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ϱ-IDEALS IN THE LATTICE OF SEMI ϱ-IDEALS

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0. INTRODUCTION

In the present paper we continue the investigation (see [2], [3], [6] and [7]) of ideals of binary relational systems, ϱ-ideals in brief.

The aim of this paper is to apply some methods and theorems of the general lattice theory to the poset of all ideals of a given binary relational system, ordered by set inclusion. However, there is an obstruction that the poset of all ϱ-ideals is not a lattice in general. Therefore, the concept of ϱ-ideal is first weakened to the concept of semi ϱ-ideal. Let us note that this concept is well-known and frequently used under the notation hereditary subset or semiideal, whenever ϱ is a partial ordering.

The poset of semi ϱ-ideals is always a lattice, moreover, it is an algebraic lattice, and so ϱ-ideals are considered in the lattice of all semi ϱ-ideals as elements of special kind. Now it is possible to apply methods of the general lattice theory to deduce some properties of ϱ-ideals with respect to the lattice of semi ϱ-ideals.

In order not to have to interrupt the discussion later, we recall in section 1 some definitions and basic properties of ϱ-ideals that will be needed in this paper.

In section 2 we characterize join irreducible, complete-join irreducible and directly irreducible ϱ-ideals in the lattice of semi ϱ-ideals.

In section 3 we use the results of section 2 to derive some properties of ϱ-ideals satisfying the Ascending Chain Condition. Also some connections between the ACC and the join irreducibility of ϱ-ideals are investigated.

The last section deals with the binary relational systems isomorphic to their ϱ-ideal posets and to their semi ϱ-ideal lattices. In [5], D. Higgs has solved the problem of G. Grätzer. As an application of theorems of section 4, we give a slight extension of D. Higgs' Theorem.

1. PRELIMINARIES

By a *binary relational system* is meant a pair $\langle A, \varrho \rangle$, where ϱ is a binary relation on a nonempty set A. Let $a, b \in A$, denote by $U_\varrho(a, b)$ the set $\{x \in A;\ a\ \varrho\ x \text{ and } b\ \varrho\ x\}$.

The following two properties of a subset X of A will be employed frequently in this paper:

(I_1) For every element $a \in A$ and every element $x \in X$, $a \varrho x$ implies $a \in X$;

(I_2) For every elements $x, y \in X$ the set $U_\varrho(x, y) \cap X$ is nonvoid.

Let us recall that

(i) A nonvoid subset X of A satisfying (I_1) and (I_2) is called *ϱ-ideal* of $\langle A, \varrho \rangle$.

(ii) An arbitrary subset X of A satisfying (I_1) is called *semi ϱ-ideal* of $\langle A, \varrho \rangle$.

(iii) An arbitrary subset X of A satisfying (I_2) is called *ϱu-directed* subset of $\langle A, \varrho \rangle$.

Clearly, the poset of all semi ϱ-ideals of $\langle A, \varrho \rangle$, denoted by $\langle \mathfrak{S}(A), \subseteq \rangle$, is a complete sublattice of the complete lattice of all subsets of A, ordered by set-inclusion. Consequently, $\langle \mathfrak{S}(A), \subseteq \rangle$ is an algebraic lattice and the compact elements of $\mathfrak{S}(A)$ are exactly the finitely generated semi ϱ-ideals. In general, the semi ϱ-ideal generated by a subset M of A is denoted by $S(M)$, the notation $S(\{a_1, \ldots, a_n\})$ is replaced by $S(a_1, \ldots, a_n)$.

As we noted above, the poset of all ϱ-ideals, denoted by $\langle \mathscr{I}(A), \subseteq \rangle$, is not a lattice, nevertheless, for a subset M of A, $I(M)$ denotes the smallest ϱ-ideal containing the set M, whenever it exists.

Without risk of confusion we will use $I(a_1, \ldots, a_n)$ to denote $I(\{a_1, \ldots, a_n\})$. The ϱ-ideal $I(a)$ is called *principal ϱ-ideal,* the poset of all principal ϱ-ideals of $\langle A, \varrho \rangle$ is denoted by $\langle \mathscr{I}_0(A), \subseteq \rangle$.

