the case of embedding of the not subdivided edges h_1, h_2 according to Fig. 19 and Fig. 20 we recolour the edge $h_1 = au$ either by the colour 2 or by the colour 3 analogously as in the subcase 1. from part 1. To finish colouring of the edges of the set H_2 it suffices to colour the edge $uu_2(ue)$ in Fig. 19 (Fig. 20) by the colour 3(2) according to the colour of the edge h_1. All the other edges adjacent to vertex v are coloured by Fig. 19 (Fig. 20). By virtue of Lemma 2 we create neither cycle C_{12} nor cycle C_{13} in the graph L_2^+.

The edges of H_2 are coloured by required manner.

Subcase (ii). Let $fbd = 1, fac = 2$ in the graph L_2. The induced colouring of the end edges of the set H_2 is as on Figures 21, 22 and 23. We proceed as in subcase 1. from part 1 in the case when the edges of the set H_2 are coloured by Fig. 21. If we do not create a cycle $C_{12} = auvv_1bb_1x \ldots a$ by colouring the edges according to Fig. 22, then the edges of the graph L_2^+ are coloured in a desirable way. In the opposite case we change the colours of the edges in the cycle C_{12}, i.e. we change the colour 1 by the colour 2 in the whole cycle C_{12}. Furthermore we put $fv_1v = 3$, $fvv_2 = 1$. By Lemmas 1 and 2 we do not create new cycles $C_{12}^+ = \dots auvv_2c \dots$ $C_{13} = \dots auu_2d \dots$ If the vertices a, d are not connected by a path $P_{12}^1 = aa_1w \dots d$ in the graph L_2, then it is enough to colour the edges of H_2 according to Fig. 23

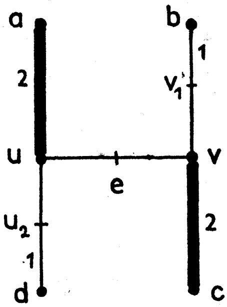

Fig. 21

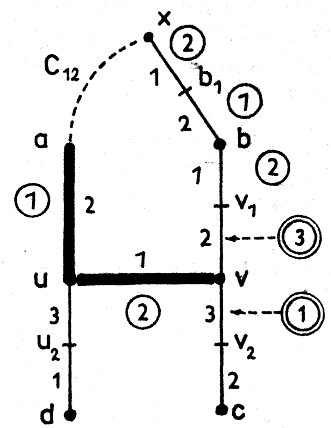

Fig. 22

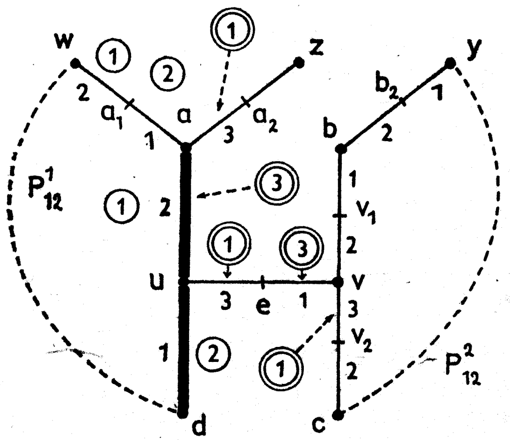

Fig. 23

and the edges of the graph L_2^+ are coloured by required manner. In the opposite case we have a cycle $C_{12}^1 = P_{12}^1 \cup \{dua\}$ in the graph L_2^+. We eliminate it from the graph L_2^+ as follows. We change the colouring of the edges in the cycle C_{12}^1 as above and by colouring of the edge a_2z we discern two cases:

Case 1. $fa_2z = 1$. We interchange the colours in the path P_{31} which begin in the vertex v_2 (the new colours are given in double circles on Fig. 23). By such recolouring we cannot form a new cycle $C_{21}^2 = bv_1vv_2c \ldots xb_1b$ in the graph L_2^+. It is possible if the vertices b, c are connected by the path $P_{21}^2 = bb_2y \ldots c$ and we have the cycle $C_{12} = P_{12}^1 \cup \{db\} \cup P_{21}^2 \cup \{ca\}$ in the graph L_2. This is impossible because, by asumption, the edges of the graph L_2 are acyclically and regularly coloured by the set of the colours X.

Case 2. $fa_2z = 2$. We proceed analogously as above. We change the colouring of the edges in the path v_2veuaa_2. Since we create neither a cycle C_{21}^2 nor (by virtue of Lemma 1) a new cycle C_{12}^+ (we admit a cycle C_{12}^1 in L_2^+) in the graph L_2^+, then the colouring of the edges of the set H_2 (and the edges of the whole graph L_2^+) is finished.

This completes the proof of Theorem 3.

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