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Abstract. The existence, unicity, continuous dependence of the solution on the given friction force, and the boundedness independent of the friction force is proved.

Keywords: Lamé system, viscoelasticity, Signorini boundary value condition, Coulomb law of friction

Classification: 35K85,49A29,49H05

The dynamic case of the contact problem with friction considered in the framework of the Coulomb law has been remaining unsolved in spite of its importance in applications (cf. [5],[6],[7]). The present paper solves the question of the existence of solutions of the problem with a given friction which is time dependent. This problem, which is an auxiliary problem to the previously mentioned one, cf. [3], was solved only for the time-independent friction-force (cf. [2]). To solve the time-dependent case, the viscoelasticity according to the suggestion of J.Nečas is included, which allows us to execute the necessary energy-type estimation. We do not need, however, the 2nd energy inequality which can hardly be derived.

1. CLASSICAL FORMULATION OF THE PROBLEM

Let $\Omega \subset R^N$, $N \geq 2$ be a domain with a Lipschitz boundary Γ, $\Gamma = \Gamma_u \cup \Gamma_c$, $\Gamma_u \cap \Gamma_c = \emptyset$, $\overline{Int\,\Gamma_\alpha} = \overline{\Gamma_\alpha}$, $\alpha = u, c$, mes $\Gamma_c < +\infty$. Put $Q = (0,T)\times\Omega$, $S_u = (0,T)\times\Gamma_u$, $S_c = (0,T)\times\Gamma_c$. For a velocity $u : Q \to R^N$ and the corresponding displacement U, $U(t,x) = \int_0^t u(\tau,x)\,d\tau$, we define the small strain tensor $e_{ij}(w) = \frac{1}{2}\left(\frac{\partial w_j}{\partial x_i} + \frac{\partial w_i}{\partial x_j}\right)$, $i,j = 1,\dots,N$, $w = u, U$, and the stress tensor

$$\tau_{ij}(u) = a^0_{ijkl}e_{kl}(U) + a^1_{ijkl}e_{kl}(u), \tag{1}$$

where the symetry $a^\varkappa_{ijkl} = a^\varkappa_{jikl} = a^\varkappa_{klij}$, $i,j,k,l = 1,\dots,N$, $\varkappa = 0,1$, holds on Q and the usual ellipticity and continuity conditions for $a_\varkappa, A_\varkappa, B \in (0,+\infty)$, $\varkappa = 0,1$, independent of $(t,x) \in Q$ and $\xi \in R^{N^2}$,

$$a_\varkappa \leq \frac{1}{|\xi|^2}a^\varkappa_{ijkl}\xi_{ij}\xi_{kl} \leq A_\varkappa, \qquad \frac{1}{|\xi|^2}\frac{\partial a^0_{ijkl}}{\partial t}\xi_{ij}\xi_{kl} < B, \tag{2}$$
$$(t,x) \in Q, \quad \xi \in R^{N^2} \setminus \{0\}, \quad \varkappa = 0,1,$$

are fulfilled. The classical formulation of the problem is the following: For given $f : Q \to R^N$, $u^0 : \Omega \to R^N$, $v^0 : S_u \to R^N$ and $G : S_c \to R^1$ look for $u : Q \to R^N$ such that all the following conditions are fulfilled:

$$
\begin{aligned}
(3) \qquad & \dot{u}_i = \frac{\partial \tau_{ij}(u)}{\partial x_j} + f_i \text{ on } Q, \quad i = 1, \dots, N, \\
& u \equiv v^0 \text{ on } S_u, \\
& u(0,.) = u^0 \text{ on } \Omega, \\
& u_n \le 0, \quad T_n(u) \le 0, \quad T_n(u)u_n = 0 \text{ on } S_c, \\
& |T_t(u)| \le -G, \quad (|T_t(u)| + G)|u_t| = 0, \\
& |T_t(u)| = -G \neq 0 \Rightarrow \underset{\lambda \equiv \lambda(x,t) \le 0}{\exists} u_t = \lambda T_t(u) \text{ on } S_c.
\end{aligned}
$$

As usual, $T(u) = (\tau_{ij}(u)n_j)$ in (3), where n is the unit outer normal vector, the terms with the subscripts t and n are the corresponding tangential and normal components, respectively. By the dot we denote the time derivate.