For arbitrary binary relational systems $\langle A, \varrho \rangle$ and $\langle B, \sigma \rangle$, a bijective mapping $h : A \to B$ is called an *isomorphism* of $\langle A, \varrho \rangle$ onto $\langle B, \sigma \rangle$ whenever $a \varrho b$ if and only, if $h(a) \sigma h(b)$ for every $a, b \in A$. Isomorphic binary relational systems $\langle A, \varrho \rangle$ and $\langle B, \sigma \rangle$ will be denoted by $\langle A, \varrho \rangle \cong \langle B, \sigma \rangle$.

2. JOIN IRREDUCIBLE, COMPLETE-JOIN IRREDUCIBLE AND DIRECTLY IRREDUCIBLE ϱ-IDEALS

For the sake of completeness we recall some definitions from the lattice theory, see, e.g., [1].

For any complete lattice L the subset A of L is called *independent* if $a \wedge \vee (A \setminus \{a\}) = 0_L$ holds for every element $a \in A$.

An element $x \in L$ is called *complete-join irreducible, join irreducible,* and *directly irreducible* if for every nonvoid, every nonvoid finite, and every independent subset X of A, respectively, $x = \vee X$ implies $x \in X$.

At first we prove the following lemma

Lemma 1. *Let X be an arbitrary ϱu-directed subset of a binary relational system $\langle A, \varrho \rangle$. Then for any finite set of semi ϱ-ideals $S_1, \ldots, S_n$, $X \subseteq \bigcup_{i \leq n} S_i$ implies $X \subseteq S_i$ for some $i = 1, \ldots, n$.*

Proof. The proof is trivial if $n = 1$ or $X = \emptyset$. So, hereafter, we will assume that $n \geqq 2$ and $X \neq \emptyset$.

Now we prove Lemma 1 by induction on n. First of all, let $n = 2$. Suppose $X \nsubseteq S_1$ and $X \nsubseteq S_2$. This means that $X \setminus S_1 \neq \emptyset$ and $X \setminus S_2 \neq \emptyset$ hold. Consequently, there are elements $x_1 \in X \setminus S_1$ and $x_2 \in X \setminus S_2$. By hypothesis, X is a ϱu-directed subset of A, i.e. the set $U_\varrho(x_1, x_2) \cap X$ is nonvoid. Choose element $x \in U_\varrho(x_1, x_2) \cap X$. Then we get $x \in S_k$ for some $k = 1, 2$ since $x \in X \subseteq S_1 \cup S_2$. On the other hand, $x \in U_\varrho(x_1, x_2)$ implies $x_1 \varrho x$ and $x_2 \varrho x$. Thus we have $x_1 \in S_k$ and $x_2 \in S_k$, a contradiction, i.e. Lemma 1 holds for $n = 2$.

Now, let us assume that Lemma 1 is also true for $n - 1$ and consider $X \subseteq \bigcup_{i \leqq n} S_i$. Clearly, $\bigcup_{i \leqq n} S_i = \bigcup_{i \leqq n-1} S_i \cup S_n$ and it can be easily seen that the set $\bigcup_{i \leqq n-1} S_i$ is a semi ϱ-ideal. Hence the conclusion is straigtforward.

Now we are ready to prove the following

Theorem 1. *Let X be an arbitrary ϱu-directed subset of $\langle A, \varrho \rangle$, and let $\mathcal{S}$ be a set of semi ϱ-ideals of $\langle A, \varrho \rangle$. If there is an element a of X such that the set $\mathcal{S}_a = \{S \in \mathcal{S};\ a \in S\}$ is finite, then $X \subseteq \bigcup\{S;\ S \in \mathcal{S}\}$ implies $X \subseteq S$ for some $S \in \mathcal{S}_a$.*

Proof. For any element $x \in X$ the set $U_\varrho(x, a) \cap X$ is nonvoid since X is ϱu-directed. Choose $t \in U_\varrho(x, a) \cap X$, i.e. $x \varrho t$ and $a \varrho t$ hold. Clearly, t is an element of some semi ϱ-ideal $S \in \mathcal{S}$. By the definition of semi ϱ-ideal, $x \varrho t$ and $a \varrho t$ imply $x, a \in S$. This means that $S \in \mathcal{S}_a$ and thus $x \in \bigcup\{S \in \mathcal{S}_a\}$ for every $x \in X$. Applying Lemma 1 to $X \subseteq \bigcup\{S;\ S \in \mathcal{S}_a\}$, we obtain $X \subseteq S$ for some semi ϱ-ideal $S \in \mathcal{S}_a$.

