Igor Kříž

Application of the factorization theorem for Stone algebras in locales [Abstract of thesis]

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ultrafilter are the basis of a proof of the generalization of the classical Gödel's completeness theorem:

A formula φ is α–theorem, $\mathcal{T} \vdash_\alpha \varphi$, iff it is α–valid in the theory $\mathcal{T}$, $\mathcal{T} \models_\alpha \varphi$.

REPRESENTATIONS OF FLOWS AND GENERALIZED RUDOLPH'S THEOREM

M.KRUTINA, Matematický ústav University Karlovy, Sokolovská 83, 186 00 Praha 8
(30.6.1988, supervisor K.Winkelbauer)

Problems connected with representations of flows are investigated in the thesis. The first part refers to the decomposition of a flow (formed by a very general contin uous group of homeomorphisms of a Polish space) into ergodic components without any substantial assistance of a measure. It is proved at the same time that the sets of quasiregular and regular points are Borel of the type $G_{\delta\sigma\delta}$, too. In the second part, there is solved the problem of the representability of a measurable flow $\{T_t\}_{t\in R}$ on a probability space by a canonical one, i.e. by a flow under the function taking only two (irrationally related) values, whose pro–images form a (two–element) generator of the basic automorphism. To this end an auxiliary invariant, the asymptotic rate of the flow, is introduced (in the case of countably generated σ–algebra in the state space). It is shown that the asymptotic rate of the flow under the function is closely related to the asymptotic rate of the basic automorphism. Thus, the use of the theorem on the minimal cardinality of the generator allows us to find the necessary and sufficient condition of the representability of an aperiodic measurable flow by means of a canonical one. This condition is just the finality of the asymptotic rate. As the asymptotic and entropy rates are equal in the case of the ergodic flow, the Rudolph's theorem was generalized to the non–ergodic case.

APPLICATION OF THE FACTORIZATION THEOREM FOR STONE ALGEBRAS IN LOCALES

I.KŘÍŽ, katedra aplikované matematiky MFF UK, Malostranské náměstí 25, Praha 1
(24.8.1988, supervisor A.Pultr)

The theory of locales (Stone spaces, Stone algebras) has been invented to study topology by means of lattice theory. One of the most important notions to be investigated is the concept of a subspace of a Stone space. As shown elsewhere, it corresponds to the concept of a quotient Stone algebra.

In the present work we prove a theorem characterizing quotient Stone algebras obtained by an arbitrary system of relations. This allows to construct spaces from generators and defining relations. Our main aim is to study the applications of this method.

The basic theorem is proved in Chapter 1. In Chapter 2 we prove Tychonoff's theorem for locales without the axioms of choice and replacement. This strengthens a result of P.T.Johnstone and solves his problem.

In Chapter 3 we construct universal regular Stone spaces of given weight. As an application, we obtain universal regular topological spaces with bases whose each linked subsystem is filtered. We also solve Rosický's problem whether regular locales form an epireflective hull of regular spaces in the negative.

In Chapter 4 we construct (hyper) completion of uniform locales by means of generators and defining relations.

The Chapter 5 is devoted to connectedness. We present a short proof of a result of Moerdijk and Wraith that connected locally connected locales are arcwise connected. However, the main result of the chapter is to construct an example of two connected locales, whose product is not connected. This strengthens our joint result with A.Pultr.

In Chapter 6 we characterize LT–groups (a result from my earlier paper) and construct a non–trivial L–group with a single point.

The Chapter 7 is an appendix containing some constructions of technical nature.

ALGEBRAIC AND COMBINATORIAL PROPERTIES OF PRODUCTS

V.PUŠ, Výzkumný ústav lesního hospodářství a myslivosti, Jíloviště–Strnady
(24.8.1988, supervisors M.Vlach, J.Nešetřil)

In 1964, P.Erdös proved the following theorem (T)

(T) Suppose that $M \subseteq N$ is a set of positive integers such that the following condition holds: every positive integer can be expressed as the product $x \cdot y$ where $x, y \in M$. Then for every positive integer p there exists a positive integer n which can be expressed as the product of two elements of M in at least p different ways.

Erdös's proof of Theorem (T) was very complicated and had a purely number–theoretical character. Thus it gave no possibility to generalize Theorem (T) to other commutative semigroups besides the semigroup $(N, .)$. But, in 1985, J.Nešetřil and V.Rödl gave another proof of Theorem (T), based on the theorem of Ramsey, which was very simple and provided a straightforward possibility of generalization to other structures. In the presented thesis some general conditions on commutative semigroups are given, under which the analogy of Theorem (T) holds.

We also examine the semigroups for which the analogy of Theorem (T) does not hold. In particular, we examine the commutative semigroups $S = (X, .)$ containing a set M such that the following condition holds: every element $n \in X$ can be expressed, except for the order, in exactly one way as the product $x \cdot y$, where $x, y \in M$.

A particular attention is devoted to the study of the cardinal multiplication of simple graphs. It is proved that for the cardinal multiplication of simple graphs the analogy of Theorem (T) holds. We also study some related problems, namely the representations of commutative semigroups by products of simple graphs and the structure of irreducible decompositions of simple graphs.

Eventually we examine the chromatic number of products of graphs and the distances in products of graphs.