We remark that in the contact problem with friction in the sense of the Coulomb law, the coefficient of the friction $\mathcal{F}$ is given instead of the friction force G and we look for a solution of (3) such that $G = \mathcal{F}T_n(u)$.

2. VARIATIONAL FORMULATION EXISTENCE AND UNICITY OF A SOLUTION

Put $\mathcal{H} = L_2(0,T; H^1(\Omega; R^N)), \mathcal{H}_0 = \{v \in \mathcal{H}; v/S_u = 0\}, \mathcal{H}_1 = H^1(Q; R^N)$. Denote by $(.,.), \langle .,. \rangle , [.,.]$ the $L_2(Q; R^N)-, L_2(\Gamma_c; R^1)-$ and $L_2(\Omega; R^N)-$ scalar products, respectively. The corresponding duality pairings based on those scalar products will be denoted in the same way. Let for $v^0 \in H^{\frac{1}{2}}(S_u; R^N)$ and $u^0 \in H^1(\Omega; R^N)$ such that $u_n^0/S_c \le 0$ the following condition hold:

(C) There is $\tilde{v}^0 \in \mathcal{H}_1$ such that $\tilde{v}^0/S_u = v^0, \tilde{v}^0(0,.) = u^0$ and $\tilde{v}^0/S_c = 0$.

Let us denote

$$B(0,T; L_2(\Omega; R^N)) := \{v : \langle 0,T \rangle \to L_2(\Omega; R^N); \sup_{t \in \langle 0,T \rangle} \|v(t,.)\|_{L_2(\Omega; R^N)} < +\infty\},$$

$$(4) \qquad a^{\varkappa}(u,v) = \int_Q a^{\varkappa}_{ijkl} e_{ij}(u) e_{kl}(v) dx\, dt \quad \text{ for } u, v \in \mathcal{H}, \varkappa = 0, 1,$$

$$(5) \; \mathcal{K} := \{v \in \tilde{v}^0 + \mathcal{H}_0; v_n/S_c \le 0\}, \qquad C^+ := \{g \in L_2(0,T; H^{-\frac{1}{2}}(\Gamma_c)); g \ge 0\}.$$

The variational formulation of the problem (G) for $u^0 \in H^1(\Omega; R^N), v^0$ defined above, $-G \in C^+$ and $f \in \mathcal{H}^*$ is the following:
Look for $u \in \mathcal{K} \cap B(0,T; L_2(\Omega; R^N))$ weakly continuous from $\langle 0,T \rangle$ into $L_2(\Omega; R^N)$ such that $\frac{\partial u}{\partial t} \in \mathcal{H}_1^*$ and for every $v \in \mathcal{H}_1 \cap \mathcal{K}$ the inequality

$$(6) \qquad (\dot{u}, v-u) + a^1(u, v-u) + a^0(U, v-u) + \langle -G, |v_t| - |u_t| \rangle \ge (f, v-u)$$

holds, where $U(t,x)=\int_0^t u(\tau,x)d\tau$ and the term $(\dot u,u)$ represents $\frac{1}{2}([u(.,T),u(.,T)]-[u^0,u^0])$.