The following corollary characterizes the join irreducible ϱ-ideals and directly irreducible ϱ-ideals. The second statement is an unpublished result of I. Chajda.

Corollary 1. *Let $\langle A, \varrho \rangle$ be an arbitrary binary relational system. Then*

(i) *Every ϱ-ideal is join irreducible element of the lattice $\mathfrak{S}(A)$;*

(ii) *Every ϱ-ideal is directly irreducible element of the lattice $\mathfrak{S}(A)$.*

Proof. Let I be an arbitrary ϱ-ideal of a binary relational system $\langle A, \varrho \rangle$.

(i) Assume that $I = \bigcup_{i \leqq n} S_i$ for semi ϱ-ideals $S_1, \ldots, S_n$. In virtue of Lemma 1, $I \subseteq \bigcup_{i \leqq n} S_i$ implies $I \subseteq S_i$ for some $i \in \{1, \ldots, n\}$. The converse inclusion is trivial, thus $I = S_i$.

(ii) Assume that $I = \bigcup\{S;\ S \in \mathcal{S}\}$ for independent subset $\mathcal{S}$ of the complete lattice $\mathfrak{S}(A)$. It is not hard to verify that $\mathcal{S}$ is independent subset of $\mathfrak{S}(A)$ if and only if $\mathcal{S}$ consists of pairwise disjoint semi ϱ-ideals. This means that the independent subset $\mathcal{S}$ satisfies the hypothesis in Theorem 1, and so we have $I \subseteq S$ for some semi ϱ-ideal $S \in \mathcal{S}$. Clearly $I = S$.

Remark. Applying Corollary 1 to the poset $\langle \mathcal{I}(A), \subseteq \rangle$, we obtain the following interesting properties of ϱ-ideals:

No ϱ-ideal of an arbitrary binary relational system $\langle A, \varrho\rangle$ can be expressed as a union of a finite set of ϱ-ideals.

No ϱ-ideal of an arbitrary binary relational system $\langle A, \varrho\rangle$ can be expressed as an arbitrary union of pairwise disjoint ϱ-ideals.

Finally, the complete-join irreducible ϱ-ideals can be characterized as follows

Theorem 2. *For any ϱ-ideal I of a binary relational system $\langle A, \varrho\rangle$ the following three conditions are equivalent;*

(1) *I is complete-join irreducible element of the lattice $\mathfrak{S}(A)$;*

(2) *$I = S(a)$ for some element $a \in I$;*

(3) *I is compact element of the lattice $\mathfrak{S}(A)$.*

Proof. (1) implies (2): It can be easily seen that $I = \bigcup\{S(a); a \in I\}$ holds for every ϱ-ideal I of a binary relational system $\langle A, \varrho\rangle$. Thus, by hypothesis, $I = \bigcup\{S(a); a \in I\}$ implies $I = S(a)$ for some element $a \in I$.

Obviously (2) implies (3).

(3) implies (1): Suppose that $I = \bigcup\{S; S \in \mathscr{S}\}$ for a set $\mathscr{S}$ of semi ϱ-ideals of $\langle A, \varrho\rangle$. By hypothesis, $I = \bigcup_{i \leq n} S_i$ holds for some semi ϱ-ideals $S_1, \ldots, S_n \in \mathscr{S}$. We conclude $I \in \mathscr{S}$.

From Theorem 2 we get the following necessary condition for complete-join irreducible ϱ-ideals.

Corollary 2. *Every complete-join irreducible ϱ-ideal is principal.*

Proof. Let I be a complete-join irreducible ϱ-ideal. By Theorem 2 (2), $I = S(a)$ holds for some element $a \in I$. Further, every ϱ-ideal is clearly semi ϱ-ideal and so we have $S(a) \subseteq J$ for every ϱ-ideal J containing the element a. Moreover, $S(a)$ is a ϱ-ideal and thus $S(a)$ is the smallest ϱ-ideal containing the element a, i.e. $I = S(a) = I(a)$.

Remark. Further characterizations of complete-join irreducible ϱ-ideals may be found in [3].

To end with this section, we would like to give some examples of complete-join irreducible ϱ-ideals.

Example. If ϱ is an equivalence relation (lattice ordering) on A then the equivalence classes (lattice ideals, respectively) are exactly the ϱ-ideals of $\langle A, \varrho\rangle$. In these two cases it is a simple matter to check that:

Every equivalence class $[a]\,\varrho$, $a \in A$, is complete-join irreducible ϱ-ideal in the lattice of all semi ϱ-ideals.

A lattice ideal I is complete-join irreducible if and only if I is principal ideal.

3. ϱ-IDEALS SATISFYING THE ASCENDING CHAIN CONDITION

In this section we apply the theorems of section 2 to the posets of ϱ-ideals satisfying the ACC. We begin with

Theorem 3. *Let $\langle A, \varrho\rangle$ be a binary relational system. Then the following conditions* (1) *and* (2) *are equivalent,* (2) *implies* (3), *and* (3) *implies* (4):

(1) *Every semi ϱ-ideal of $\langle A, \varrho\rangle$ is finitely generated;*

(2) *The lattice $\mathfrak{S}(A)$ satisfies the ACC;*

(3) *Every ϱ-ideal is complete-join irreducible element of the lattice $\mathfrak{S}(A)$;*

(4) *The poset $\langle \mathscr{I}(A), \subseteq\rangle$ satisfies the ACC.*

Proof. It is well-known (see, e.g., [1]), that a lattice L with zero satisfies the ACC if and only if L is algebraic and every element of L is compact element. Consequently, the lattice $\mathfrak{S}(A)$ satisfies the ACC if and only if every semi ϱ-ideal of $\langle A, \varrho\rangle$ is finitely generated, which proves the equivalence of (1) and (2).

(2) implies (3): By hypothesis, also every ϱ-ideal is a compact element of of the lattice $\mathfrak{S}(A)$. Thus, in virtue of Theorem 2, every ϱ-ideal is complete-join irreducible element of $\mathfrak{S}(A)$.

(3) implies (4): Suppose an increasing sequence $I_1 \subseteq I_2 \subseteq \ldots$ of ϱ-ideals. It can be easily verified, see [2], that the set union $I = \bigcup_{n<\omega} I_n$ is a ϱ-ideal of $\langle A, \varrho\rangle$. However, by hypothesis, $I = \bigcup_{n<\omega} I_n$ implies $I = I_k$ for some $k < \omega$, whence $I_n = I_{n+1}$ for all $n \geqq k$ proving the inclusion.

Theorem 4. *Let $\langle A, g\rangle$ be a binary relational system such that $I(a)$ exists for every element $a \in A$. Then the following conditions are equivalent:*

(1) *The poset $\langle \mathscr{I}(A), \subseteq\rangle$ satisfies the ACC;*

(2) *The poset $\langle \mathscr{I}_0(A), \subseteq\rangle$ satisfies the ACC;*

(3) *$\mathscr{I}(A) = \mathscr{I}_0(A)$;*

(4) *$\langle \mathscr{I}(A), \subseteq\rangle \cong \langle \mathscr{I}_0(A), \subseteq\rangle$.*

Proof. Clearly (1) implies (2).