The existence of a solution of the problem (G) will be performed via its regularization. We define

$$(7)\qquad \varphi_\varepsilon(x)=\begin{cases}|x|; & |x|\ge\varepsilon\\ -\frac{|x|^4}{8\varepsilon^3}+\frac{3|x|^2}{4\varepsilon}+\frac{3}{8}\varepsilon; & |x|\le\varepsilon\end{cases}\quad\text{for}\quad \varepsilon>0,\quad \varphi_0(x)=|x|.$$

Clearly, for every $\varepsilon\ge 0$ φ_ε is convex and for each $x\in R^N$ $0\le\varphi_\varepsilon(x)-\varphi_0(x)\le\frac{3}{8}\varepsilon$ and for every $\varepsilon>0$ $\varphi_\varepsilon\in C_2(R^N)$. We state the problem

$(G)_\varepsilon$ look for $u\in\mathcal{H}\cap B(0,T;L_2(\Omega;R^N))$ weakly continuous from $\langle 0,T\rangle$ into $L_2(\Omega;R^N)$ such that $u(0,.)=u^0$ on Ω , $u/S_u=v^0$, $\frac{\partial u}{\partial t}\in\mathcal{H}_1^*$ and for every $v\in\mathcal{H}_1\cap\mathcal{H}_0$ the following equality

$$(8)\qquad (\dot u,v)+a^1(u,v)+a^0(U,v)+\langle -G\varphi'_\varepsilon(u_t),v_t\rangle+\frac{1}{\varepsilon}\langle u_n^+,v_n\rangle=(f,v)$$

holds.

The problem $(G)_\varepsilon$ will be solved by means of the usual Galerkin approximation for $-G\in C^+\cap L_2(S_c)$. Due to the corresponding existence and unicity theorem concerning the ordinary differential equation theory, it is easy to see that the appropriate approximate finite-dimensional problems $(G)_{\varepsilon,m}$ have a unique solution $u_{\varepsilon,m}$ for every $\varepsilon>0$ and m positive integer, where $u_{\varepsilon,m}\in\tilde v^0+X_m$, X_m is the corresponding approximate space (cf. [1]). Putting $v=u_{\varepsilon,m}-\tilde v^0$ into (8), using the usual Korn-inequality and Gronwall-lemma arguments, we can derive the energy-type a priori estimation

$$(9)\quad \|u_{\varepsilon,m}\|_{B(0,T;L_2(\Omega;R^N))}+\|u_{\varepsilon,m}\|^2_{\mathcal{H}}+\frac{1}{\varepsilon}\|(u_{\varepsilon,m})_n^+\|^2_{L_2(S_c)}<$$
$$<c\equiv c(u^0,v^0,f,a^1,A_1,a_0,A_0,\mathcal{B}),$$

where c is independent of ε and m . As $-G\in C^+$ and φ_ε is convex, $\varepsilon>0$, we can derive the following inequality from (8)

(10)
$$(\dot u_{\varepsilon,m},v-u_{\varepsilon,m})+a^1(u_{\varepsilon,m},v-u_{\varepsilon,m})+a^0(U_{\varepsilon,m},v-u_{\varepsilon,m})+\langle -G,\varphi_\varepsilon(v_t)-\varphi_\varepsilon((u_{\varepsilon,m})_t)\rangle+$$
$$+\frac{1}{\varepsilon}\langle (u_{\varepsilon,m})_n^+,v_n-(u_{\varepsilon,m})_n\rangle\ge(f,v-u_{\varepsilon,m}),\quad v\in\tilde v^0+X_m.$$

Let us suppose that X_m are chosen in such a way that $X_1\subset X_2\subset\ldots$, $\overline{\bigcup_{m\in N}X_m}=\{v\in H^1(Q;R^N);v/\Gamma_u=0\}$, and $\overline{\bigcup_{m\in N}\overset{o}{X}_m}=\overset{o}{H}{}^1(\Omega;R^N)$, where $\overset{o}{X}_m=X_m\cap\overset{o}{H}{}^1(\Omega;R^N)$ (time-constant). The closures are taken in the corresponding spaces.

By means of (2) and the Hölder inequality we obtain from (8) for every $v(t,.) \equiv u_{\varepsilon,m}(t,.)+w, \quad w \in \overset{o}{X}_{m_0}$

$$(11) \quad \left|\int_\Omega (u_{\varepsilon,m}(t_1,x)-u_{\varepsilon,m}(t_2,x))w(x)dx\right| \le ((\int_{t_1}^{t_2} \|f(t,.)\|^2_{H^{-1}(\Omega;R^N)}dt)^{\frac{1}{2}} + \\ + k)\,|t_1-t_2|^{\frac{1}{2}}\|w\|_{H^1(\Omega;R^N)} \forall t_1,t_2 \in \langle 0,T\rangle, m \ge m_0, \varepsilon > 0,$$

where k is independent of ε and $m \ge m_0$ due to (9).

For fixed $\varepsilon > 0$ let us take a suitable sequence $m_k \to +\infty$ such that $u_{\varepsilon,m_k} \rightharpoonup u_\varepsilon$ in $\mathcal{H}$ and $u_{\varepsilon,m_k}(t,.) \rightharpoonup \psi_t$ in $L_2(\Omega;R^N)$ for every $t \in \mathcal{T}$, where $\mathcal{T}$ is a countable dense subset of $\langle 0,T\rangle$. The last convergence is strong in $H^{-1}(\Omega;R^N)$. For $t_i \in \mathcal{T}$, $i = 1,2$, (11) holds for ψ_{t_i} instead of $u_{\varepsilon,m}(t_i,.)$, $i=1,2$, and for every $w \in \overset{o}{H}{}^1(\Omega;R^N)$. We define ψ_t for every $t \in \langle 0,T\rangle$ such that the mapping $t \mapsto \psi_t$ is continuous from $\langle 0,T\rangle$ into $H^{-1}(\Omega;R^N)$. Let $t \in \langle 0,T\rangle \setminus \mathcal{T}$ and m_{k_r} be such a subsequence that $u_{m_{k_r}}(t,.) \rightharpoonup \omega$ in $L_2(\Omega;R^N)$. Then $u_{m_{k_r}}(t,.) \to \omega$ in $H^{-1}(\Omega;R^N)$, but from (11) $\omega = \psi_t$ in the sense of $H^{-1}(\Omega;R^N)$ and as $\overset{o}{H}{}^1(\Omega;R^N)$ is dense in $L_2(\Omega;R^N)$, $\omega = \psi_t$ in $L_2(\Omega;R^N)$. As it holds for every convergent subsequence of $\{u_{m_k}(t,.)\}$ for every $t \in \langle 0,T\rangle$, the whole $\{u_{m_k}(t,.)\}$ must tend to ψ_t for every $t \in \langle 0,T\rangle$. Therefore $\psi_t = u_\varepsilon(t,.)$ on $\langle 0,T\rangle$ and $\frac{\partial u_{\varepsilon,m_k}}{\partial t} \rightharpoonup \frac{\partial u_\varepsilon}{\partial t}$ in $\mathcal{H}_1^*$. As $-G \in C^+$ and φ_ε is convex, $\underline{\lim}_{m_k\to+\infty}\langle -G, \varphi_\varepsilon((u_{\varepsilon,m_k})_t)\rangle \ge \langle -G, \varphi_\varepsilon((u_\varepsilon)_t)\rangle$. Thus for such u_ε the inequality (10) holds for every $v \in \mathcal{H}_1 \cap \mathcal{H}_0 + \tilde{v}^0$.

Clearly, the set $\{u_\varepsilon;\ \varepsilon > 0\}$ remains bounded in $B(0,T;L_2(\Omega;R^N)) \cap \cap L_2(0,T;H^1(\Omega;R^N))$ and equicontinuous in $C_0(0,T;H^{-1}(\Omega;R^N))$. Therefore the mapping $t \mapsto u_\varepsilon(t,.)$ is $L_2(\Omega;R^N)$-weakly continuous on $\langle 0,T\rangle$ for every $\varepsilon > 0$. The term $(\dot{u}_\varepsilon, v-u_\varepsilon)$ has the following meaning: $[u_\varepsilon(T,.),v(T,.)] - \frac{1}{2}[u_\varepsilon(T,.),u_\varepsilon(T,.)] - [u^0,v(0,.)] + \frac{1}{2}\|u^0\|_{L_2(\Omega;R^N)} - (u_\varepsilon,\dot{v})$ for $v \in \mathcal{H}_1$.