(2) implies (3): Suppose the poset $\langle \{I(x); x \in I\}, \subseteq\rangle$ for an arbitrary ϱ-ideal I. By hypothesis, there is a maximal element, denoted by $I(a)$, of this poset. Further, denote by $J = \{x \in I; I(x) \neq I(a)\}$. Clearly, $I = I(a) \cup \bigcup\{I(x); x \in J\}$ holds. In virtue of Lemma 1, we get $I = I(a)$ or $I = \bigcup\{I(x); x \in J\}$. Suppose $I = \bigcup\{I(x); x \in J\}$. Then $a \in I(x)$ for some $x \in J$. This implies $I(a) \subseteq I(x)$. With respect to the maximality of $I(a)$, we have the converse inclusion which is a contradiction. Hence $I = I(a)$.

Clearly (3) implies (4) and so it remains to prove that (4) implies (1): Denote by φ an order isomorphism of $\langle \mathscr{I}(A), \subseteq\rangle$ onto $\langle \mathscr{I}_0(A), \subseteq\rangle$ and consider an increasing sequence $I_1 \subseteq I_2 \subseteq \ldots$ of ϱ-ideals of $\langle A, \varrho\rangle$. Then the principal ϱ-ideals $\varphi(I_1)$,

$i < \omega$, also form the increasing sequence $\varphi(I_1) \subseteq \varphi(I_2) \subseteq \dots$ By [2], $\bigcup_{i<\omega} I_i$, $\bigcup_{i<\omega} \varphi(I_i)$ are ϱ-ideals and, moreover, we claim that $\varphi(\bigcup_{i<\omega} I_i) = \bigcup_{i<\omega} \varphi(I_i)$ holds.

Firstly, $I_i \subseteq \bigcup_{i<\omega} I_i$ implies $\varphi(I_i) \subseteq \varphi(\bigcup_{i<\omega} I_i)$ for all $i < \omega$, and so we have $\bigcup_{i<\omega} \varphi(I_i) \subseteq \subseteq \varphi(\bigcup_{i<\omega} I_i)$.

Conversely, $\varphi(I_i) \subseteq \bigcup_{i<\omega} \varphi(I_i)$ holds for every $i < \omega$. Thus $I_i = \varphi^{-1}\varphi(I_i) \subseteq \subseteq \varphi^{-1}[\bigcup_{i<\omega}\varphi(I_i)]$ is true since φ is an isomorphism. This means that $\bigcup_{i<\omega} I_i \subseteq \subseteq \varphi^{-1}[\bigcup_{i<\omega} \varphi(I_i)]$, i.e., $\varphi(\bigcup_{i<\omega} I_i) \subseteq \bigcup_{i<\omega} \varphi(I_i)$ and so the equality $\varphi(\bigcup_{i<\omega} I_i) = \bigcup_{i<\omega} \varphi(I_i)$ is verified.

Further, $\varphi(\bigcup_{i<\omega} I_i) \in \mathscr{I}_0(A)$, i.e., $\varphi(\bigcup_{i<\omega} I_i) = I(a)$ holds for some $a \in A$ and thus also $\bigcup_{i<\omega} \varphi(I_i) = I(a)$. This implies that $a \in \varphi(I_i)$ for some $i < \omega$. Then we get $I(a) \subseteq \varphi(I_i)$ since $\varphi(I_i)$ is a ϱ-ideal of $\langle A, \varrho \rangle$. Summarizing, we have $I(a) \subseteq \varphi(I_i) \subseteq \bigcup_{i<\omega} \varphi(I_i) = I(a)$ and therefore $\varphi(I_k) = \varphi(I_{k+1})$ is true for every $k < \omega$, $k \geqq i$. Obviously, also $I_k = = I_{k+1}$ holds for every $k \geqq i$, which completes the proof.

4. SOME ISOMORPHISM THEOREMS

In [2; Proposition 11] we state that the mapping $J_0 : \langle A, \varrho \rangle \to \langle \mathscr{I}_0(A), \subseteq \rangle$, defined by $J_0 : a \mapsto I(a)$ for every $a \in A$, is an isomorphism if and only if ϱ is a partial ordering on A. The aim of this section is to give analogous characterizations for partial ordering satisfying the ACC and for complete ordering satisfying the ACC.