Now for a suitable sequence $\varepsilon_k \to 0$, we can find $u_{\varepsilon_k} \rightharpoonup u$ both in $\mathcal{H}$ and in $B(0,T;L_2(\Omega;R^N))$ in the previous meaning, i.e. $u_{\varepsilon_k}(t,.) \rightharpoonup u(t,.)$ for every $t \in \langle 0,T\rangle$. Clearly, the mapping $t \mapsto u(t,.)$ remains weakly continuous from $\langle 0,T\rangle$ into $L_2(\Omega;R^N)$. Considering

$$(12) \qquad 0 \le \langle -G, \varphi_\varepsilon(u_\varepsilon) - \varphi_0(u_\varepsilon)\rangle \le \frac{3}{8}\varepsilon\|G\|_{L_2(S_c)}, \text{mes } S_c, \quad \varepsilon > 0,$$

we can easily see that $\underline{\lim}_{k\to+\infty}\langle -G, \varphi_{\varepsilon_k}((u_{\varepsilon_k})_t)\rangle \ge \langle -G, \varphi_0(u_t)\rangle$ due to the convexity of φ_0 and the non-negativity of $-G$. Performing the limit procedure in (10), we can easily see that for such u the inequality (6) holds for every $v \in \mathcal{H}_1 \cap \mathcal{K}$. As $u \in \mathcal{K}$ (because $\|u^+_{\varepsilon,m}\|^2_{L_2(S_c)} \to 0$), u is a solution of (G). The existence theorem for $G \in C^+ \cap L_2(S_c)$ is proved.

Let $G_1, G_2 \in L_2(S_c) \cap C^+$, and let u_1, u_2 be some respective solutions of (G) for $G = G_1$ and $G = G_2$. We can suppose that v^0, f, G_1, G_2 are defined on the interval $\langle 0, T+\eta\rangle, \eta > 0$, and the solutions are calculated on the same interval. For $t \in (-\infty, 0)$ we define $u_i(t,x) = u^0(x), i = 1,2$. Let $\tilde{u}_{i,\vartheta}$ be the appropriate time mollifier of $u_i, i = 1,2$, (cf. [4]; ϑ denotes the radius of the support of the kernel).

As $\tilde{u}_{i,\vartheta}, \vartheta \in (0,\eta)$, belong to $\mathcal{H}_1 \cap \mathcal{K}$, we can put $v = \tilde{u}_{2,\vartheta}$ into the variational inequality (6) with $G = G_1$ and $v = \tilde{u}_{1,\vartheta}$ into its version for $G = G_2$. Thus we obtain

$$(13) \quad (\dot{u}_1, \tilde{u}_{2,\vartheta} - u_1) + (\dot{u}_2, \tilde{u}_{1,\vartheta} - u_2) + a^1(u_1, \tilde{u}_{2,\vartheta} - u_1) + a^1(u_2, \tilde{u}_{1,\vartheta} - u_2) + \\ + a^0(U_1, \tilde{u}_{2,\vartheta} - u_1) + a^0(U_2, \tilde{u}_{1,\vartheta} - u_2) + \langle -G_1, |(\tilde{u}_{2,\vartheta})_t| - |u_{1t}|\rangle + \\ + \langle -G_2, |(\tilde{u}_{1,\vartheta})_t| - |u_{2t}|\rangle \geq (f, u_1 - u_2 + \tilde{u}_{2,\vartheta} - \tilde{u}_{1,\vartheta}),$$

where the integration in time can be considered on $\langle 0, t_0\rangle$ for arbitrary $t_0 \in \langle 0, T\rangle$. As $(\dot{u}_1, \tilde{u}_{2,\vartheta}) + (\dot{u}_2, \tilde{u}_{1,\vartheta}) = (\dot{u}_1, \tilde{u}_{2,\vartheta}) + (\dot{\tilde{u}}_{2,\vartheta}, u_1) = [u_1(t_0,.), \tilde{u}_{2,\vartheta}(t_0,.)] - [u^0, \tilde{u}_{2,\vartheta}(0,.)]$ (the properties of mollifiers) and due to the appropriate weak continuity of the mapping $t \mapsto u_2(t,.)$, it holds

$$[u_1(t,.), \tilde{u}_{2,\vartheta}(t,.)] \to [u_1(t,.), u_2(t,.)] \quad \forall t \in \langle 0, T\rangle.$$

Since for $\vartheta \to 0 \quad |(\tilde{u}_{i\vartheta})_t| \to |u_{it}|, \quad i = 1,2$, e.g. in $L_2(S_c)$, the limit procedure in (9) yields the following inequality for $-G_1, -G_2 \in L_2(S_c) \cap C^+$:

$$(14) \\ \sup_{t\in\langle 0,T\rangle} \frac{1}{2}\|u_1(t,.) - u_2(t,.)\|^2_{L_2(\Omega;R^N)} + a^1(u_2 - u_1, u_2 - u_1) + a^0(U_2 - U_1, u_2 - u_1) \leq \\ \leq \langle G_2 - G_1, |u_{1t}| - |u_{2t}|\rangle.$$

Due to the boundedness of $\frac{\partial a^0}{\partial t}$ (cf.(2)) and the Gronwall lemma, we obtain from (14) immediately that

$$(15) \quad c_0(a_1, A_0, \mathcal{B})\|u_1 - u_2\|^2_{\mathcal{H}} + \|u_1 - u_2\|^2_{B(0,T;L_2(\Omega;R^N))} \leq \\ \leq c_1\|G_1 - G_2\|_{L_2(0,T;H^{-\frac{1}{2}}(\Gamma_c))}(\|u_1\|_{\mathcal{H}} + \|u_2\|_{\mathcal{H}}), \quad c_0, c_1 > 0,$$

using the trace theorem and the Korn inequality. Of course c_0, c_1 does not depend on $u_i, G_i, i = 1,2$. (15) yields the uniqueness of the solution of (G) for $-G \in L_2(S_c) \cap C^+$ as well as the existence of a solution of (G) for each $-G \in C^+$. In fact, $C^+ \cap L_2(S_c)$ is dense in C^+ in the norm of $L_2(0,T;H^{-\frac{1}{2}}(\Gamma_c))$. Putting $v = \tilde{v}^0$ into (G) we can easily prove the global boundedness of solutions of (G) (independent of $-G \subset C^+ \cap L_2(S_c)$) both in $\mathcal{H}$ and in $B(0,T;L_2(\Omega;R^N))$. From it and (15) the existence of a solution of (G) for a general $-G \in C^+$ is clear due to the completeness of the appropriate spaces. Moreover (15) holds for $-G_1, -G_2 \in C^+$, thus the uniqueness and the continous dependence is proved on the whole C^+ . The arguments to prove the global boundedness of solutions of (G) on C^+ are also the same as on $C^+ \cap L_2(S_c)$. Thus we have proved the following theorem:

Theorem. *Let $u^0 \in H^1(\Omega; R^N)$ such that $u^0_n/S_c \leq 0$, let $v^0 \in H^{\frac{1}{2}}(S_u; R^N)$ such that the condition (C) is fulfilled, $f \in \mathcal{H}^*$ and $-G \in C^+$. Then there exists a unique solution u of the problem (G). The operator $-G \mapsto u$ is $\frac{1}{2}$ -Hölder continuous on C^+*

as an operator from $L_2(0,T;H^{-\frac{1}{2}}(\Gamma_c))$ into $\mathcal{H}\cap B(0,T;L_2(\Omega;R^N))$ and its range is bounded in both spaces.