Theorem 5. *For any binary relational system $\langle A, \varrho \rangle$ the following three conditions are equivalent:*

(1) $\langle A, \varrho \rangle \cong \langle \mathscr{I}(A), \subseteq \rangle$;

(2) *J_0 is an isomorphism of $\langle A, \varrho \rangle$ onto $\langle \mathscr{I}(A), \subseteq \rangle$;*

(3) *$\langle A, \varrho \rangle$ is a poset satisfying the ACC.*

Proof. Clearly, (3) implies (2) and (2) implies (1).

(1) implies (3): Apparently, the isomorphism $\langle A, \varrho \rangle \cong \langle \mathscr{I}(A), \subseteq \rangle$ implies that ϱ is a partial ordering on A. Then, by [2; Proposition 11], also $\langle A, \varrho \rangle \cong \langle \mathscr{I}_0(A), \subseteq \rangle$ is true. Summarizing, we get that $\langle \mathscr{I}_0(A), \subseteq \rangle \cong \langle \mathscr{I}(A), \subseteq \rangle$. By Theorem 4, the poset $\langle \mathscr{I}_0(A), \subseteq \rangle$ satisfies the ACC and thus the poset $\langle A, \varrho \rangle$ has the same property.

The following theorem gives equivalent conditions for chains satisfying the ACC.

Theorem 6. *For any binary relational system $\langle A, \varrho \rangle$ the following three conditions are equivalent:*

(1) $\langle A, \varrho \rangle \cong \langle \mathfrak{S}(A), \subseteq \rangle$;

(2) *J_0 is an isomorphism of $\langle A, \varrho\rangle$ onto;$\langle \mathfrak{S}(A), \subseteq\rangle$;*

(3) *$\langle A, \varrho\rangle$ is a chain satisfying the ACC.*

Proof. Clearly, (3) implies (2) and (2) implies (1). It remains to prove that (1) implies (3): By the same way as in proof of Theorem 5 we get that ϱ is a partial ordering and that $\langle A, \varrho\rangle$ is isomorphic to $\langle \mathscr{I}_0(A), \subseteq\rangle$. Thus, by hypothesis, $\langle \mathscr{I}_0(A), \subseteq\rangle$ is isomorphic to $\langle \mathfrak{S}(A), \subseteq\rangle$. Now, we claim that the poset $\langle \mathfrak{S}(A), \subseteq\rangle$ is a chain.

Denote by χ an isomorphism $\chi : \langle \mathfrak{S}(A), \subseteq\rangle \to \langle \mathscr{I}_0(A), \subseteq\rangle$ and assume that S_1, S_2 are arbitrary semi ϱ-ideals of $\langle A, \varrho\rangle$. Then $\chi(S_1)$, $\chi(S_2)$ and $y(S_1 \cup S_2)$ are principal ϱ-ideals and, moreover, it is a routine to check that $\chi(S_1 \cup S_2) = \chi(S_1) \cup \cup \chi(S_2)$ is true. Applying Lemma 1, we get $\chi(S_1 \cup S_2) = \chi(S_i)$ for some $i \in \{1, 2\}$. Consequently, also $S_1 \cup S_2 = S_i$ for some $i \in \{1, 2\}$, i.e. $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$ hold.

However, this means that the poset $\langle A, \varrho\rangle$ is a chain. Now it can be easily seen that $\mathscr{I}(A) = \mathfrak{S}(A)$. Summarizing, we find that $\langle \mathscr{I}_0(A), \subseteq\rangle \cong \langle \mathscr{I}(A), \subseteq\rangle$, i.e. (by Theorem 4; the poset $\langle \mathscr{I}_0(A), \subseteq\rangle$ satisfies the ACC. Clearly, the poset $\langle A, \varrho\rangle$ has the same property.

To end with, we apply the previous theorems to binary relational systems which happen to be lattices. As a corollary of Theorem 5, we obtain D. Higgs' solution of G. Grätzer's problem:

Theorem 7. (D. Higgs [5]). *Every lattice L such that L is isomorphic to $\mathscr{I}(L)$ has principal ideals only.*

Proof. It is a direct consequence of Theorem 5 since every $\leqq$-ideal of a lattice is a lattice ideal.

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