Remark:

1. In fact, we have proved that for the solution u $\frac{\partial u}{\partial t}$ belongs to the dual space to the space $\{v\in\mathcal{H}\cap C_0(0,T;L_2(\Omega;R^N));\ \frac{\partial v}{\partial t}\in\mathcal{H}^*\}$, as all the terms

$$[u(t,.),v(t,.)],t\in\langle 0,T\rangle,\left(\frac{\partial v}{\partial t},u\right) \tag{16}$$

are reasonable for such v. Naturally, it belongs also to $L_2(0,T;H^{-1'}(\Omega;R^N))$. But the open problem is, whether $\frac{\partial u}{\partial t}\in L_2(0,T;(H^1(\Omega;R^N))^*)$.
2. The condition (C) can be replaced by the assumption $\mathcal{K}_0\neq\emptyset$, where $\mathcal{K}_0:=\{v\in\mathcal{H};v/S_u=v^0,v(0,.)=u^0,v_n/S_c\leq 0\}$. Then we take a fixed $\tilde{v}^0\in\mathcal{K}_0$ for the proof. The assertion of the theorem, however, is proved only on $C_\eta^+:=\{g\in C;\mathrm{dist}(\mathrm{supp}\,g,S_u)\geq\eta\}$ for $\eta>0$ arbitrary.
3. Let us consider the Duvaut modification of the Coulomb law of friction, where $G=\mathcal{F}T_{n\vartheta}(u)$ with $T_{n\vartheta}(u)$ being the mollifier (taken both in time and in the space variables) of $T_n(u)/S_c$ and $\mathcal{F}$ (the cœfficient of friction) such that $\mathcal{F}\in L_\infty(S_c)\cap H^\varepsilon(S_c)$ for some fixed $\varepsilon>0,\mathcal{F}\geq 0$ on S_c and $\mathrm{dist}(\mathrm{supp}\,\mathcal{F},S_u)>0$. The existence of a solution of such a contact problem with friction is an easy consequence of our theorem and the Schauder fixed point theorem. In fact, the uniform boundedness of u in $\mathcal{H}$ independent of $G\in -C^+$ yields the same boundedness of $T_n(u)$ in $H^{-\frac{1}{2}}(S_c)$, thus the well-known properties of mollifiers give us the global boundedness of $T_{n\vartheta}(u)$ in $H^\varepsilon(S_c),\varepsilon>0$, and $T_{n\vartheta}(u)\in -C^+$ for every $G\in -C^+$.
4. If $\Gamma=\Gamma_c\cup\Gamma_u\cup\Gamma_T$, where on $S_T=(0,T)\times\Gamma_T$ the stress is prescribed and $\mathrm{mes}\,\Gamma_u>0$, the proof of our theorem requires only an easy modification.

References

[1] A.Coddington, N.Levinson, *Theory of ordinary differential equations*, Mc Graw-Hill, New York, 1955.

[2] G.Duvaut, J.L.Lions, *Les inéquations en mécanique et en physique*, Dunod, Paris, 1972.

[3] J.Jarušek, *Contact problems with bounded friction. Coercive case*, Czech.Math.J., 33, 2, 1983, 237-261.

[4] J.Nečas, *Les méthodes directes en théorie des équations elliptiques*, Academia, Prague, 1967.

[5] J.Nedoma, *On the global geodynamic model of the earth and the plate tectonic hypothesis*, part I-III., Beitr. Geophys., 95, 1986, 36-62,89-105.

[6] J.Nedoma, *Tectonic evolution of collision zones I,II.*, to appear in Beitr.Geophys..

[7] J.Nedoma, *Mathematical simulation of the function of large human hips and optimal design of their total endoprothesis*, Part 2 (in Czech). Tech.rep. V-407, Inst.Comp.Sci.Czech.Acad. Sci., Prague, 1989.